Precoding Designs in Multiuser Multicell Wireless Systems: Competition and Coordination

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To my parents

Abstract

In a multicell system, universal frequency reuse can be employed in a network of neighboring cells for higher spectral efficiency. However, universal frequency reuse comes at the price of severe inter-cell interference (ICI), especially at cell-edge mobile stations (MS), which may effectively impair the overall system performance. To actively deal with the ICI, the emerging wireless communication standard advocates the concept of interference-aware multicell coordination. Known as coordinated multipoint transmission/reception (CoMP), the new paradigm allows the multicell system to actively control and even take advantage of the ICI. The objective of this research is to study the precoding perspectives in the multiuser multicell system with CoMP. Specifically, this research examines various precoding techniques and develops low-complexity and distributed algorithms in designing optimal precoders that either minimize the transmit power or maximize the sum-rate of the CoMP systems. In addition, this research brings new perspectives and understanding to the CoMP system where the interactions among the coordinated BSs are characterized under two operating modes: interference aware (IA) and interference coordination (IC).

Under the IA mode, the multicell system is said to be in *competition* where each BS selfishly adapts its precoding strategies accordingly to the ICI. Naturally, the IA mode represents a strategic noncooperative game (SNG) with the BSs being the rational players, who try to maximize a certain utility for their connected MSs. This work characterizes the SNG played among the BSs by examining the *existence* and *uniqueness* of a stable operating point of the system, which is corresponding to a Nash equilibrium (NE) of the multicell game. The convergence to the NE and its efficiency are then thoroughly analyzed.

Under the IC mode, the multicell systems are said to be in *coordination* where the transmissions from the BSs are coordinated to jointly maximize the performance gain of CoMP. Optimality and distributed implementation are key considerations for the precoding designs under the IC mode. In this work, we propose multicell precoding designs that jointly maximize the weighted sum-rate (WSR) of the coordinated multicell system. Due to the nonconvexity of the WSR optimization problems, we focus on the development of low-complexity convex approximation techniques to decompose them into a sequence of simpler convex problems, which can be solved distributively with local processing at each coordinated cell. Simulation results show significant performance improvements in terms of transmit power and achievable sum-rate by the IC mode over the IA mode.

Sommaire

Dans un système multicellulaire, la réutilisation universelle des fréquences peut être utilisée dans un réseau de cellules voisines pour une efficacité spectrale supérieure. Cependant, la réutilisation universelle des fréquences provoque de l'interférence intercellulaire (ICI) sévère, qui peuvent nuire à la performance globale du système, en particulier dans les stations mobiles (MS) qui se trouvent dans la périphérie de la cellule. Pour contrer activement l'ICI, la norme de communication sans fil émergente préconise le concept de coordination multicellulaire informée d'interférence. Connu comme coordonnée de transmission/réception multipoint (CoMP), ce nouveau paradigme permet au système multicellulaire de contrôler activement et même de profiter de l'ICI. L'objectif de cette recherche est d'étudier les méthodes de précodage dans le système multicellulaire multiutilisateur avec CoMP. Plus précisément, cette étude examine les diverses techniques de précodage et développe des algorithmes peu complexes et distribués afin de concevoir des précodeurs optimaux qui minimisent soit la puissance d'émission ou maximisent le débit cumulé des systèmes CoMP. De plus, cette recherche apporte des perspectives et des connaissances nouvelles pour le système CoMP où les interactions entre les stations de base coordonnées sont caractérisées en deux modes de fonctionnement: le mode informé d'interférence (IA) et le mode coordination d'interférence (IC).

Dans le mode IA, le système multicellulaire est en état de compétition puisque chaque BS adapte égoïstement ses stratégies de précodage en conséquence de l'ICI. Naturellement, le mode IA représente un jeu non coopératif stratégique (SNG) dans lequel les stations de base sont les acteurs rationnels, qui tentent de maximiser une certaine utilité pour leurs MSs connectées. Ce mémoire caractérise le SNG disputé entre les stations de base en examinant l'existence et l'unicité d'un point de fonctionnement stable du système, qui correspond à un équilibre de Nash (NE) du jeu multicellulaire. La convergence vers le NE et son efficacité sont ensuite soigneusement analyées.

Dans le mode IC, les systèmes multicellulaires sont censés être en coordination puisque les transmissions de la BS sont coordonnées afin de maximiser conjointement le gain de performance de CoMP. L'optimalité et la mise en oeuvre décentralisée sont des considérations fondamentales pour la conception de précodages sous le mode IC. Dans ce mémoire, nous proposons des conceptions de précodage multicellulaires qui maximisent conjointement la somme pondérée du débit (WSR) du système multicellulaire coordonné.

En raison de la non-convexité des problèmes d'optimisation WSR, nous nous concentrons sur le développement de techniques d'approximation convexes à faible complexité pour les décomposer en une série de problèmes convexes simples, qui peuvent être résolus de façon distributive avec la transformation locale au niveau de chaque cellule coordonnée. Les résultats de simulation montrent une amélioration significative des performances en termes de puissance d'émission et de somme des débits réalisables par le mode IC sur le mode IA.

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List of Acronyms

3GPP	Third Generation Partnership Project
4G	Fourth Generation
AWGN	Additive White Gaussian Noise
BC	Broadcast Channel
BD	Block-Diagonalization
bps	Bit per second
BR	Best Response
BS	Base-Station
CDMA	Code-Division Multiple-Access
CoMP	Coordinated Multipoint Transmission/Reception
CSI	Channel State Information
D.C.	Difference-of-two-convex-functions
dB	Decibel
DPC	Dirty-Paper Coding
IA	Interference Aware
IC	Interference Coordination
ICI	Inter-cell Interference
IEEE	Institute of Electrical and Electronic Engineers
IET	Institution of Engineering and Technology
ILA	Iterative Linear Approximation
IPN	Inter-cell Interference Plus Noise
IWF	Iterative Water-Filling
JP	Joint Signal Processing
KKT	Karush-Kuhn-Tucker

LTE	Long-Term Evolution
LTE-A	Long-Term Evolution - Advanced
MAC	Multiple-Access Channel
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MMSE	Minimum Mean Squared Error
MS	Mobile-Station
MSE	Mean Squared Error
NE	Nash Equilibrium
OFDM	Orthogonal Frequency Division Multiplexing
PSD	Power Spectral Density
QoS	Quality of Service
SDMA	Space-Division Multiple-Access
SIC	Successive Interference Cancellation
SINR	Signal-to-Interference-Plus-Noise-Ratio
SNG	Strategic Noncooperative Game
SNR	Signal-to-Noise-Ratio
SOC	Second-Order Cone
SOCP	Second-Order Conic Programming
WF	Water-Filling
WMMSE	Weighted Minimization of the Mean Squared Error
WSR	Weighted Sum-rate
ZF	Zero-Forcing

List of Notations

- $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex numbers, respectively.
- A column vector is formatted in lower-case and bold, *e.g.*, $\mathbf{x}$; whereas a matrix is in upper-case and bold, *e.g.*, $\mathbf{A}$.
- A matrix positive semi-definite matrix $\mathbf{A}$ is denoted as $\mathbf{A} \succeq \mathbf{0}$.
- Superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^\dagger$ stand for transpose, complex conjugate, complex conjugate transpose, and Moore-Penrose pseudo-inverse operations, respectively.
- $[\mathbf{x}]_i$ denotes the ith element of a column vector $\mathbf{x}$, whereas $[\mathbf{A}]_{i,j}$ denotes the element at row i, column j of a matrix $\mathbf{A}$.
- $\mathrm{Tr}\{\mathbf{A}\}$, $|\mathbf{A}|$ and $\|\mathbf{A}\|_F$ denote the trace, determinant, and Frobenius norm of a matrix $\mathbf{A}$, respectively.
- $[\mathbf{A}]^+$ denotes the component-wise operation $\max\{[\mathbf{X}]_{m,n}, 0\}$.
- $[\cdot]_{\mathcal{S}}$ denotes a projection onto a set $\mathcal{S}$.
- $\rho(\mathbf{A})$, denoting the spectral radius of a matrix $\mathbf{A}$, is defined as $\rho(\mathbf{A}) \triangleq \max\{|\lambda_i|\}$, where λ_i's are eigenvalues of $\mathbf{A}$.
- $\mathbf{I}_M$ stands for the $M \times M$ identity matrix.
- $\mathrm{diag}(d_1, d_2, \ldots, d_M)$ denotes an $M \times M$ diagonal matrix with diagonal elements d_1, d_2, …, d_M.
- $\mathrm{blk}\{\mathbf{A}_1, \ldots, \mathbf{A}_K\}$ denotes a square block-diagonal matrix with the main diagonal blocks as square matrices $\mathbf{A}_1, \ldots, \mathbf{A}_K$.

- $\mathbb{E}_x[\cdot]$ and $\mathrm{var}_x[\cdot]$ indicate the expectation and variance of random variable x, respectively; whereas $x^\star$ denotes the optimal value of variable x.

- $\mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian random variable with zero mean and power spectral density (PSD) σ^2.

Chapter 1

Introduction

1.1 Interference Management in Wireless Cellular Networks

A defining characteristic of a wireless channel is its broadcast nature. In addition, the limited wireless spectrum resource constrains many wireless devices to share the same communication channel, thus inducing mutual inter-user interference. As the interference restricts the reusability of the spectrum resource and the performance among communicating entities, it has always been a critical issue in the deployment of any wireless system. In a multicell environment, the inter-user interference can come from two sources: other devices in the same cell, *i.e.*, intra-cell interference, and co-channel interference from other cells, *i.e.*, inter-cell interference (ICI).

In order to better utilize the spectrum resource and control the interference, fractional frequency reuse has been widely adopted in early wireless cellular systems, such as Global System for Mobility (GSM). As illustrated in Figure 1.1 for a fractional frequency reuse system, adjacent cells are guaranteed to operate in different frequencies [1]. On the contrary, cells operating on the same frequency are sufficiently apart such that ICI is kept sufficiently low. Thus, in this conventional multicell system, the ICI is controlled by deploying the frequency reuse pattern and setting maximum power spectral density levels at all the base stations (BS). As a result, the interference management is usually relegated to a per-cell basis, and the ICI is treated as background noise. It is to be noted that while fractional frequency reuse is adequate in controlling the ICI, each cell is allowed to utilize only parts of the available spectrum. Thus, fractional frequency reuse is inefficient in spectral utilization.

To improve the spectral efficiency, current designs of wireless networks adopt universal

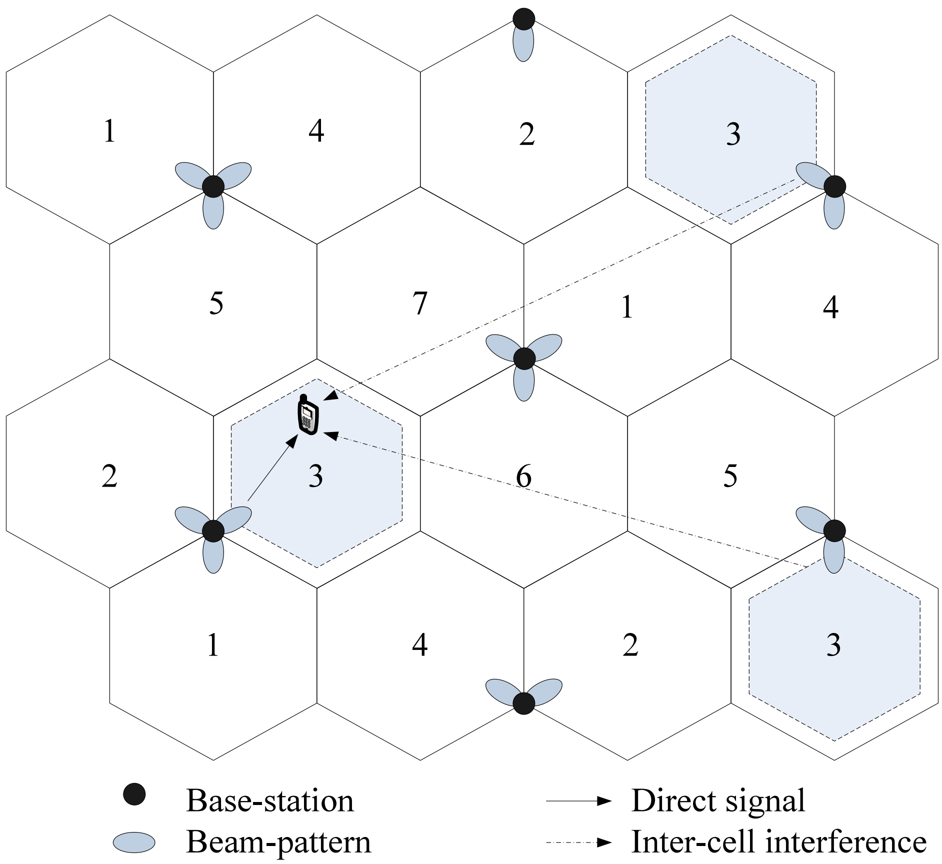

Fig. 1.1 Example of a fractional frequency reuse multicell network with a reuse factor of 7.

frequency reuse where all cells have the potential to use all available radio resources. This implementation is necessary for the current 4G and future wireless networks to cope with the fast increasing number of wireless mobile-stations (MS) and their demand for higher transmission rates. However, universal frequency reuse comes at the cost of severe ICI, especially at the cell-edge terminals. As depicted in Figure 1.2, the direct transmission to a particular MS may be strongly corrupted by the ICI from the adjacent cells. While achieving higher spectrum efficiency, universal frequency reuse may reduce the sum-rate of the network if the ICI is not adequately managed.

In recent 3GPP LTE Releases, several forms of interference avoidance and coordination techniques were proposed with the main objective to efficiently control the ICI, especially at the cell-edge MSs [2]. In inter-cell interference coordination (ICIC), such techniques as time-domain solutions (*e.g.*, subframe alignment) and frequency-domain methods (*e.g.*, channel

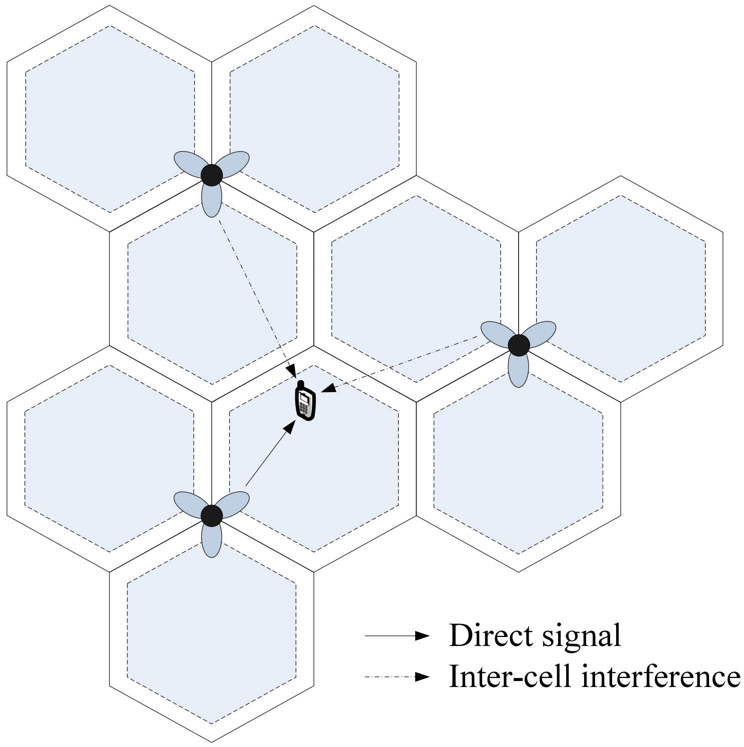

Fig. 1.2 Example of a multicell network with universal frequency reuse.

orthogonalization) explicitly control how the radio resources are utilized to moderate the ICI. This approach in dealing with the ICI might be regarded as *passive* [3]. On the contrary, the emerging wireless communication standard advocates a more *active* treatment of interference through some forms of interference-aware multicell coordination. In this more advanced coordination technique, namely coordinated multipoint transmission/reception (CoMP), the inter-cell transmission, instead of being considered as the source of interference, is taken into account as an extra means to enhance the overall system performance. The underlying concept of CoMP is quite simple: the coordinated BSs no longer adjust their parameters (such as precoding, subcarrier assignment, etc.) or decode independently of each other, but instead coordinate the precoding or decoding processes on the availability of channel state information (CSI) and the amount of information signaling over the backhaul links among the BSs [3]. One interesting aspect of exploiting coordination among the cells is that a fairly small change of infrastructure is required to implement CoMP. For this reason, it is envisioned that CoMP will be a key technology of LTE-Advanced [2, 4].

1.2 Coordinated Multipoint Transmission/Reception (CoMP)

The latest 3GPP LTE-Advanced Release considers CoMP as an enabling technology to improve coverage, throughput and efficiency [2]. Actively dealing with the ICI, this solution takes advantage of the inter-cell transmissions to enhance the overall system performance. In the downlink, CoMP coordinates the simultaneous transmissions from multiple BSs to the MSs. Such coordination is especially helpful for the cell-edge MSs, whose link conditions are usually unfavorable due to the long distances from their corresponding BSs, while being more susceptible to the ICI. In the uplink, CoMP allows for the exploitation of multiple receptions at multiple BSs to jointly decode the uplink signals from the MSs. It is mentioned in [2] that CoMP is able to significantly improve the link performance in both uplink and downlink transmissions. However, this performance enhancement may come at the cost of excessively high complexity in the joint precoding/decoding process and the demand for ideal backhaul transmissions and synchronization among the coordinated BSs.

Following the convention in literature, CoMP can be classified into the following three modes according to the extent of coordination among the cells [5]

- **Joint Signal Processing (JP):** Very tight coordination among the cells is assumed to perform joint signal processing. User data is exchanged among the coordinated BSs such that the multiple BSs are simultaneously transmitting/receiving data signals to/from the MSs within the coordinated area of multiple cells. This mode is also referred to as *Network Multiple-Input Multiple-Output (MIMO)*.

- **Interference Coordination (IC):** User data is *not* exchanged among the BSs. Each MS transmits/receives *data* signals *only* to/from its (single) serving BS, while *control* information is exchanged among the coordinated BSs to jointly control the ICI. Hereafter, the "coordination" mode or the "interference coordination" mode is referred interchangeably.

- **Interference Aware (IA):** There is no information exchange among the transmitting entities. However, the interference is estimated at each receiving entity and fed back to its corresponding transmitter. Acting as a rational entity, each BS selfishly adjusts its transmit/receive strategy according to the knowledge of the interference measured by itself or at its connected MSs. Essentially, the IA mode represents a strategic noncooperative game (SNG) with the BSs being the players. Hereafter, the "competition" mode or the "interference aware" mode is referred interchangeably.

Different CoMP modes impose different requirements on the data and signaling exchanges, as well as the CSI knowledge needed at the coordinated BSs. There is also a trade-off between the performance gain obtained by a CoMP mode and the amount of signaling overhead it placed on the backhaul links. Specifically, the higher extent of coordination among the cells will extract higher performance gains from the multicell network at the cost of higher signaling overhead.

In the JP mode, the antennas from the multiple BSs form a large single antenna array [3, 6–8]. Data streams intended for all the MSs are jointly processed and transmitted from all the antennas. Apparently, while achieving the best performance from the multicell network, such an approach is the most complex CoMP mode (*i.e.*, with the highest level of coordination). In a lesser level of coordination, the IC mode allows a BS to transmit the data only to the MSs within its cell [9, 10]. However, for both JP and IC modes, each coordinated BS may need to know the CSI of all the MSs in the system, even that of unconnected MSs in order to fully control the ICI. In addition, these two modes may require a significant amount of signaling (CSI exchange) to be exchanged among the BSs via an ideal backhaul. On the contrary, under the IA mode, each BS is only required to know the CSI of its connected MSs [11, 12]. Moreover, the IA mode, corresponding to the lowest level of coordination, only demands the signaling on a per-cell basis.

It can be seen that a CoMP system under the JP mode mimics a single large MIMO system with joint transmission/reception processing. Thus, existing research in MIMO precoding designs can be readily adopted to the JP mode. For this reason, the research in this dissertation focuses on the precoding designs for the two remaining CoMP modes: IC and IA.

1.3 Technical Challenges in Precoding Designs for a CoMP System

Conventionally, most of the related works in precoding designs focus on the single-cell setting where the ICI is simply treated as background noise at the MSs. Thus, the effect of ICI is neglected and the precoding designs are performed independently across the multiple cells. However, with universal frequency reuse, ICI is much more pronounced and should not be ignored. Consequently, existing research in single-cell precoding designs need a rework to take into account the adverse effect of ICI when applied to a multicell system. As discussed in Section 1.2, CoMP presents a paradigm shift in precoding designs from the traditional independent per-cell approach to the more coordinated multicell approach. While the potential of performance enhancement of CoMP through the means of precoding is promising, there are still several technical challenges in CoMP precoding designs.

First, it is well known that the performance of a precoding design is largely dependent on the knowledge of CSI at the transmitter. Thus, it is imperative that the transmitter, acting as a central unit, gathers the CSI to all of its connected users. However, due to the large-scale and distributed nature of the multicell system, it is much more difficult for a central unit to collect the CSI from all coordinated BSs to all MSs. Hence, instead of using a central unit in a CoMP system, the precoders should be devised in a fully distributed manner across the coordinated BSs. Essentially, each coordinated BS should design the precoders for its connected MSs using only local CSI.

Second, due to the distributed implementation in the precoding designs, the coordinated BSs may need to exchange signaling and control information. However, there are restrictions on the sharing of information among the coordinated BSs due to the limitations of the backhaul links. Thus, it is important to define and quantize the amount of message exchange, including the inter-cell messages among coordinated BSs and the intra-cell messages between a BS and its connected MS.

Third, while many precoding design problems are convex under the single-cell setting, they are nonconvex under the CoMP setting. These problems, typically difficult and computationally complex to optimally solve, pose a requirement for efficient suboptimal algorithms. Coupled with the distributed implementation requirement, it is important that certain optimization steps in the algorithms can be assigned to and executed at each BS and MS with local information. In addition, the algorithms should provide a good trade-off between the achievable performance and the computational complexity.

1.4 Thesis Contributions and Organization

CoMP is one of the promising techniques to improve the spectral efficiency and performance of cellular networks beyond what is possible with single-cell transmissions. Whether or not

CoMP can be realized to its full potential depends on how the aforementioned technical challenges are addressed. The objective of this Ph.D. research is to develop low-complexity distributed algorithms that directly address these technical challenges. Our goal is also to devise the structure of the CoMP precoders and the message exchange mechanism among the coordinated BSs and MSs that meet certain design criteria, including power minimization and sum-rate maximization. In addition, this research attempts to expose new perspectives to and understanding of the interactions among the coordinated BSs for a CoMP system under the IA and IC mode.

Under the IA mode, this work characterizes how the multicell SNG is played among the BSs and examines how each BS adaptively changes its precoding strategy accordingly to the ICI. The conditions on the existence and uniqueness of the game's NE and the convergence to the NE are subsequently studied. Under IC mode, this work proposes low-complexity convex approximation approaches to solve nonconvex problems of joint CoMP precoding designs. Our proposed solution approaches reveal not only the structure of the optimal precoders, but also the message signaling mechanism to facilitate their distributed implementation with local processing at each coordinated cell. Via extensive numerical simulations, we show that significant performance improvements in terms of transmit power and sum-rate can be achieved by jointly designing the precoders across the coordinated cells.

In this research, it is recommended that the precoding designs in a CoMP system should be performed in a fully distributed manner. However, one may argue that these distributed precoding designs may come with the issues of excessive message exchanges and synchronization among the coordinated BSs, which might be problematic in practical implementation. Instead, having a centralized unit, which obtains all the CSI knowledge, performs the precoding designs, then passes the optimized precoders to the corresponding BSs or MSs, could be more beneficial. Nonetheless, note that it is possible to deploy the controllers of the coordinated BSs at a same physical site in practice. Thus, the proposed distributed algorithms in this work can be implemented across the co-located controllers without the issues of message exchanges and synchronization. In fact, the distributed implementation of the proposed algorithms is very relevant as it allows the multiple controllers to actually share the computation loads and expedite the optimization process of the precoders.

The remainder of the dissertation is organized as follows.

Chapter 2 presents some relevant background on precoding designs in single-cell and multicell systems that are useful for the development of various precoding designs in the subsequent chapters.

Chapter 3 is concerned with the game theoretical approach in multicell precoding designs with the objective of minimizing the transmit power at the BSs. Sharing the same physical resource, the BS of each cell wishes to minimize its transmit power subject to a set of target signal-to-interference-plus-noise ratios (SINRs) at the multiple users in the cell. In this context, at first, the chapter considers a SNG where each BS greedily determines its optimal downlink beamformer strategy in a distributed manner, without any coordination among the cells. Via the game theory framework, it is shown that this game belongs to the framework of standard functions. The conditions guaranteeing the existence and uniqueness of a Nash Equilibrium (NE) in this competitive design are subsequently examined. The chapter then revisits the fully coordinated design in multicell downlink beamforming, where the optimal beamformers are jointly designed among the BSs. A comparison between the competitive and coordinated designs shows the benefits of applying the former over the latter in terms of each design's distributed implementation. Finally, in order to improve the efficiency of the NE in the competitive design, the chapter considers a cooperative game through a pricing mechanism. The pricing consideration enables a BS to steer its beamformers in a cooperative manner, which ultimately limits the interference induced to other cells. The study on the existence and uniqueness of the new game's NE is then presented. The chapter also derives a condition on the pricing factors that allows the new NE point to approach the performance established by the coordinated design, while retaining the distributed nature of the multicell game.

Chapter 4 studies multiuser precoding designs in a multicell system with the objective of maximizing the data-rate in the downlink transmission. We examine the multicell system where block-diagonalization (BD) precoding is utilized on a per-cell basis under two operating modes: *competition* and *coordination*. In the competition mode, the BS at each cell wishes to maximize the sum-rate for its connected MSs with BD precoding. In this context, the chapter considers a SNG, where each BS greedily determines its precoding strategy in a distributed manner, based on the knowledge of the ICI at its connected MSs. Via the game-theory framework, the existence and uniqueness of a NE in this SNG are subsequently studied. In the coordination mode, the BD precoders are jointly designed across the multiple BSs to maximize the network weighted sum-rate (WSR). Since this WSR maximization problem is nonconvex, we develop a low-complexity and distributed algorithm to obtain at least a locally optimal solution. Finally, we investigate the multiuser multicell system with BD-Dirty-Paper Coding (BD-DPC) being utilized at each BS on a per-cell basis. We then characterize the BD-DPC precoding game for the multicell system in the competition mode and propose an algorithm to jointly optimize BD-DPC precoders for the multicell system in the coordination mode.

Chapter 5 is concerned with the maximization of the WSR in the multicell MIMO multiple-access channel (MIMO-MAC). We consider a multicell network operating in the same frequency channel with multiple MSs per cell. Assuming that the multicell network operates in the IC mode, each BS only decodes the signals for the MSs within its cell, while the inter-cell transmissions are treated as noise. Nonetheless, the uplink precoders are jointly optimized at MSs through the coordination among the cells in order to maximize the network WSR. Since this WSR maximization problem is shown to be nonconvex, obtaining its globally optimal solution is computationally complex. Thus, our focus in this work is on the development of low-complexity algorithms to obtain at least locally optimal solutions. Specifically, we propose two iterative algorithms: one is based on successive convex approximation and the other is based on iterative minimization of weighted mean squared error. Both solution approaches then reveal the structure of the optimal uplink precoders and the message signaling mechanism to facilitate their distributed implementation among the coordinated cells. Simulation results show a significant improvement in the network sum-rate by the proposed algorithms, compared to the case with no interference coordination.

