Linear Precoding Game for MIMO MAC With Dynamic Access Point Selection

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***Abstract*—This paper examines a non-cooperative game in precoding design for MIMO multiple-access channels with dynamic access point (AP) selection. This game is first shown to be a potential game, where the potential function is the sum rate achieved by successive interference cancellation. Due to the mixed-integer nature of the optimization variable, it is challenging to directly characterize the maximum of the potential function, which are closely related to the Nash equilibrium (NE) of the game. Instead, we establish the existence and achievability of the maximum through non-decreasing and upperbounded properties of the potential function as a direct result of our proposed update scheme. A distributed algorithm is designed where each player selfishly optimizes its AP selection and linear precoding strategy in a sequential manner. Convergence is a by-product of the established properties of the potential function which are materialized by an iterative waterfilling algorithm. Numerical results show that the algorithm is able to reach fast convergence and provides a system sum rate nearing that of the optimal centralized solution.**

***Index Terms*—Resource allocation, access point selection, linear precoding, potential games, MIMO-OFDM.**

I. Introduction

With increasingly dense deployment of small cells in future networking, users are likely to be roaming in the overlapping coverage of several access points (APs), which necessitates efficient resource allocation to meet target quality of service (QoS). Specifically, it is imperative to examine how channel assignment and power allocation should be designed since they have practical implications for utility maximization and interference management.

For distributed power control, strategic non-cooperative game (SNG) theory proves to be a useful tool in that it well captures the competitive nature of each user's self-interested behavior such that large-scale distributed algorithms naturally arise. In the literature, the information-theoretic achievable rate is often chosen as the utility function which results in Nash equilibrium (NE) strategy profiles of a waterfilling form. Accordingly, the game is called a waterfilling game. Such game has been extensively studied for various system models, such as [1]–[3] in the case of Gaussian interference channel (IC), and [4], [5] with regard to the Gaussian multiple-access channel (MAC) in the presence of successive interference cancellation (SIC).

In this work, AP selection and linear precoding are jointly considered in the waterfilling game. This cross-layer resource allocation problem has also recently been investigated in [6], [7]. The work in [6] considers the interference MAC with full frequency reuse at each AP. On top of the users, the APs are also modelled as selfish players who are interested in choosing their optimal pricing matrices as a strategy for congestion control. In [7], the authors deal with the scenario where each AP is assigned a disjoint set of subcarriers. Power allocation is taken into account over parallel *single-input single-output* (SISO) flat-fading channels. Instead, we explore a *multiple-input multiple-output* (MIMO) system in which optimal linear precoding is employed to achieve spatial multiplexing and interference mitigation. Unlike the frequency domain, users' beamformers are implicitly coupled with each other through eigen-value decomposition in this scenario. Utility optimization is carried out over the mixed-integer space: AP indexes and linear precoding matrices. Since the search space grows exponentially with the number of users, an exhaustive search over the finite possibilities of AP association is computationally prohibitive for a large-scale system. We instead solve this problem in a game theoretical framework. We show that this is a potential game in that the potential function can be represented by the SIC-based achievable sum rate. Built upon the function's properties, existence and achievability of NEs are proven, which is obtained via a low-complexity, fast-convergent distributed algorithm.

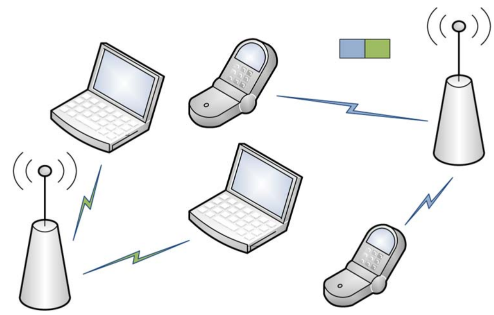

Fig. 1. System model.

