Comments and Replies

Comments on "Optimizations of a MIMO Relay Network"

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In the above paper [1], Behbahani *et al.* proposed a closed-form solution (in Section III-B-1) for the optimal relay coefficients by minimizing the minimum mean-square error (MMSE) at the destination subject to a global power constraint at the relays. We provide corrections to this closed-form solution and a few comments.

In [1], a closed-form solution to problem (14) was given in (21). However, this result is not correct as $\bar{\boldsymbol{f}}$ was mistakenly designated as an eigenvector of $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$ (cf. [1, eq. (17)]). In fact, $\bar{\boldsymbol{L}}^{*}\bar{\boldsymbol{f}}$ should be assigned to the eigenvector of $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$. To clarify this fact, let $\hat{\boldsymbol{f}} = \bar{\boldsymbol{L}}^{*}\bar{\boldsymbol{f}}$. Applying the Rayleigh–Ritz theorem [2], the objective function in problem (16) is upper-bounded as

$$\frac{\bar{\boldsymbol{f}}^{*}\boldsymbol{A}\bar{\boldsymbol{f}}}{\bar{\boldsymbol{f}}^{*}\boldsymbol{B}\bar{\boldsymbol{f}}} = \frac{\hat{\boldsymbol{f}}^{*}\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}\hat{\boldsymbol{f}}}{\hat{\boldsymbol{f}}^{*}\hat{\boldsymbol{f}}} \le \lambda_{\max} \tag{1}$$

where $\lambda_{\max}$ is the largest eigenvalue of $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$. The equality holds if $\hat{\boldsymbol{f}} = \chi \boldsymbol{c}_{\lambda_{\max}}$, where χ is a non-zero scaling factor and $\boldsymbol{c}_{\lambda_{\max}}$ is the principle eigenvector of $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$. Thus, $\bar{\boldsymbol{f}} = \bar{\boldsymbol{L}}^{-*}\hat{\boldsymbol{f}} = \chi\bar{\boldsymbol{L}}^{-*}\boldsymbol{c}_{\lambda_{\max}}$. As $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$ is rank-one, its principle eigenvector is the one corresponding to its only non-zero eigenvalue, $\bar{\boldsymbol{L}}^{-1}\boldsymbol{W}^{-1/2}\boldsymbol{D}_{\boldsymbol{h}_t}\boldsymbol{h}_s$. As a result, $\bar{\boldsymbol{f}} = \chi\boldsymbol{B}^{-1}\boldsymbol{W}^{-1/2}\boldsymbol{D}_{\boldsymbol{h}_t}\boldsymbol{h}_s$ maximizes the objective function in problem (16). The optimal $\bar{\boldsymbol{f}}$ can be expressed explicitly as

$$\bar{f}_k^{*} = \chi\frac{\sigma_{r_k}^{-1}h_{s_k}h_{t_k}}{\frac{\sigma_{v_t}^2}{p} + \frac{|h_{t_k}|^2}{\sigma_{r_k}^2}\sigma_{v_s}^2} = \chi\frac{\sigma_{r_k}h_{s_k}h_{t_k}p}{\sigma_{r_k}^2\sigma_{v_t}^2 + p|h_{t_k}|^2\sigma_{v_s}^2}. \tag{2}$$

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The correct scaling coefficient is chosen to meet the sum relay power budget at p as

$$\chi = \frac{\sqrt{p}}{\sqrt{\sum_{k=0}^{K-1}\frac{\sigma_{r_k}^2|h_{s_k}|^2|h_{t_k}|^2p^2}{\left(\sigma_{r_k}^2\sigma_{v_t}^2 + p|h_{t_k}|^2\sigma_{v_s}^2\right)^2}}}. \tag{3}$$

Finally, the optimal amplifying coefficient at relay-k is

$$f_k = \frac{\sqrt{p}h_{s_k}^{*}h_{t_k}^{*}}{\left(\sigma_{r_k}^2\sigma_{v_t}^2 + p|h_{t_k}|^2\sigma_{v_s}^2\right)\sqrt{\sum_{k=0}^{K-1}\frac{\sigma_{r_i}^2|h_{s_i}|^2|h_{t_i}|^2}{\left(\sigma_{r_i}^2\sigma_{v_t}^2 + p|h_{t_i}|^2\sigma_{v_s}^2\right)^2}}}. \tag{4}$$

Further Remarks: With the correct optimal amplifying coefficients, the maximum achievable signal-to-noise ratio (SNR) at the destination is $\sigma_s^2\lambda_{\max}$. As $\bar{\boldsymbol{L}}^{-1}\boldsymbol{A}\bar{\boldsymbol{L}}^{-*}$ is rank-one, $\lambda_{\max}$ is also its trace, i.e.,

$$\lambda_{\max} = \sum_{k=0}^{K-1}\frac{|h_{t_k}|^2|h_{s_k}|^2\sigma_{r_k}^{-2}}{|h_{t_k}|^2\sigma_{v_s}^2\sigma_{r_k}^{-2} + \frac{\sigma_{v_t}^2}{p}}. \tag{5}$$

For notational simplicity, let $a_k = \frac{\sigma_s^2|h_{s_k}|^2}{\sigma_{v_s}^2}$, and $b_k = \frac{\sigma_{v_t}^2\sigma_{r_k}^2}{\sigma_{v_s}^2|h_{t_k}|^2}$. Then, the maximum achievable SNR with a sum relay power constraint of p is given by

$$\mathrm{SNR}_{\max}(p) = \sum_{k=0}^{K-1}\frac{pa_k}{p+b_k} \tag{6}$$

which is a concave increasing function in p, and is upper-bounded by $\sum_{k=0}^{K-1}a_k$. Since a sum relay power constraint is looser than an individual power constraint at each relay, this bound is also applicable to the latter case. Thus, $\sum_{k=0}^{K-1}a_k$ is the maximum achievable SNR in any amplify-and-forward relay network.

References

[1] A. S. Behbahani, R. Merched, and A. M. Eltawil, "Optimizations of a MIMO relay network," *IEEE Trans. Signal Process.*, vol. 56, no. 10, pp. 5062–5073, Oct. 2008.

[2] R. A. Horn and C. R. Johnson, *Matrix Analysis*. New York: Cambridge Univ. Press, 1985.