Chapter 6 studies the precoding designs to maximize the WSR in a multicell MIMO broadcast channel (MIMO-BC). We consider a multicell network under universal frequency reuse with multiple MSs per cell. With IC among the multiple cells, the BS at each cell only transmits information signals to the MSs within its cell using the dirty-paper coding (DPC) technique, while coordinating the ICI induced to other cells. The main focus of this chapter is to jointly optimize the encoding covariance matrices across the BSs in order to maximize the network-wide WSR. Since this optimization problem is shown to be nonconvex, obtaining its globally optimal solution is highly intractable. To address this problem, we consider two low-complexity solution approaches with distributed implementation to obtain at least locally optimal solutions. In the first approach, by applying a successive convex approximation technique, the original nonconvex problem is decomposed into a sequence of simpler problems, which can be solved optimally and separately at each BS. In the second approach, the WSR problem is solved via an equivalent problem of weighted sum mean squared error minimization. Both solution approaches will unfold the control signaling among the coordinated BSs to allow their distributed implementation. Simulation results confirm the convergence of the proposed algorithms, as well as their superior performances over schemes with linear precoding or no interference coordination among the BSs.

Chapter 7 presents the concluding remarks and gives suggestions for further studies.

Chapter 2

Literature Review

This chapter presents some of the current state-of-the-art precoding designs in MIMO wireless communications. The first part of this chapter discusses precoding techniques in multiuser single-cell systems with two objectives: power minimization and sum-rate maximization. The second part of this chapter then presents some recent advances in multicell precoding designs.

2.1 Precoding Designs in a MIMO Single-cell System

2.1.1 Downlink Beamforming for Power Minimization

Let us consider a single cell system comprising of K single-antenna MSs, concurrently served by an M-antenna BS. Using linear precoding, the BS multiplexes several data streams in the form as $\mathbf{x} = \sum_{i=1}^{K} u_i \mathbf{w}_i$, where u_i is the signal symbol intended for user-i with unit energy, *i.e.*, $\mathbb{E}[|u_i|] = 1$, and $\mathbf{w}_i$ is an $M \times 1$ beamformer vector designed for user-i. In the downlink transmission, the received signal at user-i can be modeled as

$$y_i = \mathbf{h}_i^H \mathbf{x} + z_i = \mathbf{h}_i^H \sum_{j=1}^{K} u_j \mathbf{w}_j + z_i, \tag{2.1}$$

The materials presented in Chapter 2 have been published as a book chapter [13].

where $\mathbf{h}_i^* \in \mathbb{C}^{M\times 1}$ is the channel vector to user-i and $z_i \sim \mathcal{CN}(0, \sigma^2)$. It is easy to verify that the signal-to-interference-plus-noise ratio (SINR) is given by

$$\mathrm{SINR}_i = \frac{\left|\mathbf{h}_i^H \mathbf{w}_i\right|^2}{\sum_{j\neq i}^K \left|\mathbf{h}_i^H \mathbf{w}_j\right|^2 + \sigma^2}, \tag{2.2}$$

where $\sum_{j\neq i}^K \left|\mathbf{h}_i^H \mathbf{w}_j\right|^2$ is the inter-user interference at user-i.

Note that the performance of a system is usually quantified by the quality of service (QoS) at each MS and the power usage at the BS. In this case, the QoS is defined as the SINR, which is directly related to the achievable rate at the MS. One of the more popular design criteria in multiuser beamforming is that the BS attempts to minimize its transmit power subject to the QoS constraints at MSs. This power minimization problem is stated as

$$\begin{aligned} \underset{\mathbf{w}_1,\ldots,\mathbf{w}_K}{\text{minimize}} \quad & \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\ \text{subject to} \quad & \frac{\left|\mathbf{h}_i^H \mathbf{w}_i\right|^2}{\sum_{j\neq i}^K \left|\mathbf{h}_i^H \mathbf{w}_j\right|^2 + \sigma^2} \geq \gamma_i, \end{aligned} \tag{2.3}$$

where γ_i is the target SINR at user-i.

This optimization problem was initially considered to be nonconvex [14, 15]. Nonetheless, the problem can be optimally solved by various approaches. Uplink-downlink duality was exploited in [14–17], where the downlink problem under individual SINR constraints can be solved via the equivalent uplink problem, which is convex and much easier to solve. In another approach in [18], this nonconvex problem was relaxed into a convex semi-definite program (SDP). In more recent works [19–21], the authors formulated the downlink problem directly as a convex second-order conic program (SOCP). A simple and fast fixed-point iterative algorithm was also proposed to find the optimal downlink beamformers.

2.1.2 Uplink Precoding on the Multiple-Access Channel

This section provides a brief review on MIMO multiple-access channel (MAC) in a single-cell multiuser system. The sum-rate maximization in the MAC and optimal designs of the uplink covariance matrices will be sequentially presented. Consider a system with K MSs, each equipped with N transmit antennas, concurrently transmitting to a BS equipped with M receive antennas. The uplink transmission can be modeled as

$$\mathbf{y} = \sum_{i=1}^{K} \mathbf{H}_i \mathbf{x}_i + \mathbf{z}, \tag{2.4}$$

where $\mathbf{x}_i \in \mathbb{C}^N$ is the transmitted vector signal from user-i, $\mathbf{y} \in \mathbb{C}^M$ is the received vector signal at the BS, $\mathbf{H}_i \in \mathbb{C}^{M\times N}$ represents the channel matrix from user-i to the BS, and $\mathbf{z}$ is the AWGN at the BS with zero mean and covariance matrix $\mathbf{Z}$. We denote $\mathbf{X}_i = \mathbb{E}[\mathbf{x}_i \mathbf{x}_i^H]$ as the transmit covariance matrix of user-i.

In the MAC, the capacity-achieving decoder is the successive interference cancellation (SIC) decoder [22, 23], where the BS decodes the signals from the multiple users in sequence. After the BS decodes one user's signal, it then suppresses this user's signal from its received signal such that this user's signal no longer interferes the users being decoded later. For example, assume that the decoding order is from user-1 to user-K. Using SIC, after decoding the signal from user-1, the BS suppresses the signal from user-1 from the received signal before processing the signal from user-2. This process continues such that user-i only experiences the interferences from user-$(i+1)$ to user-K. While treating these interferences as noises, the achievable rate of user-i is [24]

$$\begin{aligned} R_i^{\mathrm{MAC}} &= \log \left| \mathbf{I} + \mathbf{H}_i^H \left(\mathbf{Z} + \sum_{j>i}^{K} \mathbf{H}_j \mathbf{X}_j \mathbf{H}_j^H \right)^{-1} \mathbf{H}_i \mathbf{X}_i \right| \\ &= \log \frac{\left| \mathbf{Z} + \sum_{j=i}^{K} \mathbf{H}_j \mathbf{X}_j \mathbf{H}_j^H \right|}{\left| \mathbf{Z} + \sum_{j>i}^{K} \mathbf{H}_j \mathbf{X}_j \mathbf{H}_j^H \right|}. \end{aligned} \tag{2.5}$$

The sum-rate in the MAC $\sum_{i=1}^{K} R_i^{\mathrm{MAC}}$ can be simplified as

$$\sum_{i=1}^{K} R_i^{\mathrm{MAC}} = \sum_{i=1}^{K} \log \frac{\left| \mathbf{Z} + \sum_{j=i}^{K} \mathbf{H}_j \mathbf{X}_j \mathbf{H}_j^H \right|}{\left| \mathbf{Z} + \sum_{j>i}^{K} \mathbf{H}_j \mathbf{X}_j \mathbf{H}_j^H \right|} = \log \left| \mathbf{Z} + \sum_{i=1}^{K} \mathbf{H}_i \mathbf{X}_i \mathbf{H}_i^H \right| - \log |\mathbf{Z}|. \tag{2.6}$$

The goal is to maximize this sum-rate. That is,

$$\begin{aligned} \underset{\mathbf{X}_1,\ldots,\mathbf{X}_K}{\text{maximize}} \quad & \log\left|\mathbf{Z}+\sum_{i=1}^{K}\mathbf{H}_i\mathbf{X}_i\mathbf{H}_i^H\right| - \log|\mathbf{Z}| \\ \text{subject to} \quad & \text{Tr}\{\mathbf{X}_i\} \le P_i,\ \forall i \\ & \mathbf{X}_i \succeq \mathbf{0},\ \forall i, \end{aligned} \tag{2.7}$$

where P_i is maximum allowable transmit power at user-i. This optimization problem is a convex problem, which allow for efficient algorithms in finding its optimal solution. In addition, as the constraints are decoupled for each of the variables $\mathbf{X}_1,\ldots,\mathbf{X}_K$, the optimization can be performed sequentially for each variable. In particular, while treating $\mathbf{Z}_i = \mathbf{Z} + \sum_{j\neq i}^{K}\mathbf{H}_j\mathbf{X}_j\mathbf{H}_j^H$ as noise, one may perform the optimization

$$\begin{aligned} \underset{\mathbf{X}_i}{\text{maximize}} \quad & \log\left|\mathbf{Z}_i+\mathbf{H}_i\mathbf{X}_i\mathbf{H}_i^H\right| \\ \text{subject to} \quad & \text{Tr}\{\mathbf{X}_i\} \le P_i \\ & \mathbf{X}_i \succeq \mathbf{0}. \end{aligned} \tag{2.8}$$

This optimization problem can be solved by applying the water-filling (WF) process as follows. First, we perform the eigen-decomposition

$$\mathbf{H}_i^H\mathbf{Z}_i^{-1}\mathbf{H}_i = \mathbf{U}_i\mathbf{D}_i\mathbf{U}_i^H, \tag{2.9}$$

where $\mathbf{U}_i$ is a semi-unitary matrix and $\mathbf{D}_i$ is diagonal matrix with non-negative eigenvalues. Then, the optimal solution $\mathbf{X}_i^\star$ is obtained from the well-known WF solution [23]

$$\mathbf{X}_i^\star = \mathbf{U}_i\left[\mu_i\mathbf{I}-\mathbf{D}_i^{-1}\right]^+\mathbf{U}_i^H, \tag{2.10}$$

where μ_i is the water-level, chosen to meet the power constraint $\text{Tr}\left\{\left[\mu_i\mathbf{I}-\mathbf{D}_i^{-1}\right]^+\right\} = P_i$. The solution to the MAC sum-rate maximization is then obtained by iteratively performing the above WF process for each of the variables $\mathbf{X}_1,\ldots,\mathbf{X}_K$ until we reach the converging state where the MAC sum-rate can no longer be improved.

2.1.3 Downlink Precoding on the Broadcast Channel

This section reviews the multiuser Gaussian vector broadcast channel (BC) in a single-cell system. We consider the same system model as in Section 2.1.2 but the transmission is on the downlink instead. Denote $\mathbf{x} = \sum_{i=1}^{K} \mathbf{x}_i$ as the transmitted signal from the BS, where $\mathbf{x}_i$ represents the transmitted signal for user-i. Further denote $\mathbf{Q}_i = \mathbb{E}[\mathbf{x}_i \mathbf{x}_i^H]$ as the transmit covariance matrix for user-i. The received signal at the ith user is given by

$$\mathbf{y}_i = \mathbf{H}_i^H \mathbf{x} + \mathbf{z}_i = \mathbf{H}_i^H \sum_{j=1}^{K} \mathbf{x}_j + \mathbf{z}_i, \tag{2.11}$$

where $\mathbf{z}_i$ is the additive Gaussian noise with the covariance $\mathbf{Z}_i$ and $\mathbf{H}_i^H \in \mathbb{C}^{N \times M}$ is the channel from the M-antenna BS to the N-antenna MS-i.

It is well known that the capacity-achieving coding scheme in the BC is "dirty-paper coding" (DPC) [25–27]. The fundamental idea of DPC is that when a transmitter knows the interference in a channel in advance, it can design a code to compensate for the interference such that the capacity of the channel is the same as if there is no interference.[1]. Assuming the encoding order from user-K to user-1, DPC is utilized such that the intended codeword for user-i does not see the intra-cell interference from user-$(i+1)$ to user-K. Thus, the achievable rate at user-i is given by

$$\begin{aligned} R_i^{\mathrm{BC}} &= \log \left| \mathbf{I} + \mathbf{H}_i \left(\mathbf{Z}_i + \sum_{j=1}^{i-1} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right)^{-1} \mathbf{H}_i^H \mathbf{Q}_i \right| \\ &= \log \frac{\left| \mathbf{Z}_i + \sum_{j=1}^{i} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|}{\left| \mathbf{Z}_i + \sum_{j=1}^{i-1} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|}. \end{aligned} \tag{2.12}$$

When the objective is to maximize the sum-rate in the BC with DPC, the optimization is given by

[1]The theory of DPC is discussed in detail in Appendix A

$$\begin{aligned}
\underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}} \quad & \sum_{i=1}^{K} \log \frac{\left|\mathbf{Z}_i + \sum_{j=1}^{i} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i\right|}{\left|\mathbf{Z}_i + \sum_{j=1}^{i-1} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i\right|} \\
\text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_i\} \leq P \\
& \mathbf{Q}_i \succeq \mathbf{0},\ \forall i,
\end{aligned} \tag{2.13}$$

where P is total allowable transmit power at the BS. Unlike the MAC problem (2.7), the optimization problem for the BC channel is nonconvex due to the inter-user interference components in the objective function. Fortunately, via the so-called BC-MAC duality [28, 29], the BC problem (2.13) can be transformed into an equivalent MAC problem

$$\begin{aligned}
\underset{\mathbf{X}_1,\ldots,\mathbf{X}_K}{\text{maximize}} \quad & \log \left|\mathbf{I} + \sum_{i=1}^{K} \tilde{\mathbf{H}}_i \mathbf{X}_i \tilde{\mathbf{H}}_i^H\right| \\
\text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{X}_i\} \leq P, \\
& \mathbf{X}_i \succeq \mathbf{0},\ \forall i,
\end{aligned} \tag{2.14}$$

where $\tilde{\mathbf{H}}_i = \mathbf{H}_i \mathbf{Z}_i^{-1/2}$. Unlike the MAC problem (2.7), the power constraint in problem (2.14) is now a single sum-power constraint on all covariance matrix variables $\mathbf{X}_i$'s. Since the transformed problem (2.14) is now convex, it can be optimally solved by efficient convex optimization methods, including the block WF algorithm [30] and the dual decomposition method [31]. The optimal solution $\mathbf{Q}_i^\star$ of the original BC problem (2.13) then can be found from the optimal solution $\mathbf{X}_i^\star$ of the equivalent MAC problem through the so-called MAC-BC transformation [28].

It is to be noted that DPC can only serves as a theoretical benchmark due to its high complexity implementation that involves random nonlinear coding. Linear transmission techniques are an attractive alternative because of their simplicity. With linear precoding, the achievable rate at MS-i is given by

$$\begin{aligned} R_i &= \log \left| \mathbf{I} + \mathbf{H}_i \left(\mathbf{Z}_i + \sum_{j \neq i}^{K} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right)^{-1} \mathbf{H}_i^H \mathbf{Q}_i \right| \\ &= \log \frac{\left| \mathbf{Z}_i + \sum_{j=1}^{K} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|}{\left| \mathbf{Z}_i + \sum_{j \neq i}^{K} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|}, \end{aligned} \tag{2.15}$$

where the inter-user interference is treated as background noise. When the goal is to maximize the sum-rate in the BC, the problem is to maximize $\sum_{i=1}^{K} R_i$. That is,

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}} \quad & \sum_{i=1}^{K} \log \frac{\left| \mathbf{Z}_i + \sum_{j=1}^{K} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|}{\left| \mathbf{Z}_i + \sum_{j \neq i}^{K} \mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i \right|} \\ \text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_i\} \leq P \\ & \mathbf{Q}_i \succeq \mathbf{0}, \ \forall i. \end{aligned} \tag{2.16}$$

Like the sum-rate maximization problem with DPC in (2.13), the above optimization is nonconvex. However, it is no longer possible to transform problem (2.16) into an equivalent convex problem in the MAC as in (2.14). Thus, it is generally difficult to obtain its globally optimal solution. Recent works in [32, 33] proposed iterative algorithms that convert this nonconvex problem into a sequence of convex mean squared error (MSE) minimization problems. These algorithms are shown to converge monotonically to a local optimal solution. However, the drawbacks of these algorithms are their high computational costs and intractable solutions. Alternately, one may apply the block diagonalization (BD) precoding [34–37] where the inter-user interference is completely eliminated, *i.e.*, $\mathbf{H}_i^H \mathbf{Q}_j \mathbf{H}_i = \mathbf{0}$, if $j \neq i$. In this case, the precoding matrix for a particular user, say user-i, $\mathbf{Q}_i$ is limited to be in the null space created by $[\mathbf{H}_1, \ldots, \mathbf{H}_{i-1}, \mathbf{H}_{i+1}, \ldots, \mathbf{H}_K]$. As a result, the objective function under BD constraints becomes convex and the optimal solution can easily be obtained in a closed-form WF solution [35]. It is to be noted that BD precoding is only restricted to the system where the number of transmit antennas at the BS is no smaller than the total number of receive antennas at all the MSs. Nonetheless, even with the BD constraints, the performance gap between BD precoding and the benchmark DPC is negligible at high SNR [35].

2.2 Precoding Designs in a MIMO Multicell System with CoMP

This section examines some recent advances in precoding designs for a MIMO multicell network employing CoMP. Note that these precoding designs share many similar properties with the precoding designs in single-cell networks. The difference here is the consideration of individual power constraint at each coordinated BS and the inherent ICI among the cells. With per-base-station power constraints, this section reviews CoMP precoding designs under the following two design criteria: (i) minimizing the transmit power at the BSs with a set of target rates at the MSs and (ii) maximizing the sum-rate at the connected MSs. Some of the concepts of game theory applicable to CoMP under IA and IC modes are presented in Appendix B.

2.2.1 Interference Aware

The study of precoding design and power control in a mutual interference network using game theory has recently attracted considerable research attention. By considering the interference network as a SNG, each player greedily adapts its strategy to maximize its own utility, given the strategies from other players [11, 38–49].[2] In general, these works focus on studying the existence and uniqueness of the stable operating point of the system, *i.e.*, the NE. The uplink power control problem in a single-cell code-division multiple-access (CDMA) data system with multiple competing users was studied in [38,39], where the utility function was defined as the ratio of throughput to transmit power. A pricing mechanism was investigated in [39] to obtain a more efficient solution of the power control game. For an orthogonal frequency division multiplexing (OFDM) system over a shared band, the work in [11] has inspired various works on the iterative water-filling (IWF) algorithm, such as [41–45,49] with sum-rate as the utility function, or [46] with transmit power as the utility function.

In multiple antenna systems, the work in [47] considered a multicell system, where each cell consisted of *one* multiple-antenna BS and *one* single antenna MS. The objective of [47] was to study the precoding beamforming vector at the two BSs in competitive and cooperative manners. In the multiuser MIMO channels, the work in [48] studied the competitive precoding design, where each player wished to maximize its mutual information. It is to

[2] Hereafter, a cell or a base-station is referred to as a player interchangeably, whereas a MS is referred to as a user.

be noted that these works only considered the system where each transmitter, i.e, BS, communicates with only *one* receiver, *i.e.*, MS. We now discuss in detail the representative works that are closely related to the multicell network with CoMP.

Consider a MIMO multicell system with Q BSs and Q MSs. In each cell, the BS is sending information only to its connected MS. Let $\Omega = \{1, \ldots, Q\}$ denote the set of cells (players). The transmission over the q-th MIMO channel with M_q transmit and N_q receive dimensions can be described by the baseband signal model

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{x}_q + \sum_{r \neq q}^{Q} \mathbf{H}_{rq}\mathbf{x}_r + \mathbf{z}_q, \tag{2.17}$$

where $\mathbf{x}_q \in \mathbb{C}^{M_q \times 1}$ is the transmitted signal vector at BS-q and $\mathbf{y}_q \in \mathbb{C}^{N_q \times 1}$ is the received signal vectors at MS-q, and $\mathbf{z}_q$ is the AWGN with a covariance matrix $\mathbf{Z}_q$. The channel matrix from the BS-r's transmitter to MS-q is represented by $\mathbf{H}_{rq} \in \mathbb{C}^{N_r \times M_q}$. Likewise, $\sum_{r \neq q}^{Q} \mathbf{H}_{rq}\mathbf{x}_r$ is the interference induced by other MIMO links at MS-q.

Denote the precoding strategy at BS-q as $\mathbf{Q}_q = \mathbb{E}\left[\mathbf{x}_q\mathbf{x}_q^H\right]$ and the precoding strategy profile at all BSs except BS-q as $\mathbf{Q}_{-q}$. While treating the ICI from other links as additive Gaussian noise, the achievable rate at the link-q between BS-q and MS-q is given by [24]

$$R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\left|\mathbf{I} + \mathbf{H}_{qq}^H\mathbf{R}_q^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q\right|, \tag{2.18}$$

where $\mathbf{R}_q(\mathbf{Q}_{-q}) = \mathbf{Z}_q + \sum_{r \neq q}^{Q} \mathbf{H}_{rq}\mathbf{Q}_r\mathbf{H}_{rq}^H$ is the interference plus noise covariance matrix at MS-q.

Depending on the strategies of the other players, which are reflected in $\mathbf{R}_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$, player-$q$ may want to maximize its achievable rate subject to a constraint on its transmit power. In that case, the utility of player-q is defined as $u_q = R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$, whereas the set of admissible strategies is defined as

$$\mathcal{P}_q = \left\{\mathbf{Q}_q \in \mathbb{C}^{M_q \times M_q}: \ \ \mathbf{Q}_q \succeq \mathbf{0},\ \mathrm{Tr}\{\mathbf{Q}_q\} \leq P_q^{\max}\right\}. \tag{2.19}$$

Naturally, the players in Ω forms a SNG with the utility u_q and the set of admissible strategies $\mathcal{P}_q$ for player-q. Mathematically, the SNG can be defined as

$$\mathcal{G}_R = \left(\Omega,\ \{\mathcal{P}_q\}_{q\in\Omega},\ \{R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})\}_{q\in\Omega}\right). \tag{2.20}$$

On the contrary, if each player attempts to minimize its transmit power subject to a constraint on its achievable rate, the utility function is defined as $u_q = -\text{Tr}\{\mathbf{Q}_q\}$, whereas the set of admissible strategies is defined as

$$\mathcal{R}_q(\mathbf{Q}_{-q}) = \left\{\mathbf{Q}_q \in \mathbb{C}^{M_q \times M_q} : \mathbf{Q}_q \succeq \mathbf{0}, R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \geq R_q^{\min}\right\}. \tag{2.21}$$

Thus, the corresponding SNG is given by

$$\mathcal{G}_P = \left(\Omega,\ \{\mathcal{R}_q(\mathbf{Q}_{-q})\}_{q\in\Omega},\ \{-\text{Tr}\{\mathbf{Q}_q\}\}_{q\in\Omega}\right). \tag{2.22}$$

For the case of multiple input single output (MISO) channels, *i.e.*, $N_q = 1, \forall q$, the NE characterizations of games $\mathcal{G}_R$ and $\mathcal{G}_P$ are quite straightforward. It is maintained in [47] that the best response (BR) beamforming strategy at each player is the maximal ratio transmitting (MRT) beamformer, *i.e.*, $\mathbf{x}_q = \mathbf{H}_q^H/\|\mathbf{H}_q\|$. This is due to the fact that an MRT beamformer maximizes the SINR at its corresponding receiver. Thus, each player only concerns with its transmit power adjustment to maximize its utility.

Let P_q be the transmit power at BS-q. In game $\mathcal{G}_R$, it is straightforward to see that each player transmits at its maximum power at the NE [47], *i.e.*, $P_q = P_q^{\max}$. Thus the NE of game $\mathcal{G}_R$ is always existent and unique. In game $\mathcal{G}_P$, player-q performs its BR strategy at a given time slot, say $t+1$, by adjusting its transmit power P_q as

$$P_q[t+1] = \frac{\mathrm{e}^{R_q^{\min}} - 1}{\mathrm{e}^{R_q[t]} - 1} P_q[t], \quad q \in \Omega, \tag{2.23}$$

where $\mathrm{e}^{R_q[t]} - 1$ is the measured SINR at MS-q at time t. Note that the above power update is well-known as the optimal power control mechanism for CDMA-based wireless networks [50]. This iteration always converges to a unique fixed-point, which is the NE of game $\mathcal{G}_P$, if such a NE exists [51]. The condition for the existence and uniqueness of game $\mathcal{G}_P$ shall be presented later as one of the research topics in Chapter 3.

For the case of multiple receive antennas, *i.e.*, $N_q > 1$, the analyses of games $\mathcal{G}_R$ and $\mathcal{G}_P$ are much more difficult. Nonetheless, various works in literature have addressed the NE analysis in these games [12, 46, 48]. For both games, the BR strategy by each player has the WF structure. Specifically, given the eigen-decomposition $\mathbf{H}_{qq}^H \mathbf{R}_q^{-1} \mathbf{H}_{qq} = \mathbf{U}_q \mathbf{D}_q \mathbf{U}_q^H$, the BR strategy at player-q is

$$\mathbf{Q}_q^\star = \mathbf{WF}_q(\mathbf{Q}_{-q}) = \mathbf{U}_q \left[\mu_q \mathbf{I} - \mathbf{D}_q^{-1}\right]^+ \mathbf{U}_q^H, \tag{2.24}$$

where μ_q is the water-level, chosen either to meet the power constraint in game $\mathcal{G}_R$ or to meet the rate constraint in game $\mathcal{G}_P$. Each player plays the WF strategy until the game converges. For this reason, these games are often referred to as IWF games in literature. The NE of this noncooperative game, which is the intersection between the BR strategies of each user, can be restated as

$$\mathbf{Q}_q^\star = \mathbf{WF}_q(\mathbf{Q}_{-q}^\star), \quad \forall q \in \Omega. \tag{2.25}$$

Using this WF structure, the sufficient conditions on the existence and uniqueness of these IWF games are presented in [12, 48]. A key observation from those conditions reveals that the game's NE is always existent and unique when the ICI is sufficiently small.

2.2.2 Interference Coordination

In the previous section, a fully decentralized approach to the CoMP under the IA mode was examined using the game theory framework and the NE of the system was characterized. However, it is well-known that the NE need not be Pareto-efficient [52]. Via the interference coordination among the BSs, significant power reduction or rate enhancement can be obtained by jointly designing all the precoders across the coordinated BSs at the same time. Pareto-efficient precoding designs have been recently proposed in [47, 53] for the multicell system with interference coordination.

When the design setting is to jointly maximize the network sum-rate, the joint optimization for CoMP under the IC mode can be stated as

$$\underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}} \quad \sum_{q=1}^{Q} \omega_q \left|\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_q^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{Q}_q\right| \tag{2.26}$$

$$\begin{aligned} \text{subject to} \quad & \text{Tr}\{\mathbf{Q}_q\} \le P_q^{\max},\ \forall q \\ & \mathbf{Q}_q \succeq \mathbf{0},\ \forall q, \end{aligned} \tag{2.27}$$

where ω_q is the non-negative weight for the link-q. For a given set of weights $\omega_1, \ldots, \omega_K$, the optimal solution to the above problem represents an optimal trade-off point between the cells' rates. Certainly, this point is Pareto-optimal, *i.e.*, one cell cannot further improve its data rate without decreasing the data rate at (one or more) other cells. Similar to the optimization for the BC in a single-cell network (2.16), problem (2.26) is nonconvex. Thus, obtaining its globally optimal solution is a highly complex process. Fortunately, the iterative algorithms developed for the BC in [32, 33], as mentioned in Section 2.1.3, can be readily applied to this multicell problem. Alternately, by successively approximating the nonconvex part in the objective function of (2.26) to a convex lower bound function, the work in [54] proposed an algorithm that converges monotonically to a local optimal solution. However, these solution approaches are centralized and might not be suitable to the multicell setup. Recent works in [55–57] proposed an interference pricing mechanism that decomposes the nonconvex problem (2.26) and solves it on a per-cell basis. The idea of this approach is as follows. To isolate the ICI terms that render problem (2.26) nonconvex, we define the weighted sum-rate at all BSs except BS-q as $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{r\neq q}^{Q} \omega_r R_r(\mathbf{Q}_q, \mathbf{Q}_{-q})$. At an instance of $(\mathbf{Q}_q, \mathbf{Q}_{-q})$, evaluated at $(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q})$, after taking the Taylor's expansion of $f_q(\cdot)$ and retaining only the linear term, the optimization (2.26) can be approximated by a set of Q per-cell problems

$$\begin{aligned} \underset{\mathbf{Q}_q}{\text{maximize}} \quad & \omega_q \left|\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_q^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{Q}_q\right| - \mathrm{Tr}\{\mathbf{A}_q \mathbf{Q}_q\} \\ \text{subject to} \quad & \mathrm{Tr}\{\mathbf{Q}_q\} \leq P_q^{\max}, \end{aligned} \tag{2.28}$$

where $\mathbf{A}_q$, obtained from the negative derivative of $f_q(\cdot)$ with respect to $\mathbf{Q}_q$ [57], is the pricing matrix charged on the ICI caused by BS-q. Note that this approximated problem only requires local channel information pertained to BS-q. In addition, it is convex on $\mathbf{Q}_q$, and its optimal solution can easily be obtained in a closed-form WF solution [57]. Thus, the problem can be solved locally at each BS. Interestingly, the algorithm was shown to converge monotonically to a local optimal solution of the original problem (2.26). It is worth mentioning that while this solution approach can be implemented in a distributed manner, it still demands message passing to calculate and exchange the parameters $\mathbf{A}_q$'s.