II. System Model and Problem Statement

Suppose there is a group of Q users in the coverage of N APs in a MIMO system. Each AP and user are equipped with M_r and M_t antennas respectively. The whole frequency band is divided into orthogonal subcarriers by orthogonal frequency division multiplexing (OFDM), and each AP is assigned a non-overlapping subset of subcarriers. This system model is illustrated in Fig. 1. Let us denote the set of subcarriers associated with AP n as C_n and the set of users connected to AP n as Q_n. Accordingly, the received signal at AP n on the kth subcarrier is given by

$$\mathbf{y}_n^k = \sum_{q\in Q_n} \mathbf{H}_{q,n}^k \mathbf{x}_q^k + \mathbf{z}_n^k$$

where $\mathbf{H}_{q,n}^k \in \mathbb{C}^{M_r\times M_t}$ is the channel coefficient matrix, $\mathbf{x}_{q,n}^k \in \mathbb{C}^{M_t\times 1}$ is the transmit signal, and $\mathbf{z}_n^k \in \mathbb{C}^{M_r\times 1}$ is the spatially

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white zero mean circularly symmetric complex Gaussian noise, i.e., $\mathbf{S}_{z,n}^k = \mathbb{E}(\mathbf{z}_n^k \mathbf{z}_n^{k\dagger}) = \sigma^2 \mathbf{I}_{M_r}$ with σ^2 the noise power. At a particular AP, single-user detection (SUD) is assumed for its associated users. This is often the case in practice due to the low implementation complexity of SUD.

Without loss of generality, let us consider the transmission from user q to AP n, and define the access profile $\mathbf{a}$ as a vector whose qth entry a_q corresponds to user q's AP index. Should user q be connected to AP n, i.e., $a_q = n$, its achievable data rate can be expressed as

$$R_q(\mathbf{S}_{q,n}, \mathbf{S}_{-q,n}; n) = \sum_{k \in C_n} \log\det \left[\mathbf{I} + \mathbf{H}_{q,n}^{k\dagger} \left(\mathbf{S}_{z,n}^k + \mathbf{S}_{-q,n}^k \right)^{-1} \mathbf{H}_{q,n}^k \mathbf{S}_{q,n}^k \right] \quad (1)$$

where $\mathbf{S}_{q,n}^k = \mathbb{E}(\mathbf{x}_{q,n}^k \mathbf{x}_{q,n}^{k\dagger})$ is the transmit covariance matrix on the kth subcarrier, the multiuser interference (MUI) covariance matrix is defined as $\mathbf{S}_{-q,n}^k = \sum_{u \in Q_n \setminus \{q\}} \mathbf{H}_{u,n}^k \mathbf{S}_{u,n}^k \mathbf{H}_{u,n}^k$, $\mathbf{S}_{q,n} = \bigcup_{k \in C_n} \{\mathbf{S}_{q,n}^k\}$ and $\mathbf{S}_{-q,n} = \bigcup_{k \in C_n} \{\mathbf{S}_{-q,n}^k\}$ are the union of transmit and MUI covariance matrices across all subcarriers respectively. To maximize its data rate, user q can optimize its transmit covariance matrices $\mathbf{S}_{q,n}$ by solving the following optimization problem:

$$\underset{\mathbf{S}_{q,n} = \bigcup_{k \in C_n} \{\mathbf{S}_{q,n}^k\}}{\text{maximize}} \quad R_q(\mathbf{S}_{q,n}, \mathbf{S}_{-q,n}; n) \quad (2)$$

$$\text{subject to} \quad \sum_{k \in C_n} \operatorname{tr}\left(\mathbf{S}_{q,n}^k \right) \le P_q \quad (3)$$

$$\mathbf{S}_{q,n}^k \succeq \mathbf{0}, \forall k \in C_n. \quad (4)$$

The optimal solution is the well-known space-frequency water-filling (SFWF):

$$\mathbf{S}_{q,n}^k = \mathbf{U}_{q,n}^k \left[\lambda \mathbf{I} - \left(\Sigma_{q,n}^k \right)^{-1} \right]^+ \mathbf{U}_{q,n}^{k\dagger}$$

where the unitary matrix $\mathbf{U}_{q,n}^k$ and the diagonal matrix $\Sigma_{q,n}^k$ are related by the eigenvalue decomposition

$$\mathbf{H}_{q,n}^{k\dagger} \left(\mathbf{S}_{z,n}^k + \mathbf{S}_{-q,n}^k \right)^{-1} \mathbf{H}_{q,n}^k = \mathbf{U}_{q,n}^k \Sigma_{q,n}^k \mathbf{U}_{q,n}^{k\dagger}$$

and the Lagrange multiplier λ is obtained to satisfy the power constraint

$$\sum_{k \in C_n} \operatorname{tr}\left(\mathbf{S}_{q,n}^k \right) = P_q.$$

When dynamic AP selection is allowed, i.e., a_q becomes an optimization variable, jointly optimizing the precoding and the AP selection becomes a mixed-integer program. Moreover, it appears that the combinatorial nature of the problem is suitable for a centralized brute-force approach. Nonetheless, the obvious drawback is the exponential growth of complexity with the number of users, which makes it inapplicable to large-scale systems. Alternatively, we propose to study this problem in the framework of game theory. In strategic game-theoretic terms, the problem is formulated as

$$\mathcal{G} = \left\{ Q, \{\chi_q\}_{q \in Q}, \left\{ R_q(\mathbf{S}_{q,a_q}, \mathbf{S}_{-q,a_q}; a_q) \right\}_{q \in Q} \right\} \quad (5)$$

where $Q = \{1, \ldots, Q\}$ is the set of users, $\chi_q = \bigcup_{a_q \in \mathcal{N}} \{\mathcal{F}_q, a_q\}$ is the strategy set for user q with $\mathcal{N} = \{1, \ldots, N\}$ denoting the set of AP indices, and $R_q(\mathbf{S}_q, \mathbf{S}_{-q}; a_q)$ is the utility function as defined by (1). Specifically in χ_q, a_q denotes the AP index, and accordingly $\mathcal{F}_q$ is the set of linear precoding schemes $\mathcal{F}_q = \{\mathbf{S}_{q,aq} : \sum_{k \in C_{a_q}} \operatorname{tr}(\mathbf{S}_{q,a_q}^k) \le P_q, \mathbf{S}_{q,a_q}^k \succeq \mathbf{0}, \forall k\}$, and the linear precoding strategies for a specific AP n is aggregately denoted as $\mathcal{F}_n = \bigcup_{q \in Q_n} \mathcal{F}_q$.

Intuitively, whenever a user switches to another AP, it will inevitably increase the level of interference to others already there. As a result, the originally optimal AP might no longer be the case for some users, who are now prompted to leave. The dynamics naturally lead to the question of the existence of an equilibrium state where none of the users has the incentive to deviate. Such an equilibrium is a NE, and it is defined as a strategy profile $\{(\mathbf{S}_{q,a_q^\star}^\star, a_q^\star)\}_{q \in Q}$ for which

$$R_q\left(\mathbf{S}_{q,a_q^\star}^\star, \mathbf{S}_{-q,a_q^\star}^\star; a_q^\star \right) \ge R_q\left(\mathbf{S}_{q,a_q'}', \mathbf{S}_{-q,a_q'}^\star; a_q' \right),$$
$$\forall \left(\mathbf{S}_{q,a_q'}', a_q' \right) \in \mathcal{X}_q, \ \forall q \in Q.$$

In other words, no single user can benefit from a unilateral deviation at NE. In what follows, we study the existence of NE and show how to achieve it in a distributed manner.

III. Potential Game Characterization

The concept of potential games is first introduced in [8] which can be conveniently characterized by the so-called potential function. The potential function captures the variation in a player's utility function by its unilateral deviation regardless of the player's index. Precisely,

Definition 1: A strategic game $\mathcal{G} = \{\Omega, \{\chi_q\}_{q \in \Omega}, \{\Phi_q\}_{q \in \Omega}\}$ is an exact potential game if there exists a function $P : \chi \to \mathbb{R}$ such that $\forall q \in \Omega$, $\Phi(\mathbf{x}_q, \mathbf{x}_{-q}) - \Phi(\mathbf{y}_q, \mathbf{x}_{-q}) = P(\mathbf{x}_q, \mathbf{x}_{-q}) - P(\mathbf{y}_q, \mathbf{x}_{-q})$ where $\mathbf{x}_q$ (and $\mathbf{y}_q$) and $\mathbf{x}_{-q}$ denote the strategy profile of user q and all the other players respectively. The function P is called the potential function.

Usefulness of a concave potential function derives from the property that its maximizers correspond to NEs of the game under mild conditions [9]. As a first step, we show that the achievable sum rate by SIC is a potential function.