When the design setting is to jointly minimize the power consumption, the joint optimization for CoMP under the IC mode can be stated as

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{minimize}} \quad & \sum_{q=1}^{Q} \omega_q \mathrm{Tr}\left\{\mathbf{Q}_q\right\} \\ \text{subject to} \quad & \left|\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_q^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{Q}_q\right| \geq R_q^{\min},\ \forall q. \end{aligned} \tag{2.29}$$

Similar to problem (2.26), this optimization is nonconvex, except for the MISO case, i.e, $N_q = 1, \forall q$. For that case, the work in [10] showed that the optimal beamformers (precoders) can be found in a distributed manner. Nonetheless, the distributed implementation of the algorithm proposed in [10] comes with several requirements, including perfect channel reciprocals, instant signaling exchanges, and synchronization among the coordinated BSs. To alleviate these drawbacks, we shall consider a new game with pricing consideration that retains the advantages of the multicell game in CoMP with IA mode later in Chapter 3.

2.3 Concluding Remarks

In summary, this section has discussed various linear and nonlinear precoding designs in a single-cell network and examined how these designs have been adopted to the context of CoMP. The key aspect of precoding designs in a CoMP system is the consideration of the effect of ICI to the overall system performance. When CoMP is deployed under the IA mode, it is important to examine how each BS strategically adapts its precoding strategy accordingly to the amount of ICI at its connected MS. When CoMP is operating in the IC mode, the main concern is the joint control of the ICI by means of precoding in order to optimize the overall system performance.

It is worth mentioning that most of the related works in literature study the CoMP system with *one* MS per cell. Only a limited number of works, such as [9,10,58], considered a CoMP system with multiple MSs at each cell for the IC mode. It is observed that no work in literature has provided a thorough overview on how the ICI affects the multiuser precoding process in a CoMP system. In addition, while research on single-cell precoding techniques is quite plentiful, as previously mentioned in this section, many of these techniques do not have a counterpart version for a CoMP system in the presence of ICI. These observations motivate us to study the precoding designs for a CoMP system under a more general setting with multiple MSs per cell, and non-homogeneous channel and noise conditions at the MSs. In this setting, each BS has to take into account the resource allocation between its connected MSs and the signaling with other coordinated BSs in the system as well. In addition, each BS should be able to determine its precoders in fully distributed manner. How these factors impact on the precoding process at each coordinated BS are the main concerns of this research.

Chapter 3

Multiuser Downlink Beamforming in Multicell Wireless Systems: A Game Theoretical Approach

3.1 Introduction

Recently, the study of power control in a mutual interference multiuser network using game theory has attracted considerable research attention. By considering the multiuser system as a strategic noncooperative game (SNG), each player (a wireless device) greedily adapts its strategy to maximize its own utility, given the strategies from other players [11, 38–48]. In general, these works focus on studying the existence and uniqueness of the stable operating point of the system, *i.e.*, the Nash Equilibrium (NE). The uplink power control problem in a single-cell code-division multiple-access (CDMA) data system with multiple competing users was studied in [38,39], where the utility function was defined as the ratio of throughput to transmit power. A pricing mechanism was investigated in [39] to obtain a more efficient solution of the power control game. In orthogonal frequency division multiplexing (OFDM) system over a shared band, the work in [11] has inspired various other works on the IWF algorithm, such as [41–45] with sum-rate as the utility function, or [46] with transmit power

The materials presented in Chapter 3 have been presented at the 2010 IEEE Global Communications Conference in Miami, FL, USA [59], the 2011 IEEE Global Communications Conference in Houston, TX, USA [60], published in the IEEE Transactions on Signal Processing [61], and accepted for publication in the IET Communications [62].

as the utility function. More recently, the IWF game has also been considered in [63] for a system utilizing both protected and shared bands.

In multiple antenna systems, the work in [47] considered a multicell system, where each cell consists of *one* multiple-antenna base-station (BS) and *one* single antenna mobile-station (MS). The objective of [47] was to study the precoding beamforming vector at the two BSs in competitive and cooperative manners. In the multiuser MIMO channels, the work in [48] studied the competitive precoding design, where each player wished to maximize its mutual information. It is to be noted that these works only considered the system where each transmitter (base-station) communicates with only *one* receiver (mobile-station).

Inspired by the mentioned works, this chapter considers a game theoretical approach to study the competitive precoding design in a multiuser multicell system, where each BS concurrently serves multiple MSs (or users). Sharing the same frequency band, the BS of each cell wishes to design the optimal downlink beamformers for its users in order to minimize its transmit power, given a set of target signal-to-interference-plus-noise ratios (SINRs) for the users in its cell. Under a similar setup, the work in [64] studied scheduling schemes to handle the ICI and provided a quality of service (QoS) guaranteed in the form of packet error rate. Multicell downlink beamforming with coordination was considered in [10], where the total weighted transmit power across multiple BSs is jointly minimized. Via the concept of uplink-downlink duality, it is shown in [10] that such a jointly optimal design can be implemented in a distributed manner under certain requirements, including perfect channel reciprocal from each BS to each MS (not necessarily in the same cell) and synchronization among the BSs. These requirements, which may be difficult to meet in practice, are the drawbacks of the distributed implementation in the coordinated design. Conversely, in the competitive design, where the multicell beamformers are devised on per-cell basis with no centralized control, these requirements can be alleviated.

Using the game-theory framework, we establish the best response strategy of a cell, given the beamforming strategies from other cells. Then, it is shown that such best response strategy is a *standard* function [51],[1] which guarantees the uniqueness of the NE and the convergence of the distributed algorithm. This is the distinction of this power minimization game, compared to typical n-person concave games in an OFDM system with WF as

[1] The framework of *standard* functions, proposed by Yates [51], shall be discussed for our context in Section 3.3.

the optimal strategy [11, 41–44]. In addition, necessary and sufficient conditions for the existence of the NE are also given. A comparison to the fully coordinated multicell downlink beamforming design is then presented in this work.

It is worth mentioning that the NE of the multicell game needs not to be Pareto-optimal, *i.e.*, it may not stay in the surface established by the coordinated design. Moreover, it may happen that a beamformer design in one cell is highly correlated to the channel of the other cells, which then causes significant ICI. To avoid this undesired effect, we consider a new multicell downlink beamforming game with pricing consideration, where each BS voluntarily attempts to minimize the interference induced to other cells. This pricing technique allows a BS to steer its beamformers in a more cooperative way, which results in a more Pareto-efficient NE. The characterization of the new game's NE reveals that under certain conditions, the new NE point is able to approach the performance established by the coordinated design, while retaining the distributed nature of the SNG.

3.2 System Model

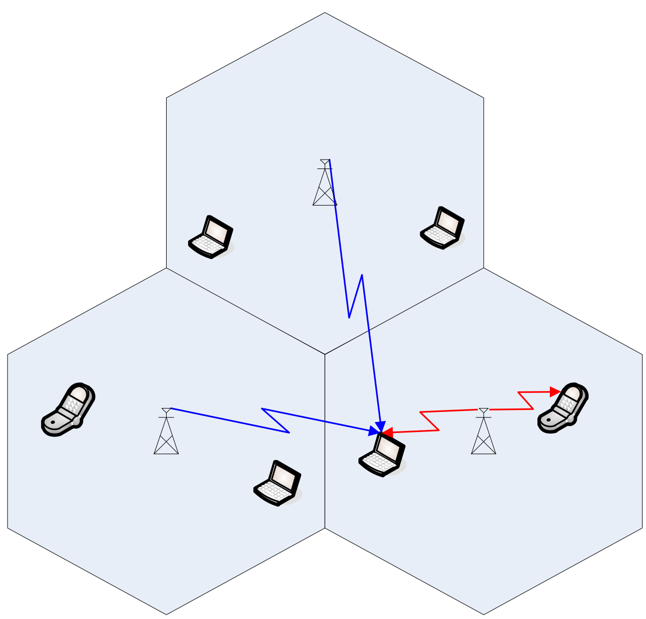

Fig. 3.1 An example of a multicell system with 3 base-stations and 2 users per cell.

We consider a multiuser downlink beamforming system with Q separate cells operating on the same frequency channel, as illustrated in Figure 3.1. In each cell, one multiple-antenna BS is concurrently sending independent information to several remote single-antenna MSs. Let $\Omega = \{1, \ldots, Q\}$ denote the set of the cells (players), and define $\Omega_{-q} = \Omega \backslash \{q\}$. For simplicity of presentation, it is assumed that each BS is equipped with M antennas, and is serving K MSs. We note that each user (MS) is now subject to the co-channel interference from other cells (ICI), in addition to the interference caused by the signals intended for other users in the same cell (intra-cell interference). In a competitive design for this multicell system, it is assumed that each BS has full knowledge of the downlink channels within its cell, but not the inter-cell channels. Thus, each BS is only able to manage the intra-cell interference. On the other hand, the BS treats the ICI as background noise. In the later parts of this work, a BS may possess the full or partial channel information to the users in other cells. The additional channel knowledge then allows a BS to control the ICI as well.

Considering the transmission at a particular cell, say cell-q, its downlink channel can be modeled as

$$y_{q_i} = \mathbf{h}_{qq_i}^H \mathbf{x}_q + \sum_{m \neq q}^{Q} \mathbf{h}_{mq_i}^H \mathbf{x}_m + z_{q_i}, \tag{3.1}$$

where $\mathbf{x}_m$ is an $M \times 1$ complex vector representing the transmitted signal at BS-m, $\mathbf{h}_{mq_i}^*$ is an $M \times 1$ complex channel vector from BS-m to user-i of cell-q, y_{q_i} represents the received signal at user-i, and z_{q_i} is $\mathcal{CN}(0, \sigma^2)$. It is assumed that the channel vector $\mathbf{h}_{qq_i}^*$ is known at both the BS and user-i of cell-q, whereas the cross-cell channel $\mathbf{h}_{mq_i}, m \neq q$ is unknown.

In a beamforming design, the transmitted signal $\mathbf{x}_q$ is of the form

$$\mathbf{x}_q = \sum_{i=1}^{K} x_{q_i} \mathbf{w}_{q_i}, \tag{3.2}$$

where x_{q_i} is a complex scalar representing the signal intended for user-i, and $\mathbf{w}_{q_i}$ is an $M \times 1$ beamforming vector for user-i. Without loss of generality, let $\mathbb{E}[|x_{q_i}|] = 1$. It is easy to verify that the SINR at user-i of cell-q is

$$\mathrm{SINR}_{q_i} = \frac{\left|\mathbf{w}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j \neq i}^{K} \left|\mathbf{w}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 + \sum_{m \neq q}^{Q} \sum_{j=1}^{K} \left|\mathbf{w}_{m_j}^H \mathbf{h}_{mq_i}\right|^2 + \sigma^2}. \tag{3.3}$$

Note that the received signal at user-i of cell-q is corrupted by the intra-cell interference $\sum_{j\neq i}^{K}\left|\mathbf{w}_{q_j}^H\mathbf{h}_{qq_i}\right|^2$, the ICI $\sum_{m\neq q}^{Q}\sum_{j=1}^{K}\left|\mathbf{w}_{m_j}^H\mathbf{h}_{mq_i}\right|^2$, as well as the AWGN. Although the channel state information from other cells is not known at both the users and BS of cell-q, each user can measure its total interference and report back to the BS. The BS, having known the channel to the users in its cell, can determine the total ICI plus AWGN at each user.

3.3 The Multicell Downlink Beamforming Game

3.3.1 Problem Formulation

In the first part of this work, we are interested in formulating the multicell downlink beamforming design within the framework of game theory. In particular, we consider a SNG, where the players are the cells and the payoff functions are the transmit powers of the BSs. More specifically, each player competes with each other by choosing the downlink beamformer design that greedily minimizes its own transmit power subject to a given set of target SINRs at the users within its cell. Each channel is assumed to vary sufficiently slowly such that it can be considered fixed while the game is being played.

Define the precoding matrix $\mathbf{W}_q = [\mathbf{w}_{q_1},\ldots,\mathbf{w}_{q_K}]$ as the strategy at BS-q, and $\mathbf{W}_{-q}$ as the precoding strategy of all the BSs, except BS-q. The transmit power at BS-q, is then given by $\|\mathbf{W}_q\|_F^2$. Further define the set of admissible beamforming strategies $\mathbf{W}_q \in \mathcal{P}_q(\mathbf{W}_{-q})$ of cell-q as

$$\mathcal{P}_q(\mathbf{W}_{-q}) = \left\{\mathbf{W}_q \in \mathbb{C}^{M\times K} : \mathrm{SINR}_{q_i}(\mathbf{W}_q,\mathbf{W}_{-q}) \geq \gamma_{q_i}, \forall i\right\},$$

where γ_{q_i} is the target SINR at user-i of cell-q.

At cell-q, denote the total ICI plus background noise (IPN) at the ith user as $r_{-q_i}(\mathbf{W}_{-q}) = \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\left|\mathbf{w}_{m_j}^H\mathbf{h}_{mq_i}\right|^2 + \sigma^2 = \sum_{m\neq q}^{Q}\left\|\mathbf{W}_m^H\mathbf{h}_{mq_i}\right\|^2 + \sigma^2$. Furthermore, denote $\mathbf{r}_{-q} = [r_{-q_1},\ldots,r_{-q_K}]^T$. Note that the set of feasible strategies $\mathcal{P}(\mathbf{W}_{-q})$ of cell-q depends on the beamforming strategies $\mathbf{W}_{-q}$ of all the other cells. Mathematically, the corresponding game has the following structure

$$\mathcal{G} = \left(\Omega,\ \left\{\mathcal{P}_q(\mathbf{W}_{-q})\right\}_{q\in\Omega},\ \left\{t_q(\mathbf{W}_q)\right\}_{q\in\Omega}\right),$$

where $t_q(\mathbf{W}_q) = \left\|\mathbf{W}_q\right\|_F^2$ is the transmit power at BS-q.[2] Given the beamforming design of the others, reflected by the IPN vector $\mathbf{r}_{-q}$, the optimal or best response strategy of the qth BS is the solution to the following optimization problem

$$\begin{aligned} \underset{\mathbf{W}_q}{\text{minimize}} \quad & \left\|\mathbf{W}_q\right\|_F^2 \\ \text{subject to} \quad & \frac{\left|\mathbf{w}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^K \left|\mathbf{w}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i},\ \forall i. \end{aligned} \tag{3.4}$$

We note that there are several numerical approaches to find the optimal solution to this downlink beamforming problem, as mentioned in Chapter 2. In a multicell configuration, the problem arisen here is when one player changes its beamforming matrix, the other players also need to change their own beamforming matrices in order to achieve its target SINRs. Our interest is to investigate whether game $\mathcal{G}$ eventually converges into a stable point, *i.e.*, an NE; and if an NE exists, whether its uniqueness hold. A feasible strategy profile $\mathbf{W}^\star = \{\mathbf{W}_q^\star\}_{q=1}^Q$ is an NE of game $\mathcal{G}$ if

$$t_q(\mathbf{W}_q^\star) \leq t_q(\mathbf{W}_q),\ \forall \mathbf{W}_q \in \mathcal{P}_q(\mathbf{W}_{-q}^\star), \quad \forall q \in \Omega. \tag{3.5}$$

At the NE point, given the beamforming matrices from other cells, a BS does not have the incentive to unilaterally change its own beamforming matrix, *i.e.*, it will consume more power to obtain the same SINR targets. In the following sections, by first studying the best response strategy of each player, the NE of game $\mathcal{G}$ is subsequently characterized.

3.3.2 The Best Response Strategy

In this section, we first present some claims to simplify the analysis of the game and characterize the best response strategy.

Claim 3.1. *If $\mathbf{W}_q^\star$ is the optimal beamforming strategy for cell-q, then $\mathbf{W}_q^\star\mathbf{R}$, where $\mathbf{R} = \mathrm{diag}\left(\mathrm{e}^{j\theta_1}, \ldots, \mathrm{e}^{j\theta_K}\right), \forall\theta_1, \ldots, \theta_K$ is also optimal.*

This claim stems from the fact that the effective SINR at each user is invariant to a constant phase change of the beamforming vector of any other user.

[2] According to recent use, this game may be referred to as a generalized Nash equilibrium problem where the admissible strategy set of a player depends on the other players' strategy [46].

Claim 3.2. *With unlimited transmit power, the feasibility of the optimization problem* (3.4) *at cell-q is only dependent on the channel* $\mathbf{h}_{qq_1}, \ldots, \mathbf{h}_{qq_K}$ *and the set of target SINRs* $\gamma_{q_1}, \ldots, \gamma_{q_K}$. *It is independent of the IPN vector* $\mathbf{r}_{-q} > 0$.

This claim comes directly from Proposition 1 in [19], which shows that the rank of the matrix $\mathbf{H}_q = [\mathbf{h}_{qq_1}, \ldots, \mathbf{h}_{qq_K}]$ determines the feasibility of the optimization problem (3.4). In addition, when $\mathbf{H}_q$ is full-rank, any set of target SINRs $\gamma_{q_1}, \ldots, \gamma_{q_K}$ is feasible. In this work, we assume that $\mathbf{H}_q$ is full-rank, $\forall q$.

It is to be noted that Claim 3.2 is only applicable where no power constraint is imposed at the BS. In practice, there exists a power bound at the BS, which then affects the feasibility of the QoS problem (3.4). In this work, we relax this power constraint with the assumption that the power limit at each BS is high enough to accommodate the transmissions to its connected MSs at the targeted QoS. Nonetheless, it is possible that a bounded feasible solution to the whole multicell system might not be found. Thus, our focus in this work is on the study of the boundedness and existence of the NE.

Claim 3.3. *For two different IPN vectors at cell-q,* $\mathbf{r}_{-q}$ *and* $\bar{\mathbf{r}}_{-q}$, *the optimal beamforming vector* $\mathbf{w}^{\star}_{q_i}$, *corresponding to* $\mathbf{r}_{-q}$, *is a scaled version of the optimal beamforming vector* $\bar{\mathbf{w}}^{\star}_{q_i}$, *corresponding to* $\bar{\mathbf{r}}_{-q}$.

Proof. It is first observed that the IPN components in $\mathbf{r}_{-q}$ are scalars, which therefore should not impact on the directions of the beamformers at BS-q. To support this observation, we revisit the dual uplink problem of problem (3.4), which turns out to be [21]

$$\begin{aligned} &\underset{\lambda_{q_1},\ldots,\lambda_{q_K}}{\text{maximize}} && \sum_{i=1}^{K} \lambda_{q_i} r_{-q_i} \\ &\text{subject to} && \sum_{j\neq i}^{K} \lambda_{q_j} \mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H + \mathbf{I} \succeq \frac{\lambda_{q_i}}{\gamma_{q_i}} \mathbf{h}_{qq_i}\mathbf{h}_{qq_i}^H, \ \forall i. \end{aligned} \tag{3.6}$$

This optimization is then equivalent to the dual uplink problem [21]

$$\begin{aligned} &\underset{\substack{\lambda_{q_1},\ldots,\lambda_{q_K} \\ \mathbf{w}_{q_1},\ldots,\mathbf{w}_{q_K}}}{\text{minimize}} && \sum_{i=1}^{K} \lambda_{q_i} r_{-q_i} \\ &\text{subject to} && \frac{\lambda_{q_i}\left|\hat{\mathbf{w}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \lambda_{q_j}\left|\hat{\mathbf{w}}_{q_i}^H \mathbf{h}_{qq_j}\right|^2 + \hat{\mathbf{w}}_{q_i}^H \hat{\mathbf{w}}_{q_i}} \geq \gamma_{q_i}, \ \forall i, \end{aligned} \tag{3.7}$$

where the optimization is over a weighted sum-power of the uplink powers λ_{q_i}'s and the receive beamformer vectors $\hat{\mathbf{w}}_{q_i}$'s. It is to be noted that the optimal uplink power $\lambda^\star_{q_i}$ is only dictated by the constraints, but not the objective function. As the constraints are met with equality at optimality, the solution of λ_{q_i} can be obtained by the fixed point iteration [51]

$$\lambda_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\mathbf{h}_{q_i}^H \left(\sum_{j=1}^K \lambda_{q_j}^{(n)} \mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H + \mathbf{I}\right)^{-1} \mathbf{h}_{q_i}}. \tag{3.8}$$

The optimal receive beamformer vector $\hat{\mathbf{w}}^\star_{q_i}$ is the minimum mean square error (MMSE) receiver, *i.e.*, $\hat{\mathbf{w}}^\star_{q_i} = \left(\sum_{j=1}^K \lambda^\star_{q_j} \mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H + \mathbf{I}\right)^{-1} \mathbf{h}_{qq_i}$. Clearly, the optimal uplink power $\lambda^\star_{q_i}$ and the optimal receive beamformer $\hat{\mathbf{w}}^\star_{q_i}$ are independent of r_{-q_i}. The optimal downlink beamformer vectors $\mathbf{w}^\star_{q_i}$ then can be found to be a scaled version of the uplink receive beamformer $\hat{\mathbf{w}}^\star_{q_i}$ [19]. Thus, both the optimal beamforming vector $\mathbf{w}^\star_{q_i}$, corresponding to $\mathbf{r}_{-q}$, and the vector $\bar{\mathbf{w}}^\star_{q_i}$, corresponding to $\bar{\mathbf{r}}_{-q}$, are scaled versions of $\hat{\mathbf{w}}_{q_i}$. This concludes the proof for this claim. □

From Claim 3.3, it is to be noted that whenever the IPN vector $\mathbf{r}_{-q}$ is changed, BS-q only needs to adjust the allocated power for each user, but not the beam patterns to its users.[3] Thus, BS-q can determine its beam pattern first, and then allocate the appropriate power to each user subject to the ICI in $\mathbf{r}_{-q}$.

The optimal beam patterns at each cell can be determined by solving problem (3.4) in the absence of ICI, which is

$$\begin{aligned} \underset{\mathbf{W}_q}{\text{minimize}} \quad & \sum_{i=1}^K \left\|\mathbf{w}_{q_i}\right\|^2 \\ \text{subject to} \quad & \frac{\left|\mathbf{w}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^K \left|\mathbf{w}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 + \sigma^2} \geq \gamma_{q_i},\ \forall i. \end{aligned} \tag{3.9}$$

This problem can be easily solved by the techniques mentioned in the previous section. The beamforming pattern for each user in cell-q is determined as $\widetilde{\mathbf{w}}_{q_i} = \mathbf{w}_{q_i}/\|\mathbf{w}_{q_i}\|$. By

[3]By beam pattern, we mean the norm-1 $\mathbf{w}_{q_i}/\|\mathbf{w}_{q_i}\|$.

Claim 3.3, we can restate the optimization problem (3.4) as

$$\underset{p_{q_1},\dots,p_{q_K}}{\text{minimize}} \quad \sum_{i=1}^{K} p_{q_i} \tag{3.10}$$

$$\text{subject to} \quad \frac{p_{q_i}\left|\widetilde{\mathbf{w}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} p_{q_j}\left|\widetilde{\mathbf{w}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i},\ \forall i,$$

$$p_i \geq 0,\ \forall i, \tag{3.11}$$

which is reduced back to a power allocation problem, where p_{q_i} is the allocated power for user-i. Denote $\mathbf{p}_q = (p_{q_i}, \dots, p_{q_K})$, $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_Q)$, and $\mathbf{p}_{-q} = (\mathbf{p}_1, \dots, \mathbf{p}_{q-1}, \mathbf{p}_{q+1}, \dots, \mathbf{p}_Q)$. Note that the strategy set of player-q is redefined as

$$\mathcal{P}_q(\mathbf{p}_{-q}) = \left\{\mathbf{p}_q \in \mathbb{R}_+^K : \mathrm{SINR}_{q_i}(\mathbf{p}_q, \mathbf{p}_{-q}) \geq \gamma_{q_i},\ \forall i\right\}. \tag{3.12}$$

After obtaining the optimal solution $p_{q_i}^\star$ of problem (3.10), the optimal beamforming vectors corresponding to $\mathbf{r}_{-q}$ are $\sqrt{p_{q_i}^\star}\,\widetilde{\mathbf{w}}_{q_i}$. The following claim presents the analytical solution to (3.10).

Claim 3.4. *The optimal solution of* (3.10)*,* $\mathbf{p}_q^\star = [p_{q_1}^\star, \dots, p_{q_K}^\star]^T$*, is given by*

$$\mathbf{p}_q^\star = \mathbf{G}_q^{-1}\mathbf{r}_{-q}, \tag{3.13}$$

where $\mathbf{G}_q \in \mathbb{R}^{K\times K}$*, defined as* $[\mathbf{G}_q]_{i,i} = (1/\gamma_{q_i})\left|\widetilde{\mathbf{w}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2$ *and* $[\mathbf{G}_q]_{i,j} = -\left|\widetilde{\mathbf{w}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2$ *if* $i \neq j$*, is invertible. Furthermore,* $\mathbf{p}_q^\star > 0$*,* $\forall \mathbf{r}_{-q} > 0$*.*

Proof. Since all the SINR constraints in (3.10) are met with equality at optimality, they are equivalent to

$$p_{q_i}^\star \frac{\left|\widetilde{\mathbf{w}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K} p_{q_j}^\star \left|\widetilde{\mathbf{w}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 = r_{-q_i},\ i = 1, \dots, K.$$

The solution of this set of K equations with K variables are the optimal solution of (3.10). Rewriting these K equations in matrix form, one has $\mathbf{G}_q\mathbf{p}_q^\star = \mathbf{r}_{-q}$. From Claim 3.2, as the problem (3.10) is always feasible $\forall \mathbf{r}_{-q} > 0$, $\mathbf{G}_q$ has to be invertible, and $\mathbf{p}_q^\star = \mathbf{G}_q^{-1}\mathbf{r}_{-q} > 0$ is uniquely defined, $\forall \mathbf{r}_{-q} > 0$. □

We proceed to denote by $\mathbf{G}_{mq} \in \mathbb{R}^{K \times K}, m \neq q$ the ICI matrix from cell-m to cell-q, where $[\mathbf{G}_{mq}]_{i,j} = \left|\widetilde{\mathbf{w}}_{m_j}^H \mathbf{h}_{mq_i}\right|^2$. Then $\mathbf{G}_{mq}\mathbf{p}_m$ is the interference vector caused by BS-m to the K users of cell-q. Thus, one has $\mathbf{r}_{-q} = \sum_{m \neq q}^{Q} \mathbf{G}_{mq}\mathbf{p}_m + \mathbf{1}\sigma^2$.