Theorem 1: The game $\mathcal{G}$ in (5) is an exact potential game. Given an access profile $\mathbf{a} = (a_1, \ldots, a_Q) \triangleq (a_q, \mathbf{a}_{-q})$, a potential function for each AP n is $P_n(\mathbf{S}_n; \mathbf{a}) = \sum_{k \in C_n} \log\det[\mathbf{I}_{M_r} + (\mathbf{S}_{z,n}^k)^{-1} \sum_{j \in Q_n} \mathbf{H}_{j,n}^k \mathbf{S}_{j,n}^k \mathbf{H}_{j,n}^{k\dagger}]$. Thus, the system potential function is $P(\mathbf{S}; \mathbf{a}) = \sum_{n \in \mathcal{N}} P_n(\mathbf{S}_n; \mathbf{a})$.

Proof: Suppose user q deviates unilaterally from AP y to AP x. Let us denote the updated access profile as $\hat{\mathbf{a}} = (a_q = x, \mathbf{a}_{-q})$ and the original as $\mathbf{a}^\star = (a_q = y, \mathbf{a}_{-q})$, to which correspond the sets of transmit covariances $\hat{\mathbf{S}} = \{\mathbf{S}_{q,a_q}\}_{a_q \in \hat{\mathbf{a}}}$ and $\mathbf{S}^\star = \{\mathbf{S}_{q,a_q}\}_{a_q \in \mathbf{a}^\star}$ respectively. On one hand, the utility function changes as $R_q(\mathbf{S}_{q,x}, \mathbf{S}_{-q,x}; x) - R_q(\mathbf{S}_{q,y}, \mathbf{S}_{-q,y}; y)$. On the other, the potential function changes as

$$\begin{aligned} P(\hat{\mathbf{S}}; \hat{\mathbf{a}}) - P(\mathbf{S}^\star; \mathbf{a}^\star) &= P_x(\mathbf{S}_{q,x}, \mathbf{S}_{-q,x}; a_q = x, \mathbf{a}_{-q}) + P_y(\mathbf{S}_{-q,y}; a_q = x, \mathbf{a}_{-q}) \\ &\quad - P_x(\mathbf{S}_{-q,x}; a_q = y, \mathbf{a}_{-q}) - P_y(\mathbf{S}_{q,y}, \mathbf{S}_{-q,y}; a_q = y, \mathbf{a}_{-q}). \end{aligned}$$

It can be seen that

$$P(\hat{\mathbf{S}}; \hat{\mathbf{a}}) - P(\mathbf{S}^\star; \mathbf{a}^\star) = R_q(\mathbf{S}_{q,x}, \mathbf{S}_{-q,x}; x) - R_q(\mathbf{S}_{q,y}, \mathbf{S}_{-q,y}; y).$$

■

Given an access profile $\mathbf{a}$, the system potential function $P(\mathbf{S};\mathbf{a}) = \sum_{n\in\mathcal{N}} P_n(\mathbf{S}_n;\mathbf{a})$ is maximized if and only if $P_n(\mathbf{S}_n;\mathbf{a})$ is maximized. Accordingly, let us denote the maximizers of $P_n(\mathbf{S}_n;\mathbf{a})$ as

$$\bar{\mathbf{S}}_n \in \arg\max_{\mathbf{S}_n\in\mathcal{F}_n(\mathbf{a})} P_n(\mathbf{S}_n;\mathbf{a}),\ n\in\mathcal{N}. \quad (6)$$

Obviously the following corollary holds.

Corollary 1: The strategies $\{\bar{\mathbf{S}}_n\}_{n\in\mathcal{N}}$ conditioned on an access profile $\mathbf{a}$ as in (6) are NEs of the potential game (5).

IV. A Sequential Approach to NE

A. Existence and Achievability of NE

The potential function is difficult, if not impossible, to be directly optimized due to its mixed-integer domain which consists of discrete AP indices and continuous power allocation matrices. Nonetheless, we prove that a NE can still be obtained by allowing *only one* user to perform strategy update in the following theorem.

Theorem 2: Starting from any initial strategy, a sequential best-response update of strategies will achieve a NE of the game ultimately.

Proof: Suppose user q decides to join AP y from AP x whereas the other users remain associated with the original APs. Let us denote the original and the updated access profile as $\mathbf{a}^\star$ and $\hat{\mathbf{a}}$ respectively. Specifically, $\mathbf{a}^\star = (a_1^\star,\ldots,a_{q-1}^\star,a_q^\star = x,a_{q+1}^\star,\ldots,a_Q^\star) \stackrel{\Delta}{=} (a_q^\star = x,\mathbf{a}_{-q}^\star)$, and $\hat{\mathbf{a}} = (a_1^\star,\ldots,a_{q-1}^\star,\hat{a}_q = y,a_{q+1}^\star,\ldots,a_Q^\star) \stackrel{\Delta}{=} (\hat{a}_q = y,\mathbf{a}_{-q}^\star)$. Suppose $\mathbf{S}^\star \in \mathcal{E}(\mathbf{a}^\star)$ and $\hat{\mathbf{S}} \in \mathcal{E}(\hat{\mathbf{a}})$ where $\mathcal{E}(\mathbf{a}) = \arg\max_{\mathbf{S}\in\mathcal{F}(\mathbf{a})} P(\mathbf{S};\mathbf{a})$, i.e., the set of linear precoding strategies that maximize the achievable sum rate conditioned on the access profile $\mathbf{a}$. The maximum achievable rate of user q after the access profile update can be expressed as $\hat{R}_q(\hat{\mathbf{S}}_{q,y},\mathbf{S}_{-q,y}^\star;y) = P_y(\hat{\mathbf{S}}_{q,y},\mathbf{S}_y^\star;\hat{\mathbf{a}}) - P_y(\mathbf{S}_y^\star;\mathbf{a}^\star)$, where the transmit covariance matrix $\hat{\mathbf{S}}_{q,y}$ is determined by $\hat{\mathbf{S}}_{q,y} = \arg\max_{\mathbf{S}_{q,y}\in\mathcal{F}_{q,y}} \hat{R}_q(\mathbf{S}_{q,y},\mathbf{S}_{-q,y}^\star;y)$. On the other hand, the rate up to which AP x can support user q is $R_q(\mathbf{S}_{q,x}^\star,\mathbf{S}_{-q,x}^\star;x) = P_x(\mathbf{S}_x^\star;\mathbf{a}^\star) - P_x(\mathbf{S}_{-q,x}^\star;\mathbf{a}^\star)$. The best-response strategy update requires that

$$\hat{R}_q\left(\hat{\mathbf{S}}_{q,x},\mathbf{S}_{-q,y}^\star;y\right) > R_q\left(\mathbf{S}_{q,x}^\star,\mathbf{S}_{-q,x}^\star;x\right)$$

or equivalently,

$$P_y\left(\hat{\mathbf{S}}_{q,y},\mathbf{S}_y^\star;\hat{\mathbf{a}}\right) - P_y\left(\mathbf{S}_y^\star;\mathbf{a}^\star\right) > P_x\left(\mathbf{S}_x^\star;\mathbf{a}^\star\right) - P_x\left(\mathbf{S}_{-q,x}^\star;\mathbf{a}^\star\right).$$

Based on the definition $\hat{\mathbf{S}} \in \mathcal{E}(\hat{\mathbf{a}})$, it holds true that $\hat{\mathbf{S}}_x \in \arg\max_{\mathbf{S}\in\mathcal{F}_x(\hat{\mathbf{a}})} P_x(\mathbf{S};\hat{\mathbf{a}})$. Since $\mathbf{S}_{-q,x}^\star \in \mathcal{F}_x(\hat{\mathbf{a}})$, $P_x(\hat{\mathbf{S}}_x;\hat{\mathbf{a}}) \geq P_x(\mathbf{S}_{-q,x}^\star;\hat{\mathbf{a}}) = P_x(\mathbf{S}_{-q,x}^\star;\mathbf{a}^\star)$. Besides, $P_y(\hat{\mathbf{S}}_y;\hat{\mathbf{a}}) \geq P_y(\hat{\mathbf{S}}_{q,y},\mathbf{S}_{-q,y}^\star;\hat{\mathbf{a}})$ because $\hat{\mathbf{S}}_y \in \arg\max_{\mathbf{S}\in\mathcal{F}(\hat{\mathbf{a}})} P_y(\mathbf{S};\hat{\mathbf{a}})$. Putting all together, we arrive at