From Claim 3.4, the best response strategy of the qth cell subject to the strategy of Ω_{-q} is

$$\mathbf{p}_q^{\star} = \mathrm{BR}_q\big(\mathbf{p}_{-q}\big) = \mathbf{G}_q^{-1}\left(\sum_{m \neq q}^{Q} \mathbf{G}_{mq}\mathbf{p}_m + \mathbf{1}\sigma^2\right), \ \forall q. \tag{3.14}$$

The NEs of game $\mathcal{G}$ can now be redefined as the intersection points of the BRs, *i.e.*,

$$\mathbf{p}_q^{\star} = \mathbf{G}_q^{-1}\left(\sum_{m \neq q}^{Q} \mathbf{G}_{mq}\mathbf{p}_m^{\star} + \mathbf{1}\sigma^2\right), \ \forall q. \tag{3.15}$$

The next lemma shows that the best response function in (3.14) is *standard* [51], which guarantees the uniqueness of the NE if such NE exists [65].

Lemma 3.1. *The best response function is a standard function.*

Proof. First, define $\mathbf{p} = [\mathbf{p}_1^T, \ldots, \mathbf{p}_Q^T]^T$, then $\mathrm{BR}_q(\mathbf{p}) \triangleq \mathrm{BR}_q\big(\mathbf{p}_{-q}\big)$. We need to show that the best response function $\mathrm{BR}_q(\mathbf{p})$ meets the three requirements of a *standard* function:

1. *Positivity:* for any $\mathbf{p} \geq 0$, as $\mathbf{G}_{mq}$ is a positive matrix, we have $\sum_{m \neq q}^{Q} \mathbf{G}_{mq}\mathbf{p}_m + \mathbf{1}\sigma^2 > 0, \forall q$. Thus, $\mathrm{BR}_q(\mathbf{p}) = \mathbf{G}_q^{-1}\left(\sum_{m \neq q}^{Q} \mathbf{G}_{mq}\mathbf{p}_m + \mathbf{1}\sigma^2\right) > 0, \forall q$, as a result from Claim 3.4.

2. *Monotonicity:* for $\mathbf{p} \geq \mathbf{p}'$, then

$$\mathrm{BR}_q(\mathbf{p}) - \mathrm{BR}_q(\mathbf{p}') = \mathbf{G}_q^{-1}\left[\sum_{m \neq q}^{Q} \mathbf{G}_{mq}\left(\mathbf{p}_m - \mathbf{p}_m'\right)\right] \geq 0, \ \forall q$$

   as a result from Claim 3.4 and each $\mathbf{p}_m - \mathbf{p}_m' \geq 0$.

3. *Scalability:* $\forall \epsilon > 1, \forall \mathbf{p} \geq 0$, one has

$$\epsilon \mathrm{BR}_q(\mathbf{p}) - \mathrm{BR}_q(\epsilon \mathbf{p}) = \epsilon \mathbf{G}_q^{-1}\mathbf{1}\sigma^2 - \mathbf{G}_q^{-1}\mathbf{1}\sigma^2 = (\epsilon - 1)\mathbf{G}_q^{-1}\mathbf{1}\sigma^2 > 0.$$

□

Since $\text{BR}_q(\mathbf{p}), q = 1, \ldots, Q$, are *standard* functions, the iteration $\mathbf{p}_q^{(t+1)} = \text{BR}_q(\mathbf{p}^{(t)}), q = 1, \ldots, Q$, will converge from any starting point $\mathbf{p}^{(0)} > 0$ to a unique fixed point (when it exists) [51], which is the NE of game $\mathcal{G}$ [65]. In addition, the iterative update can be implemented in a fully asynchronous manner among the cells. It is to be noted that due to the monotonicity of the standard function, if the fixed point does not exist, the transmit power at each BS will increase without bound. To this end, one sufficient condition and one necessary condition guaranteeing the boundedness of the NE are examined. Such conditions equivalently guarantee the existence and uniqueness of the game's NE.

3.3.3 A Sufficient Condition for the Existence and Uniqueness of the NE

In this section, we consider the best response dynamic of the game as a mapping process. A sufficient condition is then presented such that the mapping is a contraction, which guarantees the existence of the fixed-point of the mapping, *i.e.*, the NE of game $\mathcal{G}$. We summarize the obtained result in the following proposition.

Proposition 3.1. *The NE of game $\mathcal{G}$ exists if the spectral radius*

$$\text{(C)}: \quad \rho(\mathbf{S}) < 1, \tag{3.16}$$

where the square matrix $\mathbf{S} \in \mathbb{R}^{Q \times Q}$ is defined as

$$[\mathbf{S}]_{q,m} = \begin{cases} 0, & \text{if } m = q \\ \left\| \mathbf{G}_q^{-1} \mathbf{G}_{mq} \right\|_F, & \text{if } m \neq q. \end{cases} \tag{3.17}$$

Proof. Let $\mathbf{T}_q(\mathbf{p}) = \text{BR}_q(\mathbf{p})$ and $\mathbf{T}(\mathbf{p}) = \left(\text{BR}_q(\mathbf{p})\right)_{q \in \Omega}$. Since $\mathbf{T}_q(\mathbf{p})$ is a mapping from $\mathbb{R}_+^{KQ}$ onto $\mathcal{P}_q(\mathbf{p}_{-q})$, which is a subset of $\mathbb{R}_+^K$, $\mathbf{T}(\mathbf{p})$ is a mapping from $\mathbb{R}_+^{KQ}$ onto a Cartesian product of Q $\mathbb{R}_+^K$ sets, *i.e.*, $\mathbb{R}_+^{KQ}$. For some $\mathbf{w} = [w_1, \ldots, w_Q]^T > 0$, the mapping $\mathbf{T}(\mathbf{p})$ is a block-contraction of modulus α, with respect to the norm $\|\cdot\|_{\infty,\text{block}}^{\mathbf{w}}$, if there exists a non-negative constant $\alpha < 1$ such that[4]

$$\|\mathbf{T}(\mathbf{p}) - \mathbf{T}(\mathbf{p}')\|_{2,\text{block}}^{\mathbf{w}} \leq \alpha \, \|\mathbf{p} - \mathbf{p}'\|_{2,\text{block}}^{\mathbf{w}}, \; \forall \mathbf{p}, \mathbf{p}' \geq 0. \tag{3.18}$$

The condition $\alpha < 1$ is sufficient to guarantee the existence and uniqueness of the

[4] The definitions of vector norms and matrix norms, and the concept of contraction mapping are given in Appendix C.

fixed point $\mathbf{p} = \mathbf{T}(\mathbf{p})$ as well as the convergence of the mapping to the fixed point. It is worth mentioning that this property has been commonly applied to the analysis of games with best response dynamics, such as the well-known IWF game in a multi-channel system [41, 43, 44, 48]. To this end, this contraction mapping property is exploited to establish a sufficient condition on the boundedness of the power update in game $\mathcal{G}$.

Let $e_{\mathbf{T}_q} = \left\|\mathbf{T}_q(\mathbf{p}) - \mathbf{T}_q(\mathbf{p}')\right\|_2$, and $e_q = \left\|\mathbf{p}_q - \mathbf{p}'_q\right\|_2$, then

$$\begin{aligned} e_{\mathbf{T}_q} &= \left\|\mathrm{BR}_q(\mathbf{p}) - \mathrm{BR}_q(\mathbf{p}')\right\|_2 \\ &= \left\|\mathbf{G}_q^{-1}\left[\sum_{m\neq q}^{Q}\mathbf{G}_{mq}\left(\mathbf{p}_m - \mathbf{p}'_m\right)\right]\right\|_2 \\ &\leq \sum_{m\neq q}^{Q}\left\|\mathbf{G}_q^{-1}\mathbf{G}_{mq}\right\|\left\|\mathbf{p}_m - \mathbf{p}'_m\right\|_2, \end{aligned} \tag{3.19}$$

where the inequality is satisfied if the matrix norm $\|\cdot\|$ applied to $\mathbf{G}_q^{-1}\mathbf{G}_{mq}$ is consistent [66]. Here, we can use the Frobenius norm $\|\cdot\|_F$ since it is consistent and easy to compute. Define the vectors $\mathbf{e}_{\mathbf{T}} = [e_{\mathbf{T}_1}, \ldots, e_{\mathbf{T}_Q}]^T$ and $\mathbf{e} = [e_1, \ldots, e_Q]^T$. Furthermore, define the square matrix $\mathbf{S} \in \mathbb{R}^{Q\times Q}$, where

$$[\mathbf{S}]_{q,m} = \begin{cases} 0, & \text{if } \; m = q \\ \left\|\mathbf{G}_q^{-1}\mathbf{G}_{mq}\right\|_F, & \text{if } \; m \neq q \end{cases}. \tag{3.20}$$

Then, one has

$$\mathbf{e}_{\mathbf{T}} \leq \mathbf{S}\mathbf{e}. \tag{3.21}$$

Thus,

$$\|\mathbf{e}_{\mathbf{T}}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}}\|\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}}, \tag{3.22}$$

as the induced ∞-norm $\|\cdot\|_{\infty,\mathrm{mat}}^{\mathbf{w}}$ is consistent [66]. Then, one has

$$\begin{aligned} \left\|\mathbf{T}(\mathbf{p}) - \mathbf{T}(\mathbf{p}')\right\|_{2,\mathrm{block}}^{\mathbf{w}} &= \max_{q\in\Omega}\frac{\|\mathrm{BR}_q(\mathbf{p}) - \mathrm{BR}_q(\mathbf{p}')\|_2}{w_q} \\ &= \|\mathbf{e}_{\mathbf{T}}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \\ &\leq \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}}\|\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \\ &= \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}}\left\|\mathbf{p} - \mathbf{p}'\right\|_{2,\mathrm{block}}^{\mathbf{w}}. \end{aligned} \tag{3.23}$$

Thus, if $\|\mathbf{S}\|^{\mathbf{w}}_{\infty,\text{mat}} < 1$, the existence and the convergence to the fixed point are guaranteed because the mapping in (3.23) is a contraction. It is to be noted that if $\mathbf{S}$ is a non-negative matrix, there exists a positive vector $\mathbf{w}$ such that [67]

$$\|\mathbf{S}\|^{\mathbf{w}}_{\infty,\text{mat}} < 1 \quad \Longleftrightarrow \quad \rho(\mathbf{S}) < 1. \tag{3.24}$$

□

Remark 3.1: A physical interpretation of the sufficient condition (C) is as follows. Assuming the path-loss fading model $\mathbf{h}_{mq_i} = \bar{\mathbf{h}}_{mq_i} d_{mq_i}^{-\beta}$, where $\bar{\mathbf{h}}_{mq_i}$ contains normalized i.i.d. $\mathcal{CN}(0,1)$ channel gains, d_{mq_i} is the distance between BS-m to user-i of cell-q, and β is the path-loss exponent. When the distance d_{mq_i} increases, the cross channel gains $\mathbf{h}_{mq_i}$, $m \neq q$ are smaller, which shrinks the elements in the cross interference matrix $\mathbf{G}_{mq}$. Thus, the positive off-diagonal elements of $\mathbf{S}$, which are the Frobenius norm of $\mathbf{G}_q^{-1}\mathbf{G}_{mq}$'s, also become smaller. This results in a smaller spectral radius of $\mathbf{S}$. Thus, the more apart a MS from the BSs of other cells, the higher chance of $\rho(\mathbf{S})$ being less than 1, which then guarantees the existence and uniqueness of the NE.

It is to be noted that while the sufficient condition (C) is obtained from the contraction mapping property of the power update, the direct characterization of the NE can be utilized to draw the necessary condition of the NE existence. We consider this necessary condition in the following section.

3.3.4 The Necessary Condition for the Existence and Uniqueness of the NE

This section examines the necessary condition for the existence and uniqueness of the NE of game $\mathcal{G}$. Before proceeding, we provide some definitions of related mathematical terms to be used in the section.

Definition 3.1. *[68, 69] A square matrix $\mathbf{A}$ is a $\mathbf{Z}$-matrix if all of its off-diagonal elements are non-positive. A square matrix $\mathbf{A}$ is a $\mathbf{P}$-matrix if all of its principal minors are positive. A square matrix that is both a $\mathbf{Z}$-matrix and a $\mathbf{P}$-matrix is called an $\mathbf{M}$-matrix.*

Besides the above definition, there are several equivalent characterizations of an $\mathbf{M}$-matrix [68]. It is to be noted that if a matrix is an $\mathbf{M}$-matrix, it is invertible and the inverse is a positive matrix [68].

Proposition 3.2. *The NE of game $\mathcal{G}$ exists if and only if the following matrix*

$$(\mathrm{C1}): \quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & -\mathbf{G}_{21} & \dots & -\mathbf{G}_{Q1} \\ -\mathbf{G}_{12} & \mathbf{G}_2 & \dots & -\mathbf{G}_{Q2} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathbf{G}_{1Q} & -\mathbf{G}_{2Q} & \dots & \mathbf{G}_Q \end{bmatrix} \tag{3.25}$$

is an $\mathbf{M}$*-matrix.*

Proof. First, if the NE of game $\mathcal{G}$ exists, then an intersection point of the BR functions must exist. At the NE, (3.15) is equivalent to

$$\mathbf{G}_q \mathbf{p}_q^\star - \sum_{m \neq q}^{Q} \mathbf{G}_{mq} \mathbf{p}_m^\star = \mathbf{1}\sigma^2, \ \forall q.$$

Reorganizing the above set of equations into a matrix form, one has

$$\mathbf{G}\mathbf{p}^\star = \mathbf{1}\sigma^2,$$

where $\mathbf{G}$ is previously defined. Note that $\mathbf{G}$ is a $\mathbf{Z}$-matrix, as its off-diagonal elements are all non-positive. Since there exists $\mathbf{p}^\star > 0$ to make $\mathbf{G}\mathbf{p}^\star = \mathbf{1}\sigma^2 > 0$, this implies $\mathbf{G}$ being an $\mathbf{M}$-matrix by its characterization (Condition I_{28}, Theorem 6.2.3 in [68]).

Conversely, if $\mathbf{G}$ is an $\mathbf{M}$-matrix, its inverse exists and is a positive matrix [68]. Thus, there exists a vector $\mathbf{p}^\star = \mathbf{G}^{-1}\mathbf{1}\sigma^2 > 0$, and $\mathbf{p}^\star$ satisfies the condition of being an intersection point of the BR functions in (3.15). As a result, an NE must exist. □

As previously mentioned, there are various characterizations of an $\mathbf{M}$-matrix [68] that one can utilize to verify whether matrix $\mathbf{G}$ is one of the type.

3.4 A Comparison to the Coordinated Design

In Section 3.3, we considered the fully decentralized approach in the multi-cell downlink design and established the NE of the multicell precoding game. However, it is well-known that the NE need not be Pareto-efficient [52]. Via the coordination among the BSs, significant power reduction can be obtained by jointly designing all the beamformers at the same time. Nonetheless, this advantage may come with the cost of message passing among the BSs as explained later in this section. In the following, we review such a fully coordinated multicell downlink beamforming system [10], where the weighted total transmit power of all the cells is jointly minimized. A comparison between the two designs is presented in the end of the section.

Let $\mathbf{u}_{q_i}$ be the beamforming vector for user-i of cell-q with the coordinated design, and let $\mathbf{U}_q = [\mathbf{u}_{q_1}, \ldots, \mathbf{u}_{q_K}]$. The problem of jointly minimizing the weighted total transmit power of the Q cells is stated as follows

$$\begin{aligned} \underset{\mathbf{U}_1,\ldots,\mathbf{U}_Q}{\text{minimize}} \quad & \sum_{q=1}^{Q} \omega_q \big\|\mathbf{U}_q\big\|_F^2 \\ \text{subject to} \quad & \frac{\big|\mathbf{u}_{q_i}^H \mathbf{h}_{qq_i}\big|^2}{\sum_{j\neq i}^{K} \big|\mathbf{u}_{q_j}^H \mathbf{h}_{qq_i}\big|^2 + \sum_{m\neq q}^{Q}\sum_{j=1}^{K} \big|\mathbf{u}_{m_j}^H \mathbf{h}_{mq_i}\big|^2 + \sigma^2} \geq \gamma_{q_i}, \end{aligned} \tag{3.26}$$

where ω_q is the weight factor at BS-q, and $\sum_{q=1}^{Q} \omega_q = 1$. For a given $\boldsymbol{\omega} = [\omega_1, \ldots, \omega_Q] \geq 0$, the optimal solution to (3.26) represents an optimal trade-off point between the cells' power consumptions. Certainly, this point is Pareto-optimal, *i.e.*, one cannot further reduce the power consumption at one cell without increasing the power consumption at (one or more) other cells.

The optimization problem (3.26) is convex, since the objective function is convex and the SINR constraints can be transformed into convex second order conic (SOC) constraints [10]. Thus, its optimal solution can be obtained using any conic solution package or standard convex optimization algorithm. This approach, however, is fully centralized. On the other hand, by exploiting the dual problem, this problem can be solved in a distributed fashion with message passing among the BSs. Via the Lagrangian technique, the dual problem of (3.26) is equivalent to the virtual dual uplink problem [10]

$$\begin{aligned} \underset{\{\nu_{q_i}\},\{\hat{\mathbf{u}}_{q_i}\}}{\text{minimize}} \quad & \sum_{q=1}^{Q}\sum_{i=1}^{K} \nu_{q_i}\sigma^2 \\ \text{subject to} \quad & \frac{\nu_{q_i}\big|\hat{\mathbf{u}}_{q_i}^H \mathbf{h}_{qq_i}\big|^2}{\sum_{m=1}^{Q}\sum_{j=1}^{K} \nu_{m_j}\big|\hat{\mathbf{u}}_{q_i}^H \mathbf{h}_{qm_j}\big| + \omega_q \hat{\mathbf{u}}_{q_i}^H \hat{\mathbf{u}}_{q_i}} \geq \frac{\gamma_{q_i}}{1+\gamma_{q_i}}, \end{aligned} \tag{3.27}$$

where the optimization is taken over the sum transmit power of the uplink powers ν_{q_i}'s and the receive beamformer vectors $\hat{\mathbf{u}}_{q_i}$'s. Note that the optimal solution of the uplink powers ν_{q_i}'s can be obtained as a unique fixed point of the following iteration [10]

$$\nu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\mathbf{h}_{qq_i}^H \left(\boldsymbol{\Sigma}_q\big(\{\nu_{m_j}^{(n)}\}\big)\right)^{-1} \mathbf{h}_{qq_i}} \tag{3.28}$$

with

$$\boldsymbol{\Sigma}_q\big(\{\nu_{m_j}^{(n)}\}\big) = \sum_{m=1}^{Q}\sum_{j=1}^{K} \nu_{m_j}^{(n)} \mathbf{h}_{qm_j}\mathbf{h}_{qm_j}^H + \omega_q \mathbf{I}.$$

This iteration function is shown to be *standard* [10], which guarantees its convergence to a unique solution, if the problem is feasible [51]. It is to be noted that the feasibility study of this coordinated design has not been done in [10]. However, if the NE of the competitive design exists, the coordinated design must be feasible. Similar to the single-cell problem, given the optimal uplink transmit power $\nu_{q_i}^\star$, the optimal receive beamformers $\hat{\mathbf{u}}_{q_i}$ is the MMSE receiver, *i.e.*,

$$\hat{\mathbf{u}}_{q_i} = \left(\boldsymbol{\Sigma}_q\big(\{\nu_{m_j}^\star\}\big)\right)^{-1} \mathbf{h}_{qq_i}. \tag{3.29}$$

In addition, the optimal beamformer of the coordinated design, $\mathbf{u}_{q_i}$, can be found as a scaled version of $\hat{\mathbf{u}}_{q_i}$ by a factor $\sqrt{\delta_{q_i}}$ [10], *i.e.*, $\mathbf{u}_{q_i} = \sqrt{\delta_{q_i}}\hat{\mathbf{u}}_{q_i}$. As all the SINR constraints in (3.26) are met with equality at optimality, substituting $\mathbf{u}_{q_i} = \sqrt{\delta_{q_i}}\hat{\mathbf{u}}_{q_i}$, the KQ SINR constraints can be written as

$$\delta_{q_i}\frac{\left|\hat{\mathbf{u}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K} \delta_{q_j}\left|\hat{\mathbf{u}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 - \sum_{m\neq q}^{Q}\sum_{j=1}^{K} \delta_{m_j}\left|\hat{\mathbf{u}}_{m_j}^H \mathbf{h}_{mq_i}\right|^2 = \sigma^2. \tag{3.30}$$

In order to find δ_{q_i}, one needs to solve this set of KQ equations. Define a matrix $\mathbf{F}$ of size $KQ \times KQ$, and its components as

$$[\mathbf{F}]_{K(q-1)+i,K(m-1)+j} = \begin{cases} \frac{\left|\hat{\mathbf{u}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\gamma_{q_i}} & \text{, if } q=m, i=j \\ -\left|\hat{\mathbf{u}}_{m_j}^H \mathbf{h}_{mq_i}\right|^2 & \text{, otherwise} \end{cases}$$

with $i, j = 1, \ldots, K$, and $q, m = 1, \ldots, Q$. Then, one has

$$\delta = [\delta_{1_1}, \ldots, \delta_{1_K}, \ldots, \delta_{Q_1}, \ldots, \delta_{Q_K}]^T = \mathbf{F}^{-1}\mathbf{1}\sigma^2. \tag{3.31}$$

In summary, with the fully coordinated multicell system, the following three-step algorithm is needed to find the jointly optimal beamformers [10]:

(i) Apply the iteration (3.28) to find the fixed point $\nu_{q_i}^{\star}$.

(ii) Find the receive beamformer $\hat{\mathbf{u}}_{q_i}$ for the dual uplink channel as in (3.29).

(iii) Find the scaling factor δ_{q_i}.

In [10], the authors argued that the above algorithm can be implemented in a distributed manner under the condition of channel reciprocity. More specifically, when the uplink and downlink channels are reciprocal of each other, the virtual dual uplink is the real uplink. Thus, the iteration (3.28) in step (i) can be performed locally at each BS with local information. In particular, at BS-q, $\mathbf{h}_{qq_i}$ is typically known and $\mathbf{\Sigma}_q$ is effectively the covariance matrix of the received signal in the uplink direction. In step (ii), the receive beamformer $\hat{\mathbf{u}}_{q_i}$ can be easily obtained. Finally, step (iii) can be implemented iteratively where each δ_{q_i} is determined locally to meet its corresponding SINR target (assuming all other δ_{q_i}'s are fixed) until convergence. Overall, the distributed implementation of the coordinated design requires channel reciprocity and the synchronization among the BSs to be in the uplink phase or the downlink phase together [10]. It is worth mentioning that in practice the uplink and downlink channels usually operate in separate frequency bands in the frequency division duplexing (FDD) mode. Channel reciprocity is therefore hard to realize.

Obviously, if the condition on channel reciprocity is not true, each BS in the coordinated design needs to know all the channels from itself to all the MSs in the system. A message passing scheme among the cells is then required to jointly update the dual variables ν_{q_i}'s. In addition, certain synchronization among the BSs is desired, *i.e.*, step (iii). These are the main differences to the competitive design considered previously in this chapter, where the beamforming design is performed locally at each cell without any message exchanges and synchronization. These differences prompt us to investigate a new game that retains the advantages of the power minimization game $\mathcal{G}$, *i.e.*, fully distributed implementation, no message passing, and no synchronization, and possibly approaches the performance established by the coordinated design. We address this concern in the next section.

3.5 The Multicell Downlink Beamforming Game with Pricing

3.5.1 Problem Formulation

We begin this section with a numerical example of the power consumption at a two-cell system with both the coordinated and competitive designs. Considered is a system with two cells and two MSs per cell with target SINR $\gamma_{q_i} = 10$ (10dB). It is assumed that each BS is equipped with 3 antennas and the distance between the two BSs is normalized to 2. Each MS is located between the two BSs at a distance $d = 0.6$ from its connected BS. All the intra-cell and inter-cell channel coefficients are generated from i.i.d. Gaussian random variables, using the path-loss model with the path-loss exponent of 3. The background noise power σ^2 is 0.01. Figure 3.2 displays the power consumption at the two BS with the competitive design, *i.e.*, the NE point of game $\mathcal{G}$, relatively to the coordinated design. It should be noted that the power consumption at the cell is lower-bounded by the minimum power requirement to meet its users' SINR target, in the absence of ICI. It can be drawn from the figure that the NE point of the competitive design is relatively inefficient, compared to the Pareto-optimal curve established by the coordinated design.

An interesting question here is whether one can modify the utility function at each player such that the game becomes more cooperative and its equilibria possibly lie on the boundary of the Pareto-optimal trade-off surface, *e.g.*, point "$\circ$" in Figure 3.2. In this section, we study a new game with pricing consideration, namely game $\mathcal{G}'$. By introducing a pricing component to each player's utility function, the players now voluntarily cooperate with others by minimizing their inducing interference to the others as well as minimizing their own transmit power at the same time. In fact, it shall be shown that the point "$\circ$" can be obtained at the NE of the modified game $\mathcal{G}'$.

Now, suppose that BS-q has additional information about the channel to the users in

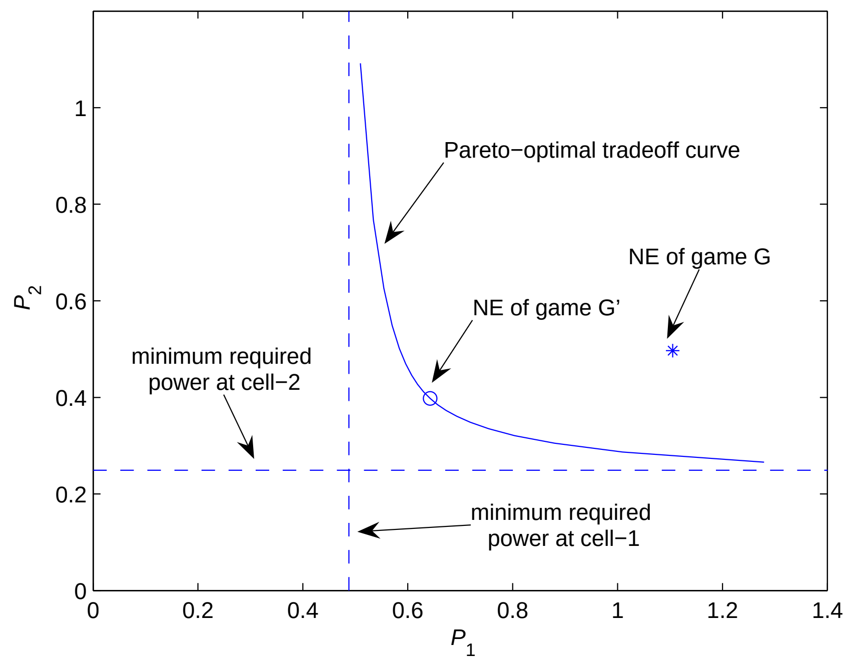

Fig. 3.2 Power consumption in two cells: competitive design vs. coordinated design.

other cells, and it performs the following optimization

$$\begin{aligned} \underset{\mathbf{V}_q}{\text{minimize}} \quad & \sum_{i=1}^{K} \|\mathbf{v}_{q_i}\|^2 + \sum_{m\neq q}^{Q} \sum_{j=1}^{K} \pi_{qm_j} \left\|\mathbf{V}_q^H \mathbf{h}_{qm_j}\right\|^2 \\ \text{subject to} \quad & \frac{\left|\mathbf{v}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \left|\mathbf{v}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i}, \ \forall i, \end{aligned} \tag{3.32}$$

where $\pi_{qm_j} \geq 0$ is the pricing factor and $\left\|\mathbf{V}_q^H \mathbf{h}_{qm_j}\right\|^2$ is the interference at user-j of cell-m, caused by BS-q.[5]

It is to be noted that unlike the coordinated design in Section 3.4, this downlink beamforming game can be implemented at the system where partial information is available. More specifically, if the channel to user-i at cell-m is known at BS-q, a pricing factor

[5] To avoid any confusion with the notation $\mathbf{w}_{q_i}$ used in Section 3.3 and $\mathbf{u}_{q_i}$ used in Section 3.4, we denote $\mathbf{v}_{q_i}$ as the beamformer for user-i of cell-q in the competitive design with pricing consideration within this section. Similarly, $\mathbf{V}_q$ is used instead of $\mathbf{W}_q$ and $\mathbf{U}_q$. Likewise, let $\mathbf{q}_q \in \mathbb{R}^K$ denote the allocated power vector for the K users in cell-q, instead of $\mathbf{p}_q$ in Section 3.3.