$$\begin{aligned} P_y(\hat{\mathbf{S}}_y;\hat{\mathbf{a}}) - P_y\left(\mathbf{S}_y^\star;\mathbf{a}^\star\right) &\geq P_y\left(\hat{\mathbf{S}}_{q,y},\mathbf{S}_{-q,y}^\star;\hat{\mathbf{a}}\right) - P_y\left(\mathbf{S}_y^\star;\mathbf{a}^\star\right) \\ &> P_x\left(\mathbf{S}_x^\star;\mathbf{a}^\star\right) - P_x\left(\mathbf{S}_{-q,x}^\star;\mathbf{a}^\star\right) \\ &\geq P_x\left(\mathbf{S}_x^\star;\mathbf{a}^\star\right) - P_x(\hat{\mathbf{S}}_x;\hat{\mathbf{a}}) \end{aligned}$$

i.e., $P_x(\hat{\mathbf{S}}_x;\hat{\mathbf{a}}) + P_y(\hat{\mathbf{S}}_y;\hat{\mathbf{a}}) > P_x(\mathbf{S}_x^\star;\mathbf{a}^\star) + P_y(\mathbf{S}_y^\star;\mathbf{a}^\star)$. Thus,

$$\sum_{n\in\mathcal{N}} P_n(\hat{\mathbf{S}};\hat{\mathbf{a}}) > \sum_{n\in\mathcal{N}} P_n(\mathbf{S}^\star;\mathbf{a}^\star).$$

This shows that the system potential function is increased after a user's unilateral deviation to a more favorable AP. Combined with the fact that the potential function is the achievable sum rate which is therefore upper bounded, we conclude that the maximum of the potential function will be ultimately achieved regardless of the initial access profile. In view of Corollary 1, the corresponding maximizing strategy is a NE. ■

B. Distributed Algorithm

Motivated by the conclusion from the previous section, in this section, a distributed algorithm is designed such that convergence to a NE is guaranteed.

Algorithm Sequential AP selection and linear precoding

Initialize:
- Randomize the initial access profile $\mathbf{a}^{(0)} = (a_1^{(0)},\ldots,a_Q^{(0)})$
- Perform iterative SFWF to obtain a set of transmit covariance matrices $\mathbf{S}^{(0)} = \{\mathbf{S}_{1,a_1}^{(0)},\ldots,\mathbf{S}_{Q,a_Q}^{(0)}\}$
- Generate a sequential order for strategy update

Repeat: For the strategy update of user $q \in \mathcal{Q}$,
- Rate estimation: Estimates the maximum transmission rate provided by each AP, $R_q^{(n)} \forall n = 1,\ldots,N$
- AP selection: Chooses the AP n_q that enables the best transmission rate, i.e.,

$$n_q = \arg\max_{n=1,\ldots,N} R_q^{(n)}$$

- Linear precoding: Updates the transmit covariance matrix by iterative SFWF along with the other users affected by the decision

Until: Each user $q \in \mathcal{Q}$ remains associated with the same AP n_q for the last $\mathcal{T}$ iterations.

Theorem 3: The algorithm converges to a NE where the sum rate on each channel is maximized by the accessing users' egotistic linear precoding strategy.

Proof: The convergence proof in [10] says that the sequential iterative SFWF maximizes the potential function for a single AP. Thus, the SFWF-based strategies in the algorithm maximize the system potential function. In other words, the increasing property (cf. Theorem 2) of the potential function holds in this case. In conjunction with its boundedness, the algorithm converges to a NE. ■

V. Numerical Results

In this section, the proposed algorithm is numerically evaluated from two aspects, i.e., convergence and NE efficiency. In the case of convergence evaluation, the system is set up as follows: There are 4 APs serving 20 users, and each AP is assigned a disjoint set of 4 subcarriers. Both APs and users are equipped with 4 antennas. The transmit power constraint per user is 20 dB. It is assumed that the locations of APs and users are uniformly distributed in an area of 10 m × 10 m with a path loss exponent of 2. For assessment of NE efficiency, given the complexity of exhaustive search, sum rates of 8 users are obtained and compared in the presence of one, two, three, and four APs. Results are obtained by averaging over 1000 independently and identically distributed (i.i.d.) Rayleigh fading channel realizations.