$\pi_{qm_j} > 0$ is set to motivate cell-q to adopt a more sociable strategy by steering its beamformers to the directions that cause less interference (damage) to other cells. Otherwise, the pricing factor π_{qm_j} is set to 0. Certainly, the nature of the game being played among the players is no longer purely competitive. Through pricing, it is possible to improve system performance by inducing cooperation among the players, and yet maintaining the decentralized nature of the game. In general, the pricing factors π_{qm_j} should be tuned such that a largest possible enhancement in the overall system is obtained [39]. In a dynamic pricing scheme, the pricing factors can be jointly decided and constantly exchanged among the BSs. However, in order to reduce the system overhead, it is assumed that the prices are chosen a *priori* and remain fixed during the game being played. This assumption may be motivated by a system with a system designer, who informs the prices to the players in advance.

The game with pricing consideration is practically the same game as $\mathcal{G}$ with different payoff function. Mathematically, the new game is defined as

$$\mathcal{G}' = \left(\Omega,\ \{\mathcal{P}_q(\mathbf{V}_{-q})\}_{q\in\Omega},\ \{s_q(\mathbf{V}_q)\}_{q\in\Omega}\right),$$

where $s_q(\mathbf{V}_q) = t_q(\mathbf{V}_q) + \sum\limits_{m\neq q}^{Q}\sum\limits_{j=1}^{K}\pi_{qm_j}\left\|\mathbf{V}_q^H\mathbf{h}_{qm_j}\right\|^2$ is the utility function at player-q. Our interest in this part is to study whether game $\mathcal{G}'$ eventually converges to an NE and whether the NE is unique. A feasible strategy profile $\{\mathbf{V}_q^\star\}_{q=1}^Q$ is an NE of game $\mathcal{G}'$ if

$$s_q(\mathbf{V}_q^\star) \leq s_q(\mathbf{V}_q),\ \forall\mathbf{V}_q \in \mathcal{P}_q(\mathbf{V}_{-q}^\star), \quad \forall q \in \Omega. \tag{3.33}$$

3.5.2 Existence and Uniqueness of the Nash Equilibrium

This section studies the existence and uniqueness of the NE of the new game $\mathcal{G}'$. First of all, given the strategy of other player $\mathbf{V}_{-q}$, we study the optimal strategy for player-q, *i.e.*, solving the optimization problem (3.32). Note that problem (3.32) is convex, as the constraints are SOC and the objective function is quadratic. This useful observation enables us to find its optimal solution via convex optimization. In addition, uplink-downlink duality can be exploited to devise the optimal solution for this problem. The following theorem establishes the analytical steps to find the solution.

Theorem 3.1. *The optimal transmit beamforming problem* (3.32) *can be solved via a dual* *virtual uplink channel*

$$\begin{aligned} \underset{\substack{\mu_{q_1},\ldots,\mu_{q_K}\\ \hat{\mathbf{v}}_{q_1},\ldots,\hat{\mathbf{v}}_{q_K}}}{\text{minimize}} \quad & \sum_{i=1}^{K} \mu_{q_i} r_{-q_i} \\ \text{subject to} \quad & \frac{\mu_{q_i}\left|\hat{\mathbf{v}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K}\mu_{q_j}\left|\hat{\mathbf{v}}_{q_i}^H \mathbf{h}_{qq_j}\right|^2 + \hat{\mathbf{v}}_{q_i}^H \boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big)\hat{\mathbf{v}}_{q_i}} \geq \gamma_{q_i}, \forall i, \end{aligned} \tag{3.34}$$

where $\boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) = \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\pi_{qm_j}\mathbf{h}_{qm_j}\mathbf{h}_{qm_j}^H + \mathbf{I}$ *is treated as the noise covariance matrix at the BS, and the optimization is taken over the weighted sum-power of the uplink power* μ_{q_i} *and the receive beamformer vectors* $\hat{\mathbf{v}}_{q_i}$*. The optimal* $\mathbf{v}_{q_i}$ *is a scaled version of the optimal* $\hat{\mathbf{v}}_{q_i}$*.*

Proof. The proof of this theorem is based on the Lagrangian technique and similar to the one in [21]. The Lagrangian of (3.32) can be reorganized as

$$\mathcal{L}_q(\mathbf{V}_q, \boldsymbol{\mu}_q) = \sum_{i=1}^{K}\mu_{q_i} r_{-q_i} + \sum_{i=1}^{K}\mathbf{v}_{q_i}^H\left(\boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) - \frac{\mu_{q_i}}{\gamma_{q_i}}\mathbf{h}_{qq_i}\mathbf{h}_{qq_i}^H + \sum_{j\neq i}^{K}\mu_{q_j}\mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H\right)\mathbf{v}_{q_i},$$

where $\boldsymbol{\mu}_q = [\mu_{q_1}, \ldots, \mu_{q_K}]^T$'s are the Lagrangian multipliers associated with SINR constraints. The dual objective function is defined as

$$g_q(\boldsymbol{\mu}_q) = \min_{\mathbf{V}_q} \mathcal{L}_q(\mathbf{V}_q, \boldsymbol{\mu}_q).$$

Obviously, if $\boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) - \frac{\mu_{q_i}}{\gamma_{q_i}}\mathbf{h}_{qq_i}\mathbf{h}_{qq_i}^H + \sum_{j\neq i}^{K}\mu_{q_j}\mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H$ is not positive semi-definite, there exists a set of $\mathbf{v}_{q_i}$ to make g_q unbounded from below. Thus, the dual problem of (3.32) is

$$\begin{aligned} \underset{\mu_{q_1},\ldots,\mu_{q_K}}{\text{maximize}} \quad & \sum_{i=1}^{K}\mu_{q_i} r_{-q_i} \\ \text{subject to} \quad & \sum_{j=1}^{K}\mu_{q_j}\mathbf{h}_{qq_j}\mathbf{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) \succeq \left(1 + \frac{1}{\gamma_{q_i}}\right)\mu_{q_i}\mathbf{h}_{qq_i}\mathbf{h}_{qq_i}^H, \ \forall i. \end{aligned} \tag{3.35}$$

Similar to the technique used to solve the traditional downlink beamforming problem

[19], the dual problem (3.35) is equivalent to the virtual uplink problem given in (3.34). Once again, the fixed point iteration can be utilized to find the optimal solution of μ_{q_i} as follows

$$\mu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\mathbf{h}_{qq_i}^H \left(\sum_{j=1}^{K} \mu_{q_j}^{(n)} \mathbf{h}_{qq_j} \mathbf{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) \right)^{-1} \mathbf{h}_{qq_i}}. \tag{3.36}$$

Using the *standard* function property, this iteration is guaranteed to converge to a unique fixed-point if the primal problem (3.32) is feasible.[6]

The optimal receive beamformer for the virtual uplink channel is indeed the MMSE receiver

$$\hat{\mathbf{v}}_{q_i} = \left(\sum_{j=1}^{K} \mu_{q_j}^{(n)} \mathbf{h}_{qq_j} \mathbf{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\big(\{\pi_{qm_j}\}\big) \right)^{-1} \mathbf{h}_{qq_i}. \tag{3.37}$$

In addition, using the similar technique as in [21], it can be shown that $\mathbf{v}_{q_i}$ is a scaled version of $\hat{\mathbf{v}}_{q_i}$, *i.e.*, $\mathbf{v}_{q_i} = \sqrt{\varepsilon_{q_i}}\hat{\mathbf{v}}_{q_i}$ where $\sqrt{\varepsilon_{q_i}}$ is the scaling factor. One can find the scaling factor by noticing that all the SINR constraints in (3.32) are met with equality at optimality. Substitute $\mathbf{v}_{q_i} = \sqrt{\varepsilon_{q_i}}\hat{\mathbf{v}}_{q_i}$ into the SINR constraints, one can rewrite them as

$$\varepsilon_{q_i} \frac{\left|\hat{\mathbf{v}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K} \varepsilon_{q_j} \left|\hat{\mathbf{v}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2 = r_{-q_i},\ i = 1, \ldots, K. \tag{3.38}$$

Define $\boldsymbol{\varepsilon}_q = [\varepsilon_{q_1}, \ldots, \varepsilon_{q_K}]^T$, then $\boldsymbol{\varepsilon}_q = \mathbf{E}^{-1}\mathbf{r}_{-q}$, where $\mathbf{E} \in \mathbb{R}^{K\times K}$ is defined as $[\mathbf{E}]_{i,i} = (1/\gamma_{q_i})\left|\hat{\mathbf{v}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2$ and $[\mathbf{E}]_{i,j} = -\left|\hat{\mathbf{v}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2$ if $i \neq j$. □

Having solved problem (3.32), it is clear that its solution resembles the solution of the typical downlink beamforming problem (3.4). Thus, many properties of problem (3.4)'s solution, *i.e.*, Claims 3.1-3.4, also hold. We summarize this observation in the following lemma.

Lemma 3.2. *Given fixed pricing factors π_{qm_j}'s, Claims 3.1-3.4 associated with the solution of problem* (3.4) *are also applicable to the solution of problem* (3.32).

Proof. Claim 3.1 is straightforward. As the feasibility depends only on the constraints, if problem (3.4) is feasible, problem (3.32) is also feasible. Thus, Claim 3.2 also holds.

[6] The proof for this function to be *standard* is similar to the ones in [19].

Claim 3.3 comes directly from the fact that the solution $\mathbf{v}_{q_i}$ of problem (3.4) is a scaled version of $\hat{\mathbf{v}}_{q_i}$ given in (3.37). Claim 3.4 also holds, following the same procedure given in Section 3.3.2. That is, given $\mathbf{r}_{-q}$ as the total interference induced by Ω_{-q} plus background noise and $\widetilde{\mathbf{v}}_{q_i}$ as the beam pattern corresponding to π_{qm_j}, the optimal allocated power vector $\mathbf{q}_q \in \mathbb{R}^K$ for the K users at BS-q is $\mathbf{q}_q = \mathbf{K}^{-1}\mathbf{r}_{-q}$, where $\mathbf{K} \in \mathbb{R}^{K\times K}$ is defined as $[\mathbf{K}]_{i,i} = (1/\gamma_{q_i})\left|\widetilde{\mathbf{v}}_{q_i}^H \mathbf{h}_{qq_i}\right|^2$ and $[\mathbf{K}]_{i,j} = -\left|\widetilde{\mathbf{v}}_{q_j}^H \mathbf{h}_{qq_i}\right|^2$ if $i \neq j$. □

Denote $\mathbf{K}_{mq} \in \mathbb{R}^{K\times K}, m \neq q$ as the ICI matrix, where $[\mathbf{K}_{mq}]_{i,j} = |\widetilde{\mathbf{v}}_{m_j}^H \mathbf{h}_{mq_i}|^2$. Then, one has $\mathbf{r}_{-q} = \sum_{m\neq q}^{Q} \mathbf{K}_{mq}\mathbf{q}_m + \mathbf{1}\sigma^2$. From Lemma 3.2, subject to the strategy of Ω_{-q}, the best response strategy of the player-q with pricing consideration is

$$\mathbf{q}_q^\star = \mathrm{BR}'_q\big(\mathbf{q}_{-q}\big) = \mathbf{K}_q^{-1}\left(\sum_{m\neq q}^{Q} \mathbf{K}_{mq}\mathbf{q}_m + \mathbf{1}\sigma^2\right). \tag{3.39}$$

Lemma 3.3. *With pricing consideration, the best response function of player-q is standard.*

Proof. The proof is the same as that in Lemma 3.1. □

Since the best response function $\mathrm{BR}'_q\big(\mathbf{q}_{-q}\big)$ is *standard*, from any starting point $\mathbf{q}^{(0)}$, the iteration $\mathbf{q}_q^{(t+1)} = \mathrm{BR}'_q\big(\mathbf{q}_{-q}^{(t)}\big)$ will surely converge to a fixed point (if it exists), which is the NE of game $\mathcal{G}'$. The necessary condition for the existence of the NE in game $\mathcal{G}'$ is similar to that of game $\mathcal{G}$, established in Proposition 3.2, *i.e.*, the matrix $\mathbf{K} \in \mathbb{R}^{KQ\times KQ}$, in the same form as $\mathbf{G}$ in (3.25) with $\mathbf{K}_q$ and $\mathbf{K}_{mq}$ replacing $\mathbf{G}_q$ and $\mathbf{G}_{mq}$, is an $\mathbf{M}$-matrix. Likewise, thanks to Proposition 3.1, if $\rho(\mathbf{S}') < 1$, where $\mathbf{S}' \in \mathbb{R}^{K\times K}$ is defined as $[\mathbf{S}']_{qq} = 0$ and $[\mathbf{S}']_{q,m} = \left\|\mathbf{K}_q^{-1}\mathbf{K}_{mq}\right\|_F$ if $m \neq q$, game $\mathcal{G}'$ is also guaranteed to admit a unique NE.

To this point, one may wonder how efficient the NE of game $\mathcal{G}'$ is compared to the NE of game $\mathcal{G}$ and the Pareto-optimal trade-off curve. Although this work does not present a concrete proof to the claim that the NE of game $\mathcal{G}'$ is more efficient than that of game $\mathcal{G}$, all simulations show that with right pricing factors, this claim is true. In fact, with a certain pricing scheme deployed at all the BSs, the NE of game $\mathcal{G}'$ is able to approach the Pareto-optimal trade-off curve. The characterization of this pricing scheme is given in the following.

Theorem 3.2. *Given the weight vector $\boldsymbol{\omega}$ for the coordinated design, and suppose that $\nu_{q_i}^\star$'s are the dual variables that satisfy the iteration* (3.28)*, if the weight factors for game $\mathcal{G}'$ are* *set as*

$$\pi_{qm_j} = \frac{\nu^{\star}_{m_j}}{\omega_q}, \quad \forall i, \ \forall q, \tag{3.40}$$

the NE of game $\mathcal{G}'$ is exactly the solution of the coordinated design. That is, the NE lies on the Pareto-optimal trade-off curve.

Proof. When the weight factors for game $\mathcal{G}'$ are set as in (3.40), the fixed-point iteration (3.36) becomes

$$\omega_q \mu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\mathbf{h}_{qq_i}^H \left(\sum_{j=1}^{K} \omega_q \mu_{q_j}^{(n)} \mathbf{h}_{qq_j} \mathbf{h}_{qq_j}^H + \omega_q \boldsymbol{\Upsilon}_q \big(\{ \nu^{\star}_{m_j} / \omega_q \} \big) \right)^{-1} \mathbf{h}_{qq_i}}. \tag{3.41}$$

Comparing to fixed-point iteration (3.28), it is obvious that the unique fixed-point of the above iteration satisfies $\omega_q \mu^{\star}_{q_i} = \nu^{\star}_{q_i}$. As a result, $\hat{\mathbf{v}}_{q_i} = \omega_q \hat{\mathbf{u}}_{q_i}$. That is, the beam pattern set for the users of game $\mathcal{G}'$ is the same as the beam pattern set in the coordinated design. Thus, it is left to determine that the beamformers of the two designs are indeed the same. Note that the coordinated design determines the scaling factor δ_{q_i} to $\hat{\mathbf{u}}_{q_i}$ by either using matrix inversion, c.f. equation (3.31), or each MS sets a δ_{q_i} to meet its corresponding SINR constraint assuming all other δ_{q_i}'s are fixed [10]. The convergence of the second method can be proved by the *standard* function technique [10], which effectively explains the convergence to a unique fixed-point. On the other hand, at each time instance, BS-q in game $\mathcal{G}'$ determines the allocated powers (equivalently the scaling factors to $\hat{\mathbf{v}}_{q_i}$) to satisfy the SINR constraints at its users. Due to the uniqueness of the fixed-point, the game played in $\mathcal{G}'$ has to converge to the same solution as the coordinated design. □

Theorem 3.2 is significant in the sense that the fully coordinated design can still be interpreted as a competitive game with the *right* pricing scheme. Our next task is to study how to implement such a pricing scheme, under two game scenarios: (i) game with complete information and (ii) game with incomplete information.

In a game with complete information, it is assumed that the system designer knows all the game parameters, including the channels and the QoS requirements. Each BS is also assumed to fully know its channels to all the MSs. The designer then can exactly decide the optimal dual variables $\nu^{\star}_{q_i}$'s, which are used to determine the optimal prices in (3.40). As stated in Theorem 3.2, the game with pricing consideration will be Pareto-optimal.

Interestingly, it has been recently shown in [70] that pricing can allow the system designer to locate the NE point to any feasible point in a broad class of power allocation games under complete information. Our result given in Theorem 2 has established a similar result for the beamforming and power allocation game in a multicell system.

In a game with incomplete information, it is assumed that neither the BSs nor the system designer fully know all the channels. Thus, implementing the pricing scheme (3.40) is no longer possible. In this case, the designer may help the BSs to search for good pricing factors. Here, we employ the same mechanism exploited in [39] due to its simplicity. More specifically, the designer lets the BSs play game $\mathcal{G}$ (no pricing) and obtain the NE. Then, each BS sets its pricing factors π_{qm_j} to a same value $c > 0$ (initially large), which is informed by the designer, and game $\mathcal{G}'$ is played between the BSs. After dividing the pricing factor c by a positive factor of Δc, game $\mathcal{G}'$ is played again and its NE is re-measured. The procedure is repeated if the sum of the utility functions at the new equilibrium is smaller than that of the previous instance. Otherwise, the procedure is stopped and all the pricing factors are set to the same factor, called c_{BEST}. As shall be shown in the simulation, this technique performs very well in improving the NE's efficiency.

3.6 Numerical Results

This section presents some numerical results to validate our findings. In particular, we compare the feasibility of the coordinated design and the probability of existence of an NE of games $\mathcal{G}$ and $\mathcal{G}'$. Also compared are the average total transmit powers (of all the cells) of the three designs. We consider a multicell network as illustrated in Figure 3.3, composed of 3 cells with 2 users per cell. It is assumed that the BSs are equidistant, and their distance is normalized to 2. The distance between a MS and its serving BS is also set the same, at d. Of the two MSs at each cell, one is located close to the borders with other cells, whereas the second one is far away. The same target SINRs are set at each MS, either $\gamma_{q_i} = 0$ dB or $\gamma_{q_i} = 10$ dB. The AWGN power spectral density σ^2 is set at 0.01. The channels from a BS to a MS are generated from i.i.d. Gaussian random variables using the path-loss model with the path-loss exponent of $\beta = 3$ and the reference distance of 1 corresponding to MSs at the cell-edge. As we vary the distance d, $10,000$ channel realizations at each value d are used to plot the probability of the existence of a stable operating point in Figure 3.4 and the average total transmit power in Figure 3.5.

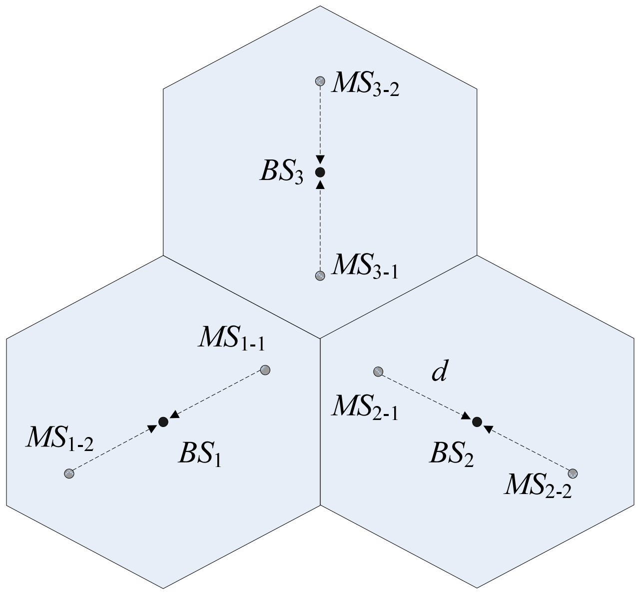

Fig. 3.3 A multicell system configuration with 3 cells, 2 users per cell. Of the two users at each cell, one stays close to the borders with other cells, one is far away.

In the competitive design with pricing consideration, it is assumed that each BS also knows the channels to the MSs that are close to the cell boundary at the other two cells. For example, base-station BS-1 knows its channels to MS-2_1 and MS-3_1. Certainly, these two MSs are subject to a much higher ICI level from cell-1 than the others. Each BS then takes advantage of this extra information to improve efficiency of the NE of game $\mathcal{G}'$ with pricing. The pricing scheme for games with incomplete information as discussed in Section 3.5 is applied in this simulation.

Figure 3.4 displays the probability of existence of a stable operating point as the function of the MS-BS distance d by evaluating whether condition (C) is satisfied and numerically examining the convergence of game $\mathcal{G}$ (which matches with condition (C1)), the coordinated design, and game $\mathcal{G}'$. From the figure, as the MSs get closer to their BS, a higher probability of the existence of a stable operating point for all three designs is observed. This is due to the fact that the stronger intra-cell channels allow a BS to transmit at a lower power level to meet its target SINRs, which then causes lower ICI. With the competitive design, a lower level of ICI certainly guarantees a higher probability of existence of an NE. On the other hand, with the pricing consideration, the whole system may further reduce the ICI.

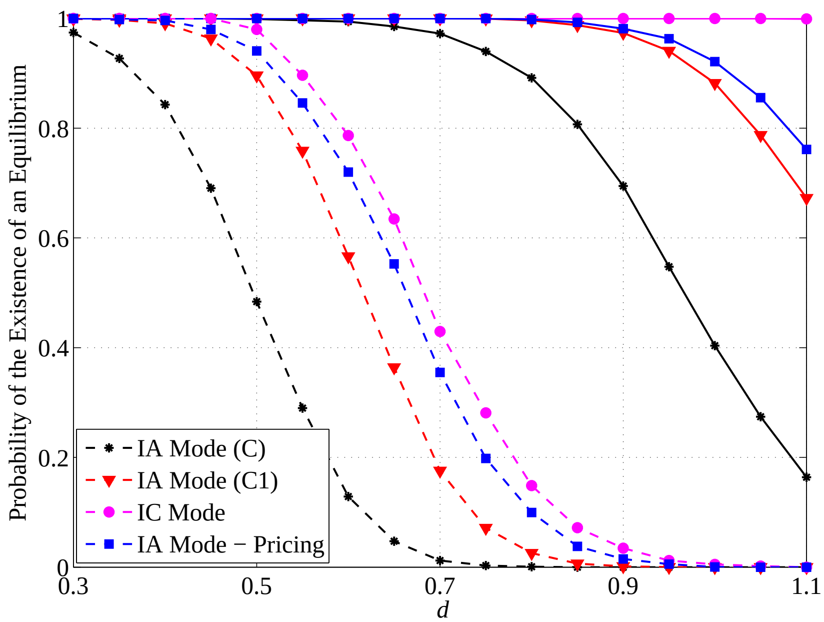

Fig. 3.4 Probability of existence of a stable operating point versus d by evaluating conditions (C) and (C1) and numerically examining the convergence of game $\mathcal{G}$, the coordinated design, and game $\mathcal{G}'$ to meet the target SINRs: $\gamma_{q_i} = 10$ dB (dashed lines) and $\gamma_{q_i} = 0$ dB (solid lines).

As a result, the existence probability of game $\mathcal{G}'$ is higher than that of game $\mathcal{G}$. Finally, with the coordinated design, where the ICI is fully managed, the feasibility of finding a solution that meets all the target SINRs is certainly higher than finding one in both games $\mathcal{G}$ and $\mathcal{G}'$. Note that that conditions (C) and (C1) can be easily examined to verify the existence of the NE of the competitive design. In case of not meeting conditions (C) or (C1), one may attempt to switch the network into the design with pricing consideration or the fully coordinated design to improve the convergence probability of the whole system. Figure 3.4 also shows that a lower target SINR, which requires lower transmit power (lower ICI), would induce a higher chance of finding the solutions in all three designs.

Of all cases where game $\mathcal{G}$ converges, the total transmit power P_{total} at the 3 BSs with three designs are averaged and compared in Figure 3.5 in the form of $P_{\text{total}}/\sigma^2$. Note that the weight factors ω_q's of the coordinated design are set equal to each other to minimize the design's sum transmit power. At small inter-cell MS-BS distances, it can be seen that the power usages of all the designs are very low and their difference is rather marginal.

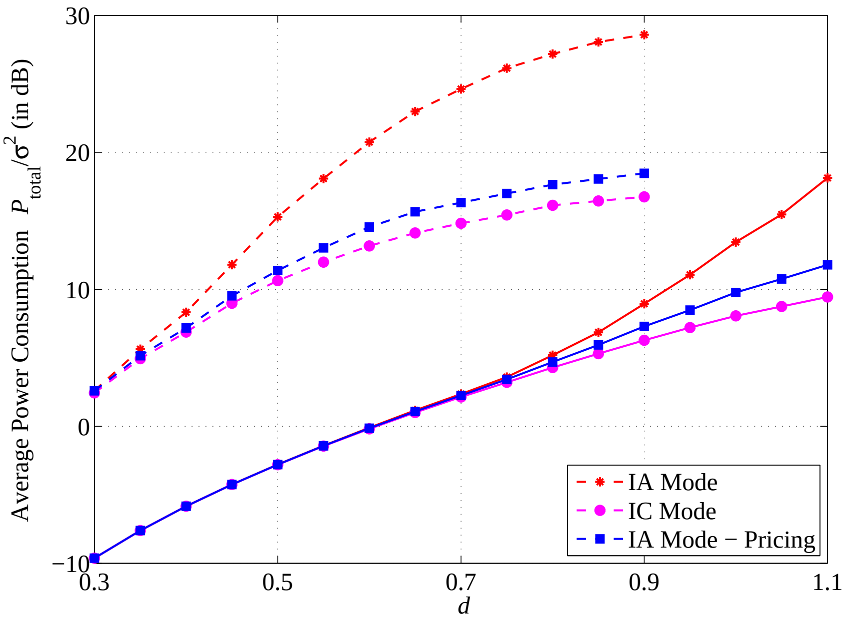

Fig. 3.5 Average total transmit power versus d of the competitive design, the coordinated design, and the competitive design with pricing consideration to meet the target SINRs: $\gamma_{q_i} = 10$ dB (dashed lines) and $\gamma_{q_i} = 0$ dB (solid lines).

Again, this is due to the fact that the intra-cell channels are strong and the ICI is too small. However, as d increases, the effect of ICI becomes significant. Since the competitive design does not attempt to control the ICI, its NE point becomes inefficient compared to the Pareto-optimal frontier established by the coordinated design. On the other hand, should a BS know the channels to the MSs at other cells, it can alter its strategy by playing the game with pricing consideration $\mathcal{G}'$. In fact, using the aforementioned procedure to determine the pricing factor, the NE of game $\mathcal{G}'$ is almost Pareto-optimal. It is worth noting that this result is obtained even thought each BS does not possess full channel knowledge from itself to all the users.

To illustrate the convergence behaviors of the multicell downlink beamforming games $\mathcal{G}$ and $\mathcal{G}'$ and compare the transmit powers of the two designs, we select two examples and display them in Figures 3.6 and 3.7. In both games, it is assumed that all the BSs perform simultaneous power update at each time instance. The transmit power of each cell is then displayed after each iteration. The system configuration is the one in Figure 3.3, with $\gamma_{q_i} = 10$ dB and $d = 0.6$.

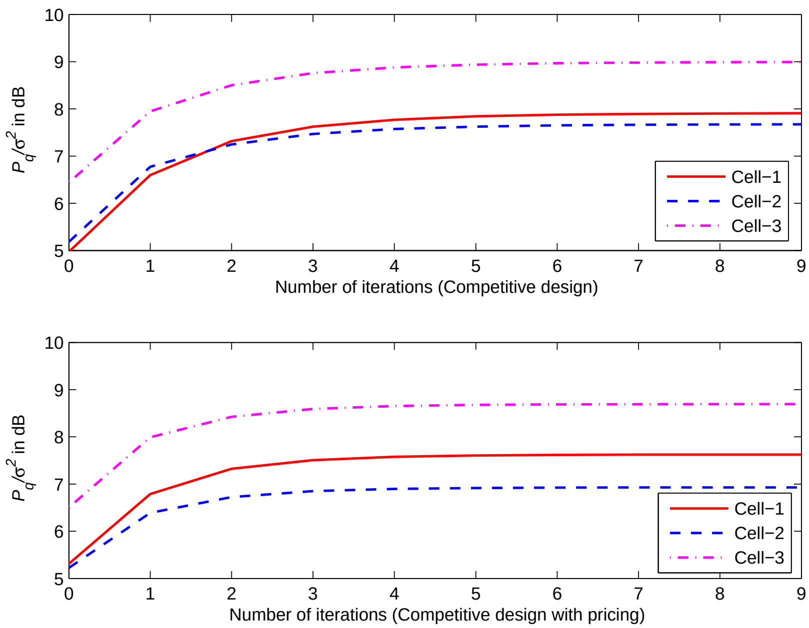

Fig. 3.6 A converging example of the downlink beamforming games $\mathcal{G}$ and $\mathcal{G}'$ in a multicell system: the sum power of each cell versus the number of iterations with $\rho(\mathbf{S}) = 0.7332$, $\rho(\mathbf{S}') = 0.6256$, and the corresponding matrices $\mathbf{G}$ and $\mathbf{K}$ are $\mathbf{M}$-matrices.

In the first example, $\rho(\mathbf{S})$ and $\rho(\mathbf{S}')$ are calculated at 0.7332 and 0.6256, respectively. The power updates of both games displayed in Figure 3.6 clearly show the convergence of the two designs. Figure 3.6 also shows the benefit of using the design with pricing consideration, where the transmit power at each cell is reduced, compared to that of the purely competitive design. For this particular example, the price is set at 0.1388.

In the second example, $\rho(\mathbf{S})$ and $\rho(\mathbf{S}')$ are calculated at 2.0016 and 0.9815, respectively. In can be seen from Figure 3.7 that the power updates of game $\mathcal{G}$ do not converge. Interestingly, with the price set at 0.551, the design with pricing consideration eventually converges. This behavior clearly indicates the benefit of adopting a more cooperative strategy at each cell by exploiting the extra channel information to other cells.

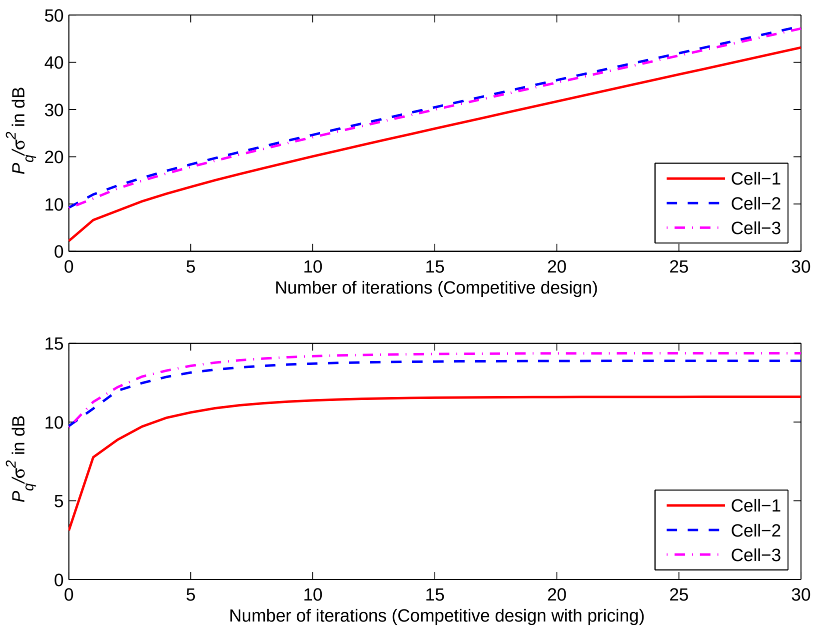

Fig. 3.7 An example of the downlink beamforming game that diverges in game $\mathcal{G}$ and but converges in game $\mathcal{G}'$: the sum power of each cell versus the number of iterations with $\rho(\mathbf{S}) = 2.0016$, and $\rho(\mathbf{S}') = 0.9815$. In this case, the corresponding matrix $\mathbf{K}$ is an $\mathbf{M}$-matrix, but $\mathbf{G}$ is not.

3.7 Concluding Remarks

This chapter has studied the problem of downlink beamformer designs in a multicell system via game theory. Given the QoS requirements at the users in its cell, each BS determines its optimal downlink beamformer strategy in a distributed manner, without any coordination among the cells. At first, we considered a fully competitive game, where each BS greedily minimizes its transmit power. We have examined necessary and sufficient conditions guaranteeing the existence and uniqueness of the NE of the game. In addition, a comparison between the competitive and coordinated designs was also presented. Finally, to improve the efficiency of the competitive game, a new game with pricing consideration was studied. Through the pricing mechanism, each BS can steer its beamformers in a more cooperative way to reduce its induced ICI to other cells. If each BS knows its channels to all MSs in the network, the new game is capable of obtaining the Pareto-optimal performance as the coordinated design, while retaining the distributed nature of a multicell game. Interestingly, the multicell beamforming game with pricing consideration can also be implemented with partial inter-cell channel knowledge.

Chapter 4

Block-Diagonalization Precoding in Multiuser Multicell MIMO Systems: Competition and Coordination

4.1 Introduction

In Chapter 3, we investigated multicell precoding designs with the objective of minimizing the transmit power at the BSs subject to SINR constraints at the MSs. In this chapter, we consider the multicell precoding designs to maximize the achievable data-rate of the MSs with power constraints at the BSs. In a MIMO system, space-division multiple-access (SDMA) can be applied at the BS to concurrently multiplex data streams for multiple MSs. With appropriate downlink precoding techniques at the BS, SDMA can significantly improve the system's spectral efficiency. Downlink precoding for a MIMO system has been an active area of resaerch for many years. Dirty-paper coding (DPC) [25–28] has been proved to be the capacity-achieving multiuser precoding strategy. However, due to its high complexity implementation that involves random nonlinear encoding and decoding, DPC can only serves a theoretical benchmark. Consequently, linear precoding techniques, such as zero-forcing (ZF) and block-diagonalization (BD) [34–37], become appealing alternatives

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due to their simplicity and good performance. With BD precoding, the transmitted signal from the BS intended for a particular MS is restricted to be in the null space created by the downlink channels associated with all the other MSs. Therefore, all inter-user interference at the MSs can be fully suppressed.

In a multicell system, the multiple BSs can fully cooperate to form a single large system with distributed antenna elements. This form of cooperation, also termed as *network MIMO*, requires all the BSs to share the CSI and data information among themselves via backhaul links as well as to coordinate their concurrent data information transmissions to all the MSs. Under the network MIMO context, BD precoding was investigated in [74, 75] with per-base-station power constraints. Specifically, the works in [74, 75] considered a multicell system where all the BSs form a single large BD precoder to remove all intra-cell and inter-cell interferences. A modified BD multicell precoding scheme was proposed in [76] to improve the performance of BD precoding in the low-to-medium signal-to-noise (SNR) region. While extracting a good performance from the multicell network, the benefits of network MIMO come at the expense of high complexity in joint precoding/decoding and ideal backhaul transmissions among the BSs for data and control signaling exchange [3].

Different from the studies in [74–76], this chapter investigates the multicell system where BD precoding is applied on a per-cell basis. Specifically, we consider the BD precoding schemes for the multicell system under the two operating modes: interference aware (IA) and interference coordination (IC) [5]. Under these two operating modes, each BS is required to transmit information data *only* to the MSs within its cell limits. The reason that we choose to study the BD precoding on a per-cell basis is due to simple implementation and good performance at the high SNR region [35].

Under the IA mode, each interference-aware MS shall measure the level of ICI and feed back this information to its connected BS [5]. Given the strategies from other BSs reflected by the ICI, each BS selfishly adjusts its precoding strategy to maximize the sum-rate for its connected MSs. Thus, the multicell system is said to be in *competition* since the BSs are competing with each other for the radio resource. Naturally, the IA mode represents a strategic noncooperative game (SNG) with the BSs being the rational players. The study of precoding design for the multicell system under the IA mode using game theory is plentiful in literature [12, 47, 48, 61, 71]. In general, these works focus on studying the existence and uniqueness of the game's Nash equilibrium (NE). The work in [47] studied the precoding game of a two-cell MISO system. In the multicell MIMO system, [12, 48] studied the competitive precoding design, where each cell selfishly maximizes its mutual information. It is to be noted that [12, 47, 48] only considered the system where each BS communicates with only one MS. For the case of multiple MSs per cell, [61] studied a multicell SNG where each BS selfishly minimizes its transmit power. The work in [71] investigated a SNG where each BS utilizes ZF precoding to maximize the sum-rate to the MSs within its cell. Both studies in [61, 71] only considered the case of single-antenna MSs. Different from these works, the consideration of BD precoding in this chapter allows us to examine a multicell SNG under a more general setting where there are multiple MSs per cell and each MS is equipped with multiple antennas. In order to characterize the multicell BD precoding game, we first present the best response strategy at the each BS in a closed-form WF solution. We then show that this WF best response strategy can be interpreted as a projection onto a closed and convex set. This interpretation shall allow us to study the uniqueness of the game's NE later on. It shall be shown that the game's NE always exists and is guaranteed to be unique under a certain condition on the ICI.

Under the IC mode, while each BS only transmits data information to the MSs within its cell limits, the precoders from all BSs are jointly designed to fully control the ICI [5]. Thus, the multicell system is said to be in *coordination* since the transmissions from the BSs are coordinated. In this work, we examine the BD precoding strategy that jointly maximizes the weighted sum-rate (WSR) of the multicell system under the IC mode. Since this WSR maximization problem is shown to be nonconvex, it is generally difficult and computationally complex to find its globally optimal solution. Thus, our focus is on proposing a low-complexity algorithm to approximate the nonconvex WSR maximization into a sequence of simpler convex problems. We then show that each of the simpler problems can be solved separately at the corresponding BS. In particular, each BS attempts to optimize its BD precoder to maximize the sum-rate for its connected MSs while doing its best in limiting the ICI induced to other cells through an interference-penalty mechanism. Simulation results show a significant improvement in the network sum-rate by the IC mode over the IA mode at the high ICI region.

In the latter part of this chapter, we examine the precoding design in a multiuser multicell where Block-Diagonalization - Dirty-Paper Coding (BD-DPC) is utilized in a per-cell basis. In BD-DPC, the information signals sent to the multiple users are encoded in sequence such that the receiver at any user does not see any inter-user interference due to the use of BD and DPC at the BS [26]. Thus, BD-DPC can take advantage of

DPC to enhance the performance of BD precoding. Under the IA mode, we attempt to characterize the NE of the BD-DPC multicell precoding game by examining the conditions for its existence and uniqueness. It will be shown that the game may have multiple NEs, depending on the encoding order in the BD-DPC precoding design at each BS. In addition, the condition for the uniqueness of the BD-DPC multicell precoding game is generally stricter than that of the BD one. Under the IC mode, we propose a numerical algorithm to maximize the WSR of the multicell system with BD-DPC precoding. In a conventional single-cell system, BD-DPC precoding can yield a better sum-rate performance over the BD precoding. Our thorough numerical simulations then confirm this observation for the multicell system under both IA and IC modes.

4.2 System Model

We consider a multiuser multicell downlink system with Q separate cells operating on the same frequency channel. At a particular cell, say cell-q, a multiple-antenna BS is concurrently sending independent information streams to K remote MSs, each equipped with multiple receive antennas. Let M and N be the numbers of antennas of the BS and the ith MS at cell-q, respectively. Denote $\mathbf{x}_q \in \mathbb{C}^{M\times 1}$ as the transmitted signal vector from BS-q. Assuming linear precoding at the BS, $\mathbf{x}_q$ can be represented as $\mathbf{x}_q = \sum_{i=1}^{K} \mathbf{W}_{q_i}\mathbf{s}_{q_i}$, where $\mathbf{W}_{q_i} \in \mathbb{C}^{M\times D_{q_i}}$ is the precoding matrix and $\mathbf{s}_{q_i} \in \mathbb{C}^{D_{q_i}\times 1}$ is the data symbol vector intended for MS-i. Without loss of generality, we assume $\mathbb{E}\left[\mathbf{s}_{q_i}\mathbf{s}_{q_i}^H\right] = \mathbf{I}, \forall i, \forall q$.

Let $\mathbf{H}_{rq_i} \in \mathbb{C}^{N\times M_r}$ model the channel coefficients from BS-r to MS-i of cell-q, and $\mathbf{z}_{q_i}$ model the zero-mean complex additive Gaussian noise vector with an arbitrary covariance matrix $\mathbf{Z}_{q_i}$. The transmission to MS-i at cell-q can be modeled as

$$\begin{aligned}\mathbf{y}_{q_i} &= \sum_{r=1}^{Q} \mathbf{H}_{rq_i}\mathbf{x}_r + \mathbf{z}_{q_i} \\ &= \mathbf{H}_{qq_i}\mathbf{W}_{q_i}\mathbf{s}_{q_i} + \mathbf{H}_{qq_i}\sum_{j\neq i}^{K}\mathbf{W}_{q_j}\mathbf{s}_{q_j} + \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K}\mathbf{W}_{r_j}\mathbf{s}_{r_j} + \mathbf{z}_{q_i}.\end{aligned} \tag{4.1}$$

It is observed from (4.1) that the received signal at MS-i of cell-q comprises of 4 components: the useful information signal $\mathbf{H}_{qq_i}\mathbf{W}_{q_i}\mathbf{s}_{q_i}$, the intra-cell interference $\mathbf{H}_{qq_i}\sum_{j\neq i}^{K}\mathbf{W}_{q_j}\mathbf{s}_{q_j}$, the ICI $\sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K}\mathbf{W}_{r_j}\mathbf{s}_{r_j}$, and the Gaussian noise $\mathbf{z}_{q_i}$. In this work, it is assumed that each MS can measure its total interference and noise power perfectly and constantly report back to its connected BS. The BS then utilizes this information to design its precoders accordingly for its connected MSs.

In the competitive design of this system model, it is assumed that each BS only possesses full knowledge of the downlink channels to the MSs in its cell, but not the channels to the MSs in other cells. As a result, the BS cannot control its induced ICI to other cells, which is then treated as background noise at the MSs. On the contrary, in the coordinated design of this system model, the BS also possesses the CSI to the MSs in the other cells. This additional channel knowledge allows the BS to control the ICI as well. Note that the BS can always fully manage the intra-cell interference within its cell by performing certain precoding techniques on a per-cell basis. In this work, we focus on the precoding technique that completely suppresses the intra-cell interference, namely BD precoding for multiple-antenna MSs [35]. To implement the BD precoding on a per-cell basis, it is assumed that the total number of receive antennas at the MSs does not exceed the number of transmit antennas at their connected BS, *i.e.*, $KN \leq M, \forall q$. If the number of receive antennas at a cell exceeds the number of transmit antennas, the BS can select a subset of MSs beforehand using low-complexity selection techniques such as [77, 78].

Let $\mathbf{Q}_{q_i} = \mathbf{W}_{q_i}\mathbf{W}_{q_i}^H$ be the transmit covariance matrix intended for MS-i of cell-q, and $\mathbf{Q}_q = \{\mathbf{Q}_{q_i}\}_{i=1}^K$ be the precoding profile for K MSs of cell-q. Likewise, let $\mathbf{Q}_{-q} = \{\mathbf{Q}_1, \ldots, \mathbf{Q}_{q-1}, \mathbf{Q}_{q+1}, \ldots, \mathbf{Q}_Q\}$ denote the precoding profile of all cells except cell-q. Denote by $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ the covariance matrix of the total IPN at MS-i of cell-q, which is defined as

$$\mathbf{R}_{q_i}(\mathbf{Q}_{-q}) = \sum_{r \neq q}^{Q} \mathbf{H}_{rq_i} \left(\sum_{j=1}^{K} \mathbf{Q}_{r_j} \right) \mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i}. \tag{4.2}$$

With BD precoding applied an a per-cell basis at BS-q, the achievable data rate R_{q_i} to MS-i is then given by

$$R_{q_i}(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log \left| \mathbf{I} + \mathbf{H}_{qq_i}^H \mathbf{R}_{q_i}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq_i} \mathbf{Q}_{q_i} \right|. \tag{4.3}$$

4.3 The Multicell Block-Diagonalization Precoding - Competitive Design

4.3.1 Problem Formulation

This section examines the multicell BD precoding under the competition mode, *i.e.*, the IA mode, where each BS selfishly designs its BD precoders without any coordination among the cells. We are interested in formulating this competitive multicell BD precoding design using the game-theory framework. In particular, we consider a SNG, where the players are the cells and the payoff functions are the sum-rates of the cells. In each cell, the BS strategically adapts its BD precoder on a per-cell basis to greedily maximize the sum-rate to its connected MSs, subject to a constraint on its transmit power.

Let $\Omega = \{1, \ldots, Q\}$ be the set of Q players. Define $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{i=1}^{K} R_{q_i}(\mathbf{Q}_q, \mathbf{Q}_{-q})$ as the payoff function of player-q. Then, given a strategy profile $\mathbf{Q}_{-q}$ from other players, player-q selfishly maximizes its payoff function by solving the following optimization problem

$$\begin{aligned} \underset{\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}}{\text{maximize}} \quad & R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{subject to} \quad & \sum_{i=1}^{K} \text{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q \\ & \mathbf{H}_{qq_j}\mathbf{Q}_{q_i}\mathbf{H}_{qq_j}^H = \mathbf{0}, \forall j \neq i \\ & \mathbf{Q}_{q_i} \succeq \mathbf{0}, \ \forall i, \end{aligned} \tag{4.4}$$

where P_q is the power budget at BS-q. To achieve the maximum sum data-rate at cell-q, it is assumed that the IPN matrix $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ is perfectly measured at the corresponding MS-i and reported back to its connected BS. Clearly, the optimization problem (4.4) shows that the optimal strategy of player-q does depend on the strategies of others. It is to be noted that the optimization (4.4) is carried with only local information (intra-cell CSI and signaling between the MSs and its connected BS). Thus, the BD precoding game is implemented in a fully distributed manner without any signaling exchanges among the BSs.

Due to the constraints $\mathbf{H}_{qq_i}\mathbf{Q}_{q_j}\mathbf{H}_{qq_i}^H = \mathbf{0}, \forall j \neq i$, each column of the precoder matrix $\mathbf{W}_{q_i}$ must be in the null space created by $\hat{\mathbf{H}}_{q_i} = [\mathbf{H}_{qq_1}^T, \ldots, \mathbf{H}_{qq_{i-1}}^T, \mathbf{H}_{qq_{i+1}}^T, \ldots, \mathbf{H}_{qq_K}^T]^T$. Suppose that one performs the singular value decomposition of the $(K-1)N \times M$ matrix $\hat{\mathbf{H}}_{q_i}$ as

$$\hat{\mathbf{H}}_{q_i} = \mathbf{U}_{q_i} \mathbf{\Sigma}_{q_i} \mathbf{V}_{q_i}^H = \mathbf{U}_{q_i} \begin{bmatrix} \tilde{\mathbf{\Sigma}}_{q_i}, & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_{q_i}^H \\ \hat{\mathbf{V}}_{q_i}^H \end{bmatrix}, \tag{4.5}$$

where $\tilde{\mathbf{\Sigma}}_{q_i}$ is diagonal, $\mathbf{U}_{q_i}$ and $\mathbf{V}_{q_i}$ are unitary matrices, and $\hat{\mathbf{V}}_{q_i}$ is formed by the last $\hat{N} \triangleq M-(K-1)N$ columns of $\mathbf{V}_{q_i}$. Then, any precoding covariance matrix $\mathbf{Q}_{q_i}$ formulated as $\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H$, where $\mathbf{D}_{q_i} \succeq \mathbf{0}$ is an arbitrary $\hat{N} \times \hat{N}$ matrix, would make $\mathbf{H}_{qq_j}\mathbf{Q}_{q_i}\mathbf{H}_{qq_j}^H = \mathbf{0}, \forall j \neq i$. Thus, the set of admissible strategies for player-q can be defined as follows:

$$\mathcal{S}_q = \left\{ \mathbf{Q}_{q_i} \in \mathbb{S}^{M \times M_q} : \mathbf{Q}_{q_i} = \hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H, \mathbf{D}_{q_i} \succeq \mathbf{0}, \sum_{i=1}^{K} \mathrm{Tr}\left\{\mathbf{D}_{q_i}\right\} \leq P_q \right\}. \tag{4.6}$$

Mathematically, the game has the following structure

$$\mathcal{G} = \left(\Omega, \; \{\mathcal{S}_q\}_{q\in\Omega}, \; \{R_q\}_{q\in\Omega} \right). \tag{4.7}$$

A NE of game $\mathcal{G}$ is defined when

$$R_q\left(\mathbf{Q}_q^\star, \mathbf{Q}_{-q}^\star\right) \geq R_q\left(\mathbf{Q}_q, \mathbf{Q}_{-q}^\star\right), \; \forall \mathbf{Q}_q \in \mathcal{S}_q, \quad \forall q \in \Omega. \tag{4.8}$$

At an NE, given the precoding strategy from other cells, a BS does not have the incentive to unilaterally change its precoding strategy, *i.e.*, it shall achieve a lower sum-rate with the same power constraint.

4.3.2 Characterization of the BD Precoding Game's Nash Equilibrium

In this section, we study the two most fundamental questions in analyzing a SNG: the existence and uniqueness of the game's NE. The NE characterization allows us to predict a stable outcome of the noncooperative BD precoding design in game $\mathcal{G}$. The existence of a pure NE in game $\mathcal{G}$ can be deduced straightforwardly from the work in [79] for N-person quasi-concave games. First, the strategy set $\mathcal{S}_q$ for player-q defined in (4.6) is compact and convex, $\forall q$. Second, the utility function $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is a continuous function in the profile of strategies $\mathcal{S}_q$ and concave in $\mathbf{Q}_{q_1}, \ldots, \mathbf{Q}_{q_K}$. Thus, Theorem 1 in [79] indicates that there always exists at least one pure NE in game $\mathcal{G}$.

In order to study the uniqueness of an NE in game $\mathcal{G}$, we first investigate the best response strategy at each player. As defined in $\mathcal{S}_q$, the best response strategy of player-q must be in the form $\mathbf{Q}_{q_i} = \hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H, \forall i$. Let $\mathbf{D}_q \triangleq \text{blk}\{\mathbf{D}_{q_i}\}$, $\mathbf{D} = \{\mathbf{D}_q\}_{q\in\Omega}$. Then, the best response strategy $\mathbf{D}_q$ at BS-q can be obtained from the following optimization problem

$$\begin{aligned}
\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_K}}{\text{maximize}} \quad & \sum_{i=1}^{K} \log\left|\mathbf{I} + \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\right| \\
\text{subject to} \quad & \sum_{i=1}^{K} \text{Tr}\{\mathbf{D}_{q_i}\} \le P_q \\
& \mathbf{D}_{q_i} \succeq \mathbf{0}, \forall i,
\end{aligned} \tag{4.9}$$

where $\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q})$ is defined as

$$\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q}) = \mathbf{R}_{q_i}(\mathbf{Q}_{-q}) = \sum_{r\neq q}^{Q} \mathbf{H}_{rq_i}\left(\sum_{j=1}^{K}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\right)\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i}. \tag{4.10}$$

By eigen-decomposing $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i} = \hat{\mathbf{U}}_{q_i}\mathbf{\Lambda}_{q_i}\hat{\mathbf{U}}_{q_i}^H$, the optimal solution to problem (4.9) can be easily obtained from the WF procedure

$$\mathbf{D}_{q_i} \triangleq \mathbf{WF}_{q_i}(\mathbf{D}_{-q}) = \hat{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I} - \mathbf{\Lambda}_{q_i}^{-1}\right]^{+}\hat{\mathbf{U}}_{q_i}^H, \tag{4.11}$$

where the water-level μ_q is adjusted to meet the power constraint $\sum_{i=1}^{K}\text{Tr}\left\{\left[\mu_q\mathbf{I} - \mathbf{\Lambda}_{q_i}^{-1}\right]^{+}\right\} = P_q$. Note that as $\hat{\mathbf{V}}_{q_i}$ only depends on in-cell channels at cell-q, BS-q only needs to strategically adapt its precoding matrices $\mathbf{D}_{q_i}, \forall i$ as in (4.11).

While the best response strategy of each player can be obtained in a closed-form solution in (4.11), the nonlinear structure of the WF operator is problematic in analyzing the uniqueness of the game's NE. Fortunately, the WF operator can be interpreted as a projection onto a closed set [48]. As studied in [48] for the case of single-user MIMO WF, this interpretation is also applicable to the multiuser WF case considered in problem (4.9).

Theorem 4.1. *The optimization problem* (4.9) *is equivalent to the following optimization* *problem*

$$\begin{aligned}
\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_K}}{\text{minimize}} \quad & \sum_{i=1}^{K} \left\| \mathbf{D}_{q_i} + \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \right\|_F^2 \\
\text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{D}_{q_i} \right\} = P_q \\
& \mathbf{D}_{q_i} \succeq \mathbf{0},
\end{aligned} \tag{4.12}$$

where $\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})}$ *is the projection onto the null space of* $\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}$, *and* c_q *is an arbitrarily large constant satisfying* $c_q \geq P_q + \max_{\forall i,\forall k}[\mathbf{\Lambda}_{q_i}]_{kk}^{-1}$.

Proof. The proof for this theorem is similar to that of Lemma 1 in [48] for the case of single-user MIMO WF. Given that $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} = \hat{\mathbf{U}}_{q_i} \mathbf{\Lambda}_{q_i} \hat{\mathbf{U}}_{q_i}^H$, one has

$$\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} = \hat{\mathbf{U}}_{q_i} \mathbf{\Lambda}_{q_i}^{-1} \hat{\mathbf{U}}_{q_i}^H. \tag{4.13}$$

Note that $\hat{\mathbf{U}}_{q_i}$ is a $\hat{N} \times N$ unitary matrix, *i.e.*, $\hat{\mathbf{U}}_{q_i}^H \hat{\mathbf{U}}_{q_i} = \mathbf{I}$, and $\mathbf{\Lambda}_{q_i}$ is a $N \times N$ diagonal matrix. By the assumption that $KN \leq M$, *i.e.*, $N \leq \hat{N}$, one may form a unitary matrix $\check{\mathbf{U}}_{q_i} = [\hat{\mathbf{U}}_{q_i}, \tilde{\mathbf{U}}_{q_i}]$, where $\tilde{\mathbf{U}}_{q_i}$ is a $\hat{N} \times (\hat{N} - N)$ matrix satisfying $\hat{\mathbf{U}}_{q_i}^H \tilde{\mathbf{U}}_{q_i} = \mathbf{0}$ and $\tilde{\mathbf{U}}_{q_i}^H \tilde{\mathbf{U}}_{q_i} = \mathbf{I}$. In addition, $\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}) = \mathcal{N}(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i})$ implies that $\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} = \tilde{\mathbf{U}}_{q_i} \tilde{\mathbf{U}}_{q_i}^H$. Thus, for a given c_q, one has

$$\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} = \check{\mathbf{U}}_{q_i} \check{\mathbf{\Lambda}}_{q_i}^{-1} \check{\mathbf{U}}_{q_i}^H,$$

where $\check{\mathbf{\Lambda}}_{q_i} = \mathrm{blk}\{\mathbf{\Lambda}_{q_i}, (1/c_q)\mathbf{I}\}$.

The optimization problem (4.12) then can be rewritten as

$$\begin{aligned}
\underset{\check{\mathbf{D}}_{q_1},\ldots,\check{\mathbf{D}}_{q_K}}{\text{minimize}} \quad & \sum_{i=1}^{K} \left\| \check{\mathbf{D}}_{q_i} + \check{\mathbf{\Lambda}}_{q_i}^{-1} \right\|_F^2 \\
\text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\left\{ \check{\mathbf{D}}_{q_i} \right\} = P_q, \\
& \check{\mathbf{D}}_{q_i} \succeq \mathbf{0},
\end{aligned} \tag{4.14}$$

where $\check{\mathbf{D}}_{q_i} \triangleq \check{\mathbf{U}}_{q_i}^H \mathbf{D}_{q_i} \check{\mathbf{U}}_{q_i}$. Due to the fact the objective function is lower-bounded by diagonal matrices $\{\check{\mathbf{D}}_{q_i}\}$, the optimal solution set $\{\check{\mathbf{D}}_{q_i}\}$ to problem (4.14) has to be diagonal. Thus, the optimization (4.14) can be reduced to

$$\begin{aligned} \underset{\check{\mathbf{D}}_{q_1},\ldots,\check{\mathbf{D}}_{q_K}}{\text{minimize}} \quad & \sum_{i=1}^{K}\sum_{k}\left(\left[\check{\mathbf{D}}_{q_i}\right]_{k,k}+\left[\check{\mathbf{\Lambda}}_{q_i}^{-1}\right]_{k,k}\right)^2 \\ \text{subject to} \quad & \sum_{i=1}^{K}\sum_{k}[\check{\mathbf{D}}_{q_i}]_{k,k}=P_q,\ [\check{\mathbf{D}}_{q_i}]_{k,k}\geq 0, \end{aligned} \tag{4.15}$$

whose (unique) optimal solution has the WF structure such that $\check{\mathbf{D}}_{q_i} = \left[\mu_q\mathbf{I}-\check{\mathbf{\Lambda}}_{q_i}^{-1}\right]^+$, where μ_q is the water-level to meet the power constraint $\sum_{i=1}^{K}\operatorname{Tr}\left\{\check{\mathbf{D}}_{q_i}\right\}=P_q$. Thus, the optimal solution to the original problem (4.12) is given by

$$\mathbf{D}_{q_i}=\check{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I}-\operatorname{blk}\left\{\mathbf{\Lambda}_{q_i}^{-1},c_q\mathbf{I}\right\}\right]^+\check{\mathbf{U}}_{q_i}^H=\hat{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I}-\mathbf{\Lambda}_{q_i}^{-1}\right]^+\hat{\mathbf{U}}_{q_i}^H,$$

if c_q is chosen to be large enough such that $[\mu_q-c_q]^+=0$. As suggested in [48], choosing $c_q\geq P_q+\max_{\forall i,\forall k}[\mathbf{\Lambda}_{q_i}]_{kk}^{-1}$ is sufficient to meet this requirement. This concludes the proof for Theorem 4.1. $\square$

From Theorem 4.1, the WF solution in (4.11) is indeed the solution of the optimization problem (4.12). Thus, the block-diagonal WF solution $\mathbf{WF}_q(\mathbf{D}_{-q})\triangleq\operatorname{blk}\left\{\mathbf{WF}_{q_i}(\mathbf{D}_{-q})\right\}$, can be interpreted as a projection

$$\mathbf{WF}_q(\mathbf{D}_{-q})=\left[-\operatorname{blk}\left\{\left(\mathbf{V}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\right)^\dagger+c_q\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})}\right\}\right]_{\mathcal{D}_q},$$

where $\mathcal{D}_q\triangleq\left\{\mathbf{D}_{q_i}\in\mathbb{S}^{\hat{N}\times\hat{N}}:\sum_{i=1}^{K}\operatorname{Tr}\{\mathbf{D}_{q_i}\}=P_q,\ \mathbf{D}_i\succeq\mathbf{0},\forall i\right\}$ is a closed and convex set.

Define the multicell mapping $\mathbf{WF}(\mathbf{D})=\{\mathbf{WF}_q(\mathbf{D}_{-q})\}_{q\in\Omega}$. Let $e_{\mathrm{WF}_q}=\left\|\mathbf{WF}_q\left(\mathbf{D}_{-q}^{(1)}\right)-\mathbf{WF}_q\left(\mathbf{D}_{-q}^{(2)}\right)\right\|_F$ and $e_q=\left\|\mathbf{D}_q^{(1)}-\mathbf{D}_q^{(2)}\right\|_F$, for any given $\mathbf{D}^{(1)}\neq\mathbf{D}^{(2)}$ and $\mathbf{D}_q^{(1)},\mathbf{D}_q^{(2)}\in\mathcal{D}_q,\forall q$.

Then,

$$
\begin{aligned}
e_{\mathrm{WF}_q} &= \Bigg\| \Bigg[-\mathrm{blk}\left\{ \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \right\} \Bigg]_{\mathcal{D}_q} \\
&\quad - \Bigg[-\mathrm{blk}\left\{ \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \right\} \Bigg]_{\mathcal{D}_q} \Bigg\|_F \\
&\le \Bigg\| \mathrm{blk}\left\{ -\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} \right\} \\
&\quad - \mathrm{blk}\left\{ -\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} \right\} \Bigg\|_F && \text{(4.16a)} \\
&\le \sum_{i=1}^{K} \left\| \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} - \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\dagger} \right\|_F && \text{(4.16b)} \\
&\le \sum_{i=1}^{K} \left\| \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \left(\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) - \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \right) \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \right\|_F && \text{(4.16c)} \\
&= \sum_{i=1}^{K} \left\| \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \left[\sum_{r \neq q}^{Q} \mathbf{H}_{rq_i} \left[\sum_{j=1}^{K} \hat{\mathbf{V}}_{r_j} \left(\mathbf{D}_{r_j}^{(1)} - \mathbf{D}_{r_j}^{(2)} \right) \hat{\mathbf{V}}_{r_j}^H \right] \mathbf{H}_{rq_i}^H \right] \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \right\|_F \\
&= \sum_{i=1}^{K} \left\| \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \left[\sum_{r \neq q}^{Q} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_{r} \left(\mathbf{D}_{r}^{(1)} - \mathbf{D}_{r}^{(2)} \right) \hat{\mathbf{V}}_{r}^H \mathbf{H}_{rq_i}^H \right] \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \right\|_F \\
&\le \sum_{i=1}^{K} \sum_{r \neq q}^{Q} \left\| \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_{r} \left(\mathbf{D}_{r}^{(1)} - \mathbf{D}_{r}^{(2)} \right) \hat{\mathbf{V}}_{r}^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \right\|_F && \text{(4.16d)} \\
&\le \sum_{i=1}^{K} \sum_{r \neq q}^{Q} \rho \left(\hat{\mathbf{V}}_{r}^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_{r} \right) \left\| \mathbf{D}_{r}^{(1)} - \mathbf{D}_{r}^{(2)} \right\|_F && \text{(4.16e)} \\
&= \sum_{r \neq q}^{Q} [\mathbf{S}]_{q,r}\, e_r, && \text{(4.16f)}
\end{aligned}
$$

where $\hat{\mathbf{V}}_r \triangleq [\hat{\mathbf{V}}_{r_1}, \ldots, \hat{\mathbf{V}}_{r_K}]$ and $\mathbf{S} \in \mathbb{C}^{Q \times Q}$ is defined as

$$
[\mathbf{S}]_{q,r} = \begin{cases} \sum_{i=1}^{K} \rho \left(\hat{\mathbf{V}}_{r}^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger H} \hat{\mathbf{V}}_{q_i}^{\dagger} \mathbf{H}_{qq_i}^{\dagger} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_{r} \right), & \text{if } r \neq q \\ 0, & \text{if } r = q. \end{cases} \tag{4.17}
$$

Note that the inequality (4.16a) holds due to the non-expansive property of the projection onto a closed and convex set [67], inequalities (4.16b) and (4.16d) hold because $\mathbf{X} = \mathrm{diag}\{\mathbf{X}_1, \ldots, \mathbf{X}_K\}$ implies that $\|\mathbf{X}\|_F \leq \sum_{i=1}^{K} \|\mathbf{X}_i\|_F$, inequality (4.16c) holds due to the reverse order law for the Moore-Penrose pseudo-inverse [48], and inequality (4.16e) holds because the Frobenius norm is consistent [66].

Define the vectors $\mathbf{e}_{\mathrm{WF}} = [e_{\mathrm{WF}_1}, \ldots, e_{\mathrm{WF}_Q}]^T$ and $\mathbf{e} = [e_1, \ldots, e_Q]^T$. The set of inequalities (4.16f) implies that

$$\mathbf{0} \leq \mathbf{e}_{\mathrm{WF}} \leq \mathbf{S}\mathbf{e}. \tag{4.18}$$

Given a vector $\mathbf{w} = [w_q, \ldots, w_Q]^T > 0$, the mapping $\mathbf{WF}(\mathbf{D})$ is a block-contraction with respect to the norm $\|\cdot\|_{F,\mathrm{block}}^{\mathbf{w}}$, if there exists a non-negative constant $\alpha < 1$, such that

$$\left\|\mathbf{WF}(\mathbf{D}^{(1)}) - \mathbf{WF}(\mathbf{D}^{(2)})\right\|_{F,\mathrm{block}}^{\mathbf{w}} \leq \alpha \left\|\mathbf{D}^{(1)} - \mathbf{D}^{(2)}\right\|_{F,\mathrm{block}}^{\mathbf{w}}, \quad \forall \mathbf{D}^{(1)}, \mathbf{D}^{(2)}. \tag{4.19}$$

From the inequality (4.18), one has

$$\|\mathbf{e}_{\mathrm{WF}}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} \|\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}}, \tag{4.20}$$

as the induced ∞-norm $\|\cdot\|_{\infty,\mathrm{mat}}^{\mathbf{w}}$ is consistent [66]. Then,

$$\begin{aligned} \left\|\mathbf{WF}(\mathbf{D}^{(1)}) - \mathbf{WF}(\mathbf{D}^{(2)})\right\|_{F,\mathrm{block}}^{\mathbf{w}} &= \max_{q \in \Omega} \frac{\left\|\mathbf{WF}_q(\mathbf{D}^{(1)}) - \mathbf{WF}_q(\mathbf{D}^{(2)})\right\|_2}{w_q} \\ &= \|\mathbf{e}_{\mathrm{WF}}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \\ &\leq \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} \|\mathbf{e}\|_{\infty,\mathrm{vec}}^{\mathbf{w}} \\ &= \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} \left\|\mathbf{D}^{(1)} - \mathbf{D}^{(2)}\right\|_{F,\mathrm{block}}^{\mathbf{w}}. \end{aligned} \tag{4.21}$$

Thus, if $\|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1$, the $\mathbf{WF}(\mathbf{D})$ mapping is a contraction, which implies the uniqueness of the NE in game $\mathcal{G}$ [67]. In addition, the condition $\|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1$ is also sufficient to guarantee the convergence of the NE from any starting precoding strategy $\mathbf{D}_q \in \mathcal{D}_q$. Note that if $\mathbf{S}$ is a non-negative matrix, there always exists a positive vector $\mathbf{w}$ satisfying [67]

$$(\mathrm{C}): \quad \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1 \quad \Longleftrightarrow \quad \rho(\mathbf{S}) < 1. \tag{4.22}$$

Remark 4.1: Assuming the path loss fading model in this multicell system, a physical interpretation of the sufficient condition (C) is as follows. When the intra-cell BS-MS distance gets smaller relatively to the distance between the BSs, the ICI becomes less dominant. Thus, the positive off-diagonal elements of $\mathbf{S}$ also become smaller. This results in a smaller spectral radius of $\mathbf{S}$. Therefore, as the MSs are getting closer to its connected BS, the probability of meeting condition (C) is higher, which then guarantees the uniqueness of the NE.

4.4 The Multicell Block-Diagonalization Precoding - Coordinated Design

4.4.1 Problem Formulation

In Section 4.3, we examined the fully decentralized approach in the multicell BD precoding design and characterized the NE of the system. However, it is well-known that the NE need not be Pareto-efficient [52]. Via coordination among the BSs, significant network sum-rate improvement can be obtained by jointly designing all the precoders at the same time. Nonetheless, this advantage may come with the expense of message passing among the coordinated BSs as explained later in this section. We investigate the coordinated multicell BD precoding design in order to jointly maximize the network WSR through the following optimization

$$\begin{aligned} \underset{\mathbf{Q}_1,\dots,\mathbf{Q}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \log\left|\mathbf{I} + \mathbf{H}_{qq_i}^H \mathbf{R}_{q_i}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq_i} \mathbf{Q}_{q_i}\right| \\ \text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_{q_i}\} \le P_q, \forall q \\ & \mathbf{Q}_{q_i} \succeq \mathbf{0},\ \forall i, \forall q \\ & \mathbf{H}_{qq_j} \mathbf{Q}_{q_i} \mathbf{H}_{qq_j}^H = \mathbf{0}, \forall j \neq i, \forall q, \end{aligned} \tag{4.23}$$

where $\omega_q \ge 0$ denotes the non-negative weight associated with BS-q. Since the BD constraints can be removed by formulating the precoding covariance matrix $\mathbf{Q}_{q_i}$ as $\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H$ (where $\mathbf{D}_{\mathbf{q}_i}$ is an arbitrary $\hat{N} \times \hat{N}$ and $\hat{\mathbf{V}}_{q_i}$ given in (4.5)), the optimization problem (4.23) can be restated as

$$\begin{aligned} \underset{\mathbf{D}_1,\ldots,\mathbf{D}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \log \left| \mathbf{I} + \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \mathbf{D}_{q_i} \right| \\ \text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{D}_{q_i}\} \leq P_q, \forall q \\ & \mathbf{D}_{q_i} \succeq \mathbf{0}, \ \forall i, \forall q. \end{aligned} \tag{4.24}$$

It is observed that problem (4.24) is nonconvex due to presence of $\mathbf{D}_{r_j}$'s in the ICI term $\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q})$'s. Thus, it is generally difficult and computationally complex to find the globally optimal solution to problem (4.24). Consequently, we focus on proposing a low-complexity algorithm that can obtain at least a locally optimal solution.

4.4.2 The Iterative Linear Approximation (ILA) Solution Approach

This section presents a solution approach to the nonconvex problem (4.24) by considering it as a difference of convex (DC) program [80]. Specifically, by iteratively isolating and approximating the nonconvex part of the objective function into linear terms, one can decompose the DC program into multiple convex optimization problems. This DC solution approach, termed as the iterative linear approximation (ILA) algorithm, has been utilized in a recent work [57] in order to maximize the multicell network sum-rate with one MS per cell.

Denote $f_q(\mathbf{D}_q, \mathbf{D}_{-q}) = \sum_{r\neq q}^{Q} \omega_r \sum_{j=1}^{K} R_{r_j}(\mathbf{D}_q, \mathbf{D}_{-q})$ as the WSR of all cells except cell-q. Note that $f_q(\mathbf{D}_q, \mathbf{D}_{-q})$ is nonconcave in $\mathbf{D}_{q_i}$, $i = 1, \ldots, K$. At a given value of $(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q})$, approximate f_q by using the Taylor expansion of f_q around $\bar{\mathbf{D}}_{q_i}$, $i = 1, \ldots, K$, and retaining the first linear term

$$f_q(\mathbf{D}_q, \bar{\mathbf{D}}_{-q}) \approx f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{A}_{q_i} \left(\mathbf{D}_{q_i} - \bar{\mathbf{D}}_{q_i} \right) \right\}, \tag{4.25}$$

where $\mathbf{A}_{q_i}$ is the negative partial derivative of f_q with respect to the $\mathbf{D}_{q_i}$, evaluated at $\bar{\mathbf{D}}_{q_i}$

$$\begin{aligned}
\mathbf{A}_{q_i} &= -\frac{\partial f_q}{\partial \mathbf{D}_{q_i}}\bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}} \\
&= -\sum_{r\neq q}^{Q} \omega_r \sum_{j=1}^{K} \frac{\partial R_{r_j}}{\partial \mathbf{D}_{q_i}}\bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}} \\
&= \sum_{r\neq q}^{Q} \omega_r \sum_{j=1}^{K} \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qr_j}^H \left[\hat{\mathbf{R}}_{r_j}^{-1} - \left(\hat{\mathbf{R}}_{r_j} + \mathbf{H}_{rr_j} \hat{\mathbf{V}}_{r_j} \mathbf{D}_{r_j} \hat{\mathbf{V}}_{r_j}^H \mathbf{H}_{rr_j}^H \right)^{-1} \right] \mathbf{H}_{qr_j} \hat{\mathbf{V}}_{q_i} \bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}}
\end{aligned} \tag{4.26}$$

Using (4.25), the network WSR around $(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q})$ can be approximated as $\omega_q \sum_{i=1}^{K} R_{q_i} - f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{A}_{q_i} \left(\mathbf{D}_{q_i} - \bar{\mathbf{D}}_{q_i} \right) \right\}$. Omitting the deterministic terms in the objective function, the nonconvex problem (4.24) can be approximated as

$$\begin{aligned}
\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_K}}{\text{maximize}} \quad & \omega_q \sum_{i=1}^{K} \log \left| \mathbf{I} + \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \mathbf{D}_{q_i} \right| - \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{A}_{q_i} \mathbf{D}_{q_i} \right\} \\
\text{subject to} \quad & \mathbf{D}_{q_i} \succeq \mathbf{0}, \ \forall i \\
& \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{D}_{q_i}\} \leq P_q,
\end{aligned} \tag{4.27}$$

which can be solved solely at BS-q. Thus, if the Q BSs take turns to approximate the original problem (4.24), it can be solved via Q per-cell separate problems (4.27).

It can be observed that the approximated problem (4.27) is similar to the sum-rate maximization problem with BD precoding, albeit the presence of the term $\sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{A}_{q_i} \mathbf{D}_{q_i} \right\}$. Herein, this term is the penalty charged on the ICI induced by BS-q to the MSs in other cells, whereas $\mathbf{A}_{q_i}$ acts as the interference price. If the ICI penalty term is not presented, the BS would only attempt to maximize the sum-rate for its connected MSs. As a result, the multicell system is in *competition* mode, as studied in Section 4.3. In contrast, in *coordination* mode, each BS is doing its best in limiting the ICI induced to other cells through this ICI penalty mechanism.

Since the approximated problem (4.27) is now convex, it can be readily solved by standard convex optimization techniques. In the following, we present the closed-form solution to the problem via the Lagrangian duality. The Lagrangian of problem (4.27) can be stated as

$$\mathcal{L}_q(\mathbf{D}_{q_i}, \lambda_q) = \omega_q \sum_{i=1}^{K} \log \left| \mathbf{I} + \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \mathbf{D}_{q_i} \right| - \sum_{i=1}^{K} \mathrm{Tr}\left\{ \left(\mathbf{A}_{q_i} + \lambda_q \mathbf{I} \right) \mathbf{D}_{q_i} \right\} + \lambda_q P_q, \tag{4.28}$$

where $\lambda_q \geq 0$ is the Lagrangian multiplier associated with the power constraint. The dual function is then given by

$$g_q(\lambda_q) = \max_{\mathbf{D}_{q_i} \succeq \mathbf{0}} \mathcal{L}_q(\mathbf{D}_{q_i}, \lambda_q). \tag{4.29}$$

For a given λ_q, the optimal solution to the Lagrangian (4.28) is presented in the following proposition.

Proposition 4.1. *Let $\mathbf{G}_{q_i}$ be the generalized eigen-matrix of $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i}$ and $(\mathbf{A}_{q_i} + \lambda_q \mathbf{I})$. The optimal solution, which maximizes the Lagrangian (4.28), must have the structure $\mathbf{G}_{q_i} \mathbf{P}_{q_i} \mathbf{G}_{q_i}^H, i = 1, \ldots, K$, where $\mathbf{P}_{q_i}$ is a diagonal matrix with non-negative elements.*

Proof. The proof for this proposition is similar to that of Proposition 1 in [57] for the case of single-user rate maximization with a penalty term. We omit the detailed proof of this proposition for brevity. □

Given $\mathbf{G}_{q_i}$ as the generalized eigen-matrix of $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i}$ and $(\mathbf{A}_{q_i} + \lambda_q \mathbf{I})$, one has

$$\begin{aligned} \mathbf{\Sigma}_{q_i}^{(1)} &= \mathbf{G}_{q_i}^H \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \mathbf{G}_{q_i} \\ \mathbf{\Sigma}_{q_i}^{(2)} &= \mathbf{G}_{q_i}^H (\mathbf{A}_{q_i} + \lambda_q \mathbf{I}) \mathbf{G}_{q_i}, \end{aligned} \tag{4.30}$$

where $\mathbf{\Sigma}_{q_i}^{(1)}$ and $\mathbf{\Sigma}_{q_i}^{(2)}$ are diagonal and positive semi-definite. Thus, the maximization of the Lagrangian (4.28) becomes

$$\underset{\mathbf{P}_{q_i} \succeq \mathbf{0}}{\text{maximize}} \;\; \omega_q \sum_{i=1}^{K} \log \left| \mathbf{I} + \mathbf{P}_{q_i} \mathbf{\Sigma}_{q_i}^{(1)} \right| - \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{P}_{q_i} \mathbf{\Sigma}_{q_i}^{(2)} \right\}, \tag{4.31}$$

whose optimal solution can be obtained by the well-known WF structure

$$[\mathbf{P}^{\star}_{q_i}]_{n,n} = \left[\frac{\omega_q}{\left[\mathbf{\Sigma}^{(2)}_{q_i}\right]_{n,n}} - \frac{1}{\left[\mathbf{\Sigma}^{(1)}_{q_i}\right]_{n,n}} \right]^+, i = 1, \ldots, K. \tag{4.32}$$

It remains to adjust the dual variable λ_q to impose the power constraint $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{P}^{\star}_{q_i}\} \leq P_q$ for the above WF solution. One can easily verify for the case $\lambda_q = 0$ whether $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{P}^{\star}_{q_i}\} < P_q$. If it holds, it means BS-q does not transmit at its full power limit. Otherwise, $\lambda_q > 0$ can be searched by the bisection method until $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{P}^{\star}_{q_i}\} = P_q$.

4.4.3 Convergence of the ILA Algorithm and its Distributed Implementation

This section addresses the convergence of the ILA algorithm and its distributed implementation. In order to solve the problem of Q cells in (4.24), the ILA algorithm requires each BS-q, $q = 1, \ldots, Q$, to update the parameters $\mathbf{A}_{q_i}$'s and sequentially take turns to solve its corresponding optimization (4.27). The convergence of the ILA algorithm is given in the following theorem.

Theorem 4.2. *The optimization* (4.27) *carried at any given BS-q always improves the network WSR. Thus, the Gauss-Seidel (sequential) iterative update across the Q BSs is guaranteed to converge to at least a local maximum.*

Proof. Suppose that $\mathbf{D}_q = \bar{\mathbf{D}}_q = \left\{\bar{\mathbf{D}}_{q_i}\right\}_{i=1}^{K}, \forall q$ is obtained from the previous iteration, and $\mathbf{D}^{\star}_q = \left\{\mathbf{D}^{\star}_{q_i}\right\}_{i=1}^{K}, \forall q$ is the optimal solution obtained from the optimization problem (4.27) at BS-q. Using a technique similar to the on applied in [57, 81], it can be shown that $f_q(\mathbf{D}_q, \bar{\mathbf{D}}_{-q})$ is a convex function with respect to $\mathbf{D}_q$. Thus, by the first-order condition for the convex function $f_q(\mathbf{D}_q, \bar{\mathbf{D}}_{-q})$ [82], one has

$$f_q(\mathbf{D}^{\star}_q, \bar{\mathbf{D}}_{-q}) \geq f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\big\{\mathbf{A}_{q_i}(\mathbf{D}^{\star}_{q_i} - \bar{\mathbf{D}}_{q_i})\big\}. \tag{4.33}$$

After the optimization (4.27) carried at BS-q, the network WSR is updated such that

$$\begin{aligned}\sum_{q=1}^{Q}\omega_q\sum_{i=1}^{K}R_{q_i}(\mathbf{D}_q^\star,\bar{\mathbf{D}}_{-q}) &= \omega_q\sum_{i=1}^{K}R_{q_i}(\mathbf{D}_q^\star,\bar{\mathbf{D}}_{-q}) + f_q(\mathbf{D}_q^\star,\bar{\mathbf{D}}_{-q})\\ &\geq \omega_q\sum_{i=1}^{K}R_{q_i}(\mathbf{D}_q^\star,\bar{\mathbf{D}}_{-q}) + f_q(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\big\{\mathbf{A}_{q_i}(\mathbf{D}_{q_i}^\star-\bar{\mathbf{D}}_{q_i})\big\}\\ &\geq \omega_q\sum_{i=1}^{K}R_{q_i}(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q}) + f_q(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\big\{\mathbf{A}_{q_i}(\bar{\mathbf{D}}_{q_i}-\bar{\mathbf{D}}_{q_i})\big\}\\ &= \sum_{q=1}^{Q}\omega_q\sum_{i=1}^{K}R_{q_i}(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q}),\end{aligned}\tag{4.34}$$

where the first inequality is due to the first-order condition in (4.33) and the second inequality is due to the fact that $\mathbf{D}_q^\star$ is the optimal solution of problem (4.27). Clearly, the optimization carried at a given BS-q always improves the network WSR. In the ILA algorithm, each BS, say BS-q, updates the parameters $\mathbf{A}_{q_i}$'s and sequentially takes turn to solve its corresponding optimization (4.27). Since the network WSR is upper-bounded, the Gauss-Seidel (sequential) updates across the Q BSs must converge monotonically to at least a local maximum. This concludes the proof for Theorem 4.2. □

As presented in Section 4.3, the BD precoding design for a multicell system under the IA mode can be implemented in a fully decentralized manner. Interestingly, distributed implementation can also be realized for the BD precoding design under the IC mode. Via the coordination and message exchange among the BSs, the ILA can be implemented distributively as follows. Since the optimization problem (4.27) can be executed at the corresponding BS with only local information (CSI and IPN at the connected MSs), it remains to show that the pricing factors $\mathbf{A}_{q_i}$'s can also be computed in a distributed manner through a message exchange mechanism among the BSs. It is observed from equation (4.26) that in order to compute $\mathbf{A}_{q_i}$, BS-q has to know the channels $\mathbf{H}_{qr_j}$'s to all the MSs in the other cells. This is an important requirement for BS-q to coordinate its induced ICI. In addition, BS-q needs to acquire the factor $\mathbf{B}_{r_j} = \hat{\mathbf{R}}_{r_j}^{-1} - \left(\hat{\mathbf{R}}_{r_j} + \mathbf{H}_{rr_j}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\mathbf{H}_{rr_j}^H\right)^{-1}$ from other cells. Thus, it is required that each MS computes its corresponding factor $\mathbf{B}_{r_j}$ using local measurements on the total IPN in $\hat{\mathbf{R}}_{r_j}$ and the total IPN plus signal in $\hat{\mathbf{R}}_{r_j} + \mathbf{H}_{rr_j}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\mathbf{H}_{rr_j}^H$. Each MS can calculate the factor $\mathbf{B}_{r_j}$ with local information and feed back to its connected BS. These factors $\mathbf{B}_{r_j}$'s are then exchanged among the BSs to evaluate the prices $\mathbf{A}_{q_i}$'s. This message exchange mechanism among the coordinated BSs is the distinct feature of the IC mode, compared to the IA mode.

4.5 Multicell BD-DPC Precoding: Competition and Coordination

4.5.1 BD-DPC Precoding on a Per-cell Basis

This section considers the multicell system where each BS utilizes BD-DPC to the downlink transmissions of its connected MSs. It is well-known that DPC is the capacity-achieving encoding scheme for the multiuser broadcast channel [26–28]. In [26], a suboptimal and simpler zero-forcing DPC (ZF-DPC) scheme was proposed for single-antenna receivers that takes advantage of both DPC and ZF precoding. In ZF-DPC, the information signals sent to the multiple users are encoded in sequence such that the receiver at any user does not see any inter-user interference due to the use of ZF and DPC at the BS. In this work, we apply a similar technique to the encoding process at each BS. Due to the consideration of multi-antenna receivers, the technique shall be referred to as the BD-DPC precoding.

At any BS, say BS-q, denote the encoding sequence to its K connected MSs as $\boldsymbol{\pi}_q = [\pi_q(1), \ldots, \pi_q(K)]^T$. The concept of BD-DPC can be briefly explained as follows:

- BS-q freely designs the precoder $\mathbf{W}_{\pi_q(1)}$ for MS-$\pi_q(1)$.
- BS-q, having the noncausal knowledge of the codeword intended for MS-$\pi_q(1)$, uses DPC such that MS-$\pi_q(2)$ does not see the codeword for MS-$\pi_q(1)$ as interference. At the same time, the precoder $\mathbf{W}_{\pi_q(2)}$ for MS-$\pi_q(2)$ is designed on the null space caused by $\mathbf{H}_{q\pi_q(1)}$ to eliminate its induced interference to MS-$\pi_q(1)$.
- Similarly, to encode the signal for user-i, BS-q can utilize the noncausal knowledge of the codewords for MSs $\pi_q(1), \ldots, \pi_q(i-1)$, and design $\mathbf{W}_{\pi_q(i)}$ on the null space caused by $\hat{\mathbf{H}}'_{\pi_q(i)} = [\mathbf{H}_{q\pi_q(1)}, \ldots, \mathbf{H}_{q\pi_q(i-1)}]$.

4.5.2 The Multicell BD-DPC Precoding - Competitive Design

Similar to game $\mathcal{G}$ defined in Section 4.3, we consider a new game $\mathcal{G}'$, where each BS strategically adapts its BD-DPC precoders to maximize the sum-rate to its connected

MSs. Mathematically, game $\mathcal{G}'$ can be defined as

$$\mathcal{G}' = \left(\Omega,\ \left\{\mathcal{S}'_q(\boldsymbol{\pi}_q)\right\}_{q\in\Omega},\ \left\{R_q\right\}_{q\in\Omega}\right), \tag{4.35}$$

The set of admissible strategies $\mathcal{S}'_q(\boldsymbol{\pi}_q)$ is now defined as

$$\begin{aligned}\mathcal{S}'_q(\boldsymbol{\pi}_q) = \Big\{\mathbf{Q}_{\pi_q(i)} \in \mathbb{S}^{M\times M_q} : \mathbf{Q}_{\pi_q(i)} = \hat{\mathbf{V}}_{\pi_q(i)}\mathbf{D}_{\pi_q(i)}\hat{\mathbf{V}}^H_{\pi_q(i)},\\ \mathbf{D}_{\pi_q(i)} \succeq \mathbf{0}, \sum_{i=1}^{K} \mathrm{Tr}\left\{\mathbf{D}_{\pi_q(i)}\right\} \le P_q\Big\},\end{aligned} \tag{4.36}$$

where $\hat{\mathbf{V}}_{\pi_q(i)}$ is the null space created by $\hat{\mathbf{H}}'_{\pi_q(i)}$. Due to the similarity between games $\mathcal{G}$ and $\mathcal{G}'$, the characterization for game $\mathcal{G}$ presented in Section 4.3.2 can be directly applied to game $\mathcal{G}'$. In particular, it can be concluded that there always exists at least one NE in game $\mathcal{G}'$ and the NE is unique if

$$(\mathrm{C}') : \rho(\mathbf{S}') < 1, \tag{4.37}$$

where $\mathbf{S}' \in \mathbb{C}^{Q\times Q}$ is defined as

$$[\mathbf{S}']_{q,r} = \begin{cases} \sum\limits_{i=1}^{K} \rho\left(\hat{\mathbf{V}}^H_{\pi_r}\mathbf{H}^H_{r\pi_q(i)}\mathbf{H}^{\dagger H}_{q\pi_q(i)}\hat{\mathbf{V}}^{\dagger H}_{\pi_q(i)}\hat{\mathbf{V}}^{\dagger}_{\pi_q(i)}\mathbf{H}^{\dagger}_{q\pi_q(i)}\mathbf{H}_{r\pi_q(i)}\hat{\mathbf{V}}_{\pi_r}\right), & \text{if } r \ne q \\ 0, & \text{if } r = q, \end{cases} \tag{4.38}$$

with $\hat{\mathbf{V}}_{\pi_r} \triangleq [\hat{\mathbf{V}}_{\pi_r(1)}, \ldots, \hat{\mathbf{V}}_{\pi_r(K)}]$.

Remark 4.2: Due to the dependence of the admissible strategy set $\mathcal{S}'_q(\boldsymbol{\pi}_q)$ on the encoding order $\boldsymbol{\pi}_q$ at BS-q, the characterization of game $\mathcal{G}'$ strictly depends on the encoding order at each BS. In addition, with different encoding orders at a BS, say BS-q, the optimal strategies, which maximize the sum-rate at BS-q, are also different. The condition (C$'$) for the uniqueness of game $\mathcal{G}'$ also depends on the encoding order at each BS-q. In fact, for any permutation in $\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_Q$, we have at least a different NE of game $\mathcal{G}'$. Given $K!$ encoding order permutations at BS-q, it can be concluded that game $\mathcal{G}'$ has at least $(K!)^Q$ NE points.

Remark 4.3: For a particular encoding order $\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_Q$ in game $\mathcal{G}'$, game $\mathcal{G}'$ provides a higher degree of freedom in designing the precoder at each BS. In fact, the size of matrix $\hat{\mathbf{V}}_{\pi_q(i)}$ in game $\mathcal{G}'$ is at least equal or larger than its counterpart $\hat{\mathbf{V}}_{q_i}$ in game $\mathcal{G}$. Intuitively, the off-diagonal elements of matrix $\mathbf{S}'$ are also larger than that of matrix $\mathbf{S}$. As a result, it is expected that the condition for the uniqueness of the NE in game $\mathcal{G}'$ is stricter than that in game $\mathcal{G}$.

4.5.3 The Multicell BD-DPC Precoding - Coordinated Design

In this section, we investigate the implementation of BD-DPC precoding in a multicell system under the IC mode. In this case, we consider the joint BD-DPC precoding design to maximize the network WSR as follows:

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{subject to} \quad & \mathbf{Q}_q \in \mathcal{S}'_q, \forall q. \end{aligned} \tag{4.39}$$

Similar to the optimization problem (4.23) considered in Section 4.4, the above problem is also nonconvex. Thus, we apply the same ILA algorithm proposed in Section 4.4 to solve problem (4.39). In particular, due to monotonic convergence of the ILA algorithm, we can obtain at least a locally optimal solution to the problem. The only difference here is that the solution $\mathbf{Q}_{q_i}$ of problem (4.39) must be in the form $\hat{\mathbf{V}}_{\pi_q(i)}\mathbf{D}_{q_i}\hat{\mathbf{V}}^H_{\pi_q(i)}$, where $\hat{\mathbf{V}}_{\pi_q(i)}$ is the null space created by $\hat{\mathbf{H}}'_{\pi_q(i)}$, and $\mathbf{D}_{q_i}$ is obtained from the ILA algorithm.

4.6 Simulation Results

This section presents simulation results validating our studies on the uniqueness of an NE and the convergence to the NE in games $\mathcal{G}$ and $\mathcal{G}'$. We also present the sum-rates of the multicell system under the IA and IC modes when BD, BD-DPC, or DPC precoding is applied on a per-cell basis. Under the IA mode, the sum-rates obtained from the games $\mathcal{G}$ and $\mathcal{G}'$ are compared to the one obtained from the game where DPC precoding is applied at each BS. In the same system setting, the DPC precoding [26, 28] is performed on a per-cell basis in a noncooperative manner (each BS selfishly maximizes its own sum-rate) until the multicell system converges to a stable state. Similarly, under the IC mode, we compare the network sum-rates (with equal weights) obtained from the ILA algorithm for

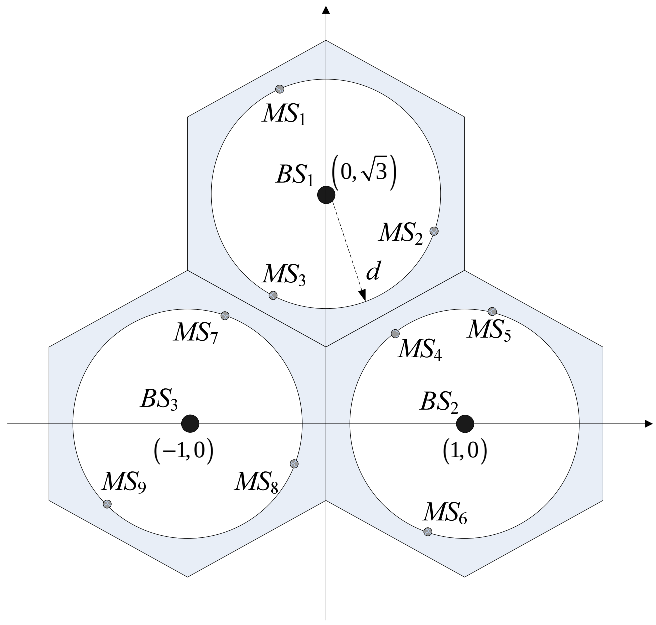

Fig. 4.1 A multicell system configuration with 3 cells, 3 users per cell.

the case of BD or BD-DPC precoding design to that of the DPC design.[1] For the BD-DPC or DPC precoding, we assume a fixed encoding order from MS-1 to MS-K at BS-q and similar orders at other BSs.

We consider a 3-cell system with 3 MSs per cell sharing the same channel frequency, as illustrated in Fig. 4.1. The numbers of antennas at each BS and each MS are set at $M_q = 8$ and $N = 2$. The same power constraint $P_q = 1$ is set at each BS, unless stated otherwise. The AWGN at each MS is set as $\mathbf{Z}_{q_i} = \sigma^2\mathbf{I}$ with $\sigma^2 = 0.01$. The distance between any two BSs is normalized to 2. In each cell, the MSs are assumed to be randomly located on a circle from its connected BS with the radius of d. The channels from a BS to a MS are generated by using the path-loss model, where the path-loss exponent is set at 3. In each figure, each plotted point is obtained by averaging over 10,000 independent channel realizations.

Fig. 4.2 displays the probability of the NE's uniqueness versus intra-cell BS-MS distance d by evaluating condition (C) for game $\mathcal{G}$ and (C$'$) for game $\mathcal{G}'$. Corresponding to a small distance d is the low-ICI region (and high signal-to-interference-plus-noise (SINR) as a

[1] To optimize the DPC in a multicell system under the IC mode, we utilize the numerical algorithm presented in Chapter 6.

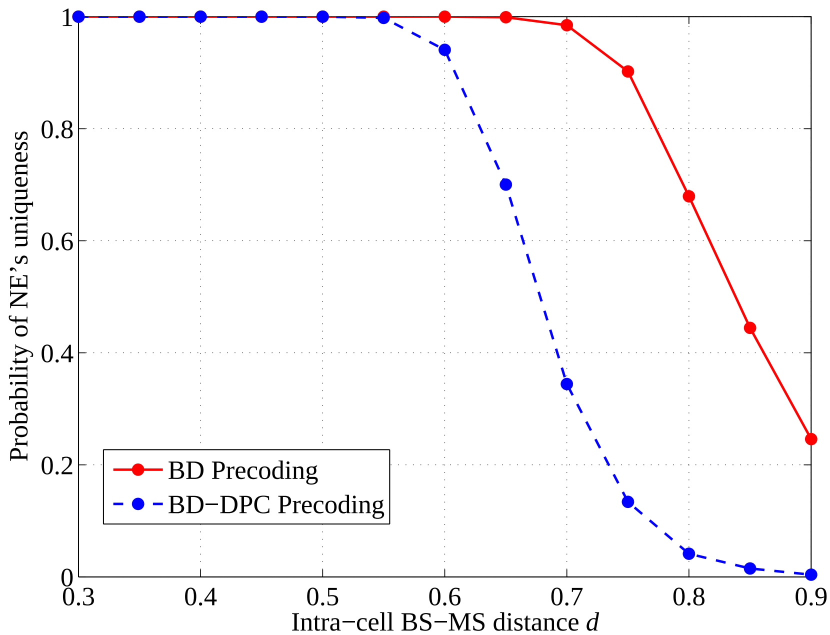

Fig. 4.2 Probability of NE's uniqueness versus the intra-cell BS-MS distance d.

result). In contrast, at high d, each MS is more susceptible to higher level of ICI (low-SINR region). As observed from the figure, the uniqueness of the NE (in both games $\mathcal{G}$ and $\mathcal{G}'$) is guaranteed if the ICI is sufficiently small, as suggested in our analytical result in Section 4.3. In addition, the condition of the NE's uniqueness in game $\mathcal{G}'$ is much tighter than that in game $\mathcal{G}$, as analyzed in Section 4.5.2.

Fig. 4.3 illustrates the network sum-rates obtained from the BD, BD-DPC, and DPC precoding versus the intra-cell BS-MS distance d under both IA and IC modes. It is observed from the figure that increasing the ICI powers results in significant sum-rate reductions in the multicell system. Under the IA mode, while the sum-rate in the DPC precoding game is always higher than the BD and BD-DPC games due to the optimality of DPC on a per-cell basis, the performance difference is rather small. In this multicell setting, the BD and BD-DPC multicell games are much simpler to analyze than the DPC precoding game. Under the IC mode, it can be observed that the BD-DPC precoder obtains a performance very close to that of the DPC precoder, especially at the high SINR (low ICI) region. Thus, it may be more advantageous to utilize BD or BD-DPC precoding over

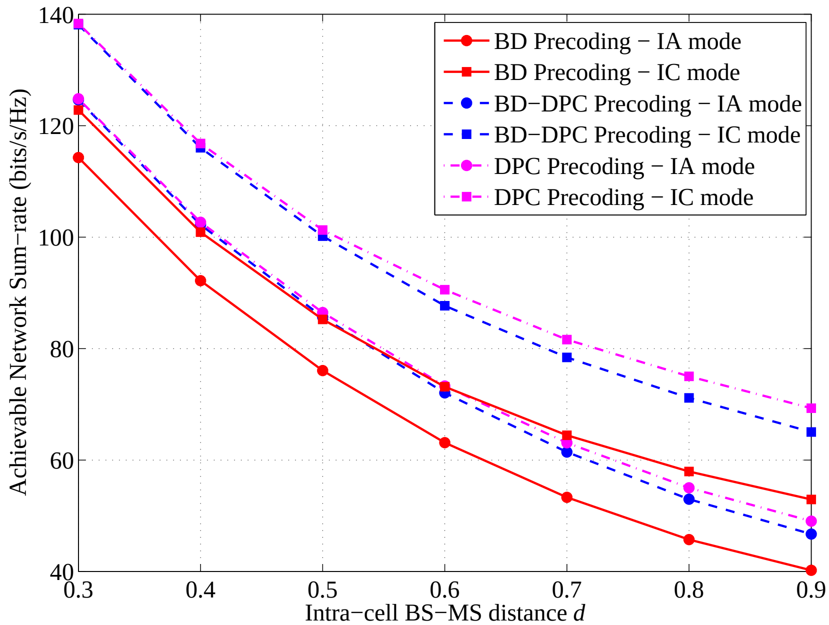

Fig. 4.3 Network sum-rates versus the intra-cell BS-MS distance d.

DPC because of their simpler implementation. In comparing the IC and IA modes, it can be observed that the network sum-rates can be significantly improved by coordinating the ICI. However, this performance advantage comes with the requirement of control signaling and CSI exchange among the coordinated BSs.

Fig. 4.4 illustrates the network sum-rates obtained from the BD, BD-DPC, and DPC precoding versus the transmit power to AWGN ratio P/σ^2 for $d = 0.7$ (assuming the same power budget P at all Q BSs). It is observed that increasing the transmit power at each BS does improve the network sum-rates in both IA and IC mode. However, at very high level of transmit power, the network sum-rates obtained from the multicell precoding games become saturated. This is due to the fact that the ICI is also increased relatively with the intra-cell information signal powers. In this case, it is desirable to coordinate and limit the amount of ICI by the IC mode. Apparently, the IC mode does perform much better than the IA mode with all 3 precoding designs at the high ICI region.

To illustrate the convergence of the multicell precoding games $\mathcal{G}$ and $\mathcal{G}'$, we select a specific channel realization and plot the achievable sum-rates versus the number of iterations of the two designs in Fig. 4.5. In both games, the BSs perform sequential precoder updates.

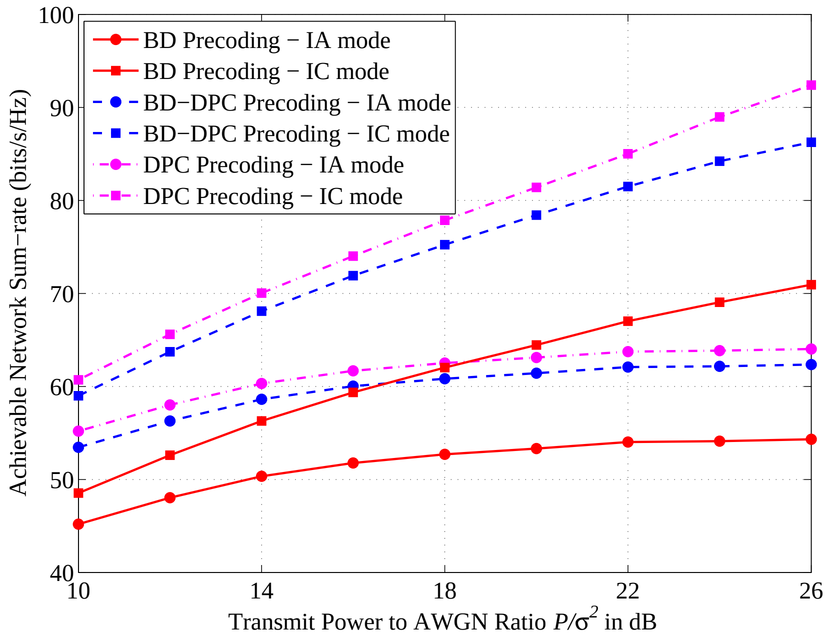

Fig. 4.4 Network sum-rates versus the transmit power to AWGN ratio at each BS for $d = 0.7$.

The network sum-rates and the sum-rates in each cell are then plotted after each instance of updating. It is observed that both games converge very quickly in a few iterations. As expected, the BD-DPC game results in a higher network sum-rate over the BD game due to the superior performance of BD-DPC precoding over BD precoding on a per-cell basis.

Finally, Fig. 4.6 illustrates the convergence of the proposed ILA algorithm to maximize the network sum-rate under the IC mode. For the same channel realization utilized to generate Fig. 4.5, we plot the network sum-rates and sum-rates in each cell after each time instance. As observed in the figure, the ILA algorithm monotonically converges with the sequential updates at the coordinated BSs for both the cases of BD and BD-DPC precoding. At each update, even though the sum-rate at one of the cells may decrease, the network sum-rate is always improved. This convergence behavior of the ILA algorithm agrees with our analysis in Theorem 4.2. As expected, the BD-DPC precoding converges to a better sum-rate than the BD precoding due to its superior performance on a per-cell basis. Compared to the convergence of the BD and BD-DPC games in Fig. 4.5, the ILA algorithm takes more iterations to converge. Nonetheless, the ILA algorithm provides

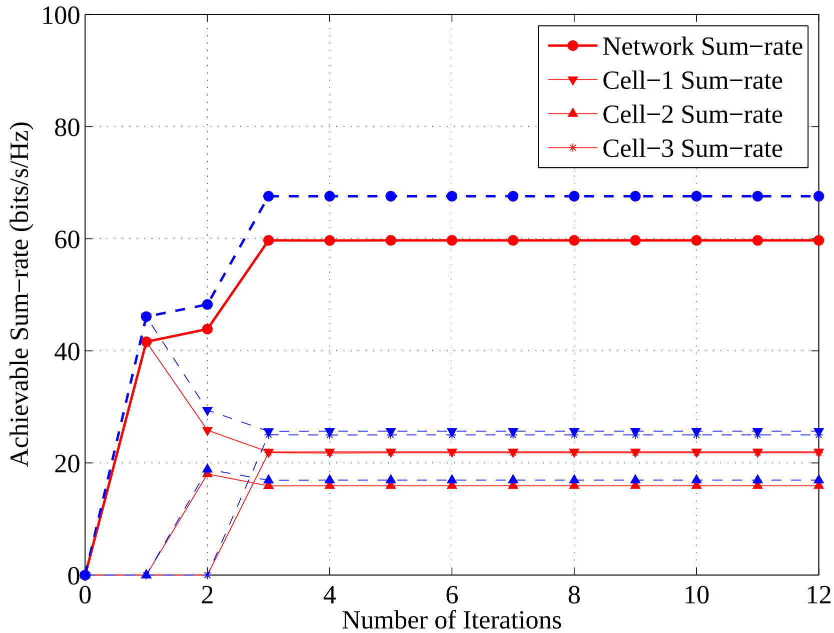

Fig. 4.5 Sum-rates versus number of iterations for $d = 0.7$ in the IA mode (*solid* lines are for the BD precoding game and *dashed-dotted* lines are for the BD-DPC precoding game).

better sum-rate performances for both BD and BD-DPC precoding.

4.7 Concluding Remarks

This chapter studied the multicell system with universal frequency reuse where BD or BD-DPC precoding is performed on a per-cell basis. When the multicell system is under *competition* mode, we investigated the conditions on the existence and uniqueness of the multicell games' NE. Simulation results confirmed that the NE of the multicell games is unique if the ICI is sufficiently small. They also indicated that the BD-DPC multicell precoding game outperforms the BD game while achieving a sum-rate very close to that of the DPC precoding game. When the multicell system is under *coordination* mode, we proposed the low-complexity and distributed ILA algorithm to obtain at least a local optimal solution to the nonconvex WSR maximization problems. Simulation results then show that the network sum-rate can be improved over the *competition* mode by coordinating the BD or BD-DPC precoders across the multicell system.

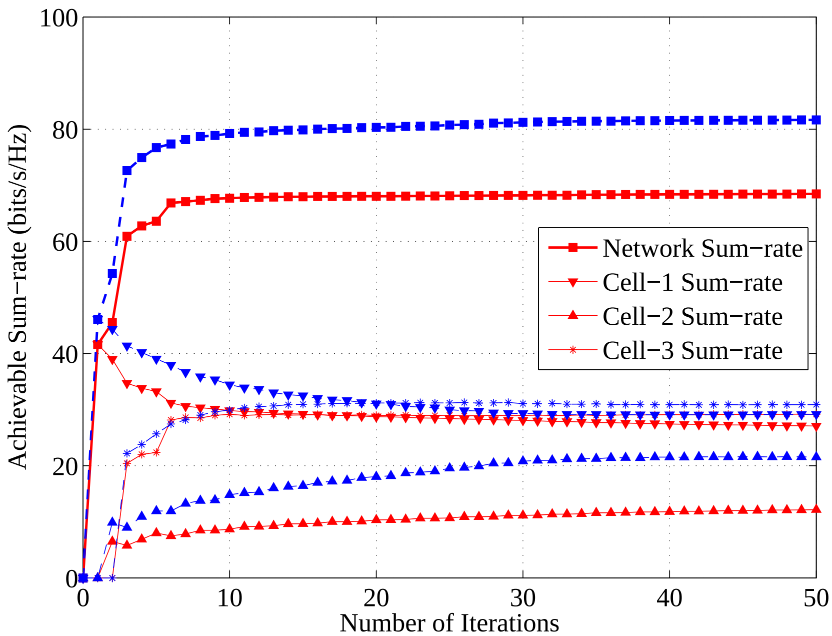

Fig. 4.6 Sum-rates versus number of iterations for $d = 0.7$ in the IC mode (*solid* lines are for the BD precoding and *dashed-dotted* lines are for the BD-DPC precoding).

Chapter 5

Sum-rate Maximization in the Multicell MIMO Multiple-Access Channel with Interference Coordination

5.1 Introduction

In the downlink direction, CoMP coordinates the simultaneous information transmissions from multiple BSs to multiple MSs, especially to the ones in the cell-edge region. In the uplink direction, CoMP allows the system to take advantage of the multiple receptions at the multiple cells to jointly decode the uplink signal from the MSs. In Chapters 3 and 4, it has been shown that significant power reduction or rate enhancement can be obtained by such a joint CoMP precoding design across the coordinated BSs in the downlink transmission. Similarly, in the uplink direction, it is expected that the system performance can be also improved by exploiting interference coordination among transmitting MSs. However, to the best of our knowledge, no work in the literature has addressed this coordinated precoding design to realize its performance enhancement in the uplink transmissions. In contrast to the downlink direction, where the coordinated precoders are designed at the

The materials presented in Chapter 5 have been presented at the 2012 IEEE Wireless Communications and Networking Conference in Paris, France [83], and accepted for publication in the IEEE Transactions on Wireless Communications [84].

BSs, it is desirable for the uplink counterpart that each MS is able to determine its precoder distributively with local information only. In this case, the role of the BSs is to exchange useful control signaling to the MSs so that each MS can optimize its precoder on its own. On the other hand, the precoder at each MS has to be devised in a coordinated manner in order to maximize the link performance to its connected BS while minimizing its induced ICI to other BSs.

In this chapter, we examine a coordinated multicell system in a general setting with multiple MSs per cell, where each MS is equipped with multiple transmit antennas. In each cell, the multiple MSs concurrently transmit information signals to its connected BS, which emulates a MIMO multiple-access channel (MAC) system. Per the *coordination* mode, the BS only decodes the signals for its connected MSs by implementing the capacity-achieving decoding technique, namely successive interference cancellation (SIC). The main interest of this chapter is to jointly design the uplink precoders at the MSs with the objective of maximizing the network WSR. Since this WSR maximization problem is shown to be nonconvex, it is generally difficult and computationally complex to find its globally optimal solution. Thus, the main focus of this chapter is on proposing low-complexity algorithms to divide the nonconvex WSR maximization into a sequence of simpler convex problems.

It is known that the resource allocation problem (power allocation, precoder design, etc) for maximizing the WSR in an interference network is a challenging task. Even if there is only one MS per cell, the WSR maximization problem turns out to be nonconvex [81]. Several works in literature have examined different numerical techniques to design the transmit precoders to maximize the WSR. Specifically, the gradient projection method was applied in [81] to search for a locally optimal transmit strategy. The works in [55, 57] applied the successive convex approximation technique to decompose the original nonconvex problem into multiple convex problems, which can be solved separately at the transmitters. In particular, each transmitter optimizes its precoder to maximize its link data rate with an interference-penalty term on the interference induced to other links [55,57]. This approach, being referred to as iterative linear approximation (ILA) [58], can be traced back to earlier works in difference of convex (DC) programming [80, 85, 86], where the nonconvex parts are linearly approximated into the penalty terms. In [55, 57, 81], by considering only one single MS per cell, the decomposed problem can be readily solved in closed-form at its corresponding BS [57].

One other distributed approach to locally solve the nonconvex WSR maximization problem in an interference network is the weighted mean squared error (WMMSE) algorithm. The main concept of the WMMSE is the transformation of the WSR maximization problem into an equivalent WMMSE problem with some specially chosen weight matrices [33]. The WMMSE problem is then solved by alternatively optimizing the weight matrices, the precoders, and the minimum mean squared error (MSE) decoders. Initially proposed in [33] for the single-cell MIMO broadcast downlink channel, the WMMSE algorithm was considered in [87] to maximize the WSR with one data stream per link. Recently, the WMMSE approach has been extended to the multicell downlink system with multiple MSs per cell in [58], where the