Convergence in terms of system sum rate and potential is demonstrated respectively in Fig. 2 and Fig. 3. All users are

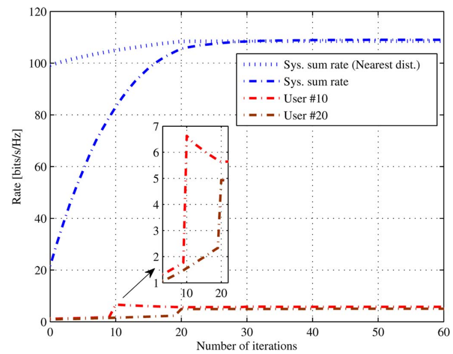

Fig. 2. Convergence of system sum rate.

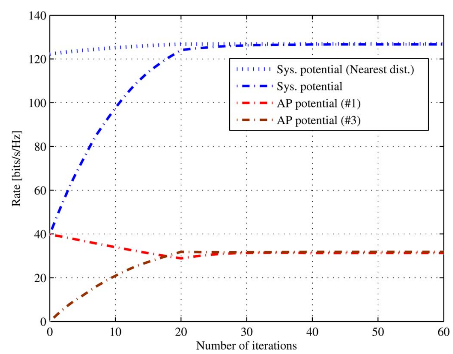

Fig. 3. Convergence of system potential.

assumed to be initially associated with the same AP and an iteration is defined as a single user's move. For comparison, the system sum rate and potential for the case where users are initially associated with the nearest APs are also included. As the algorithm progresses, all users' rates fluctuate following a similar trend. For a typical user such as user #10, we observe that before its AP selection update, its rate is slowly improving as a result of reduced interference from an increasing number of users leaving the current AP. Remarkable improvement in the rate happens with the best response based update. However, as the cell becomes more crowded, there is a marginal rate loss. Still, the user remains associated since the AP provides the best access rate. For AP #1, the potential function keeps decreasing due to users' decisions to switch, which on the contrary increases the potential of the destination AP (AP #3 in this case). Overall, the system potential is increased. This observation agrees with the conclusion in Theorem 2.

Finally, NE efficiency in terms of system sum rate is measured against that of a centralized approach with a varied number of APs in Fig. 4. The centralized approach performs a brute-force search for the largest system sum rate out of all possible access profiles. Simple analysis shows that the distributed algorithm requires a computational complexity of $O(tQ^2TKM^3)$ while the centralized exhaustive search needs that of $O(N^QQTKM^3)$, where without loss of generality, it is assumed that $M_t = M_r = M$, T is the maximum number of iterations for iterative SFWF, and t is the average number of moves that each user needs to make until convergence is reached. Interestingly, two facts about t are noted from Fig. 2 and Fig. 3: it is usually small thanks to the sequential update of strategy, and it can be further reduced by a reasonable choice of initial access profile such as one based on nearest distances. Remarkably, the distributed algorithm achieves a dramatic reduction in complexity while still being able to maintain competitive performance. However, it is worth mentioning

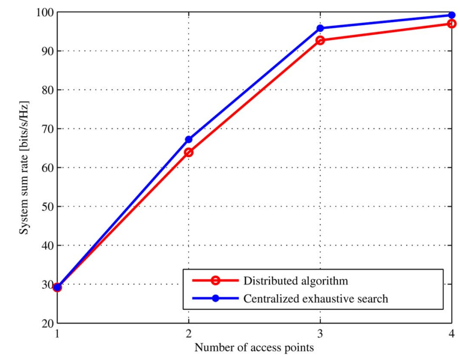

Fig. 4. NE efficiency.

that the overhead in terms of MUI matrices passed from the APs to the users required by the distributed algorithm is upper bounded by $(tQ^2 + tQ(K(N-1)+1))$ in comparison with only KQ transmit covariances fed back by the centralized approach. Even though the proposed algorithm incurs increased message passing, its distributed implementation can actually share the computations among the users. More importantly, its near-optimal performance justifies the increased overhead.

VI. Conclusion

In this paper, we investigate the problem of joint AP selection and linear precoding for MIMO-OFDM systems. The original cross-layer resource allocation problem is cast as a non-cooperative game, which is established as a potential game by the fact that the achievable system sum rate by SIC is the potential function. We prove that a sequential update of users' strategy leads to a NE, which forms the basis for design of a waterfilling-based distributed algorithm. The fast rate of convergence and near-optimal efficiency are corroborated by numerical results.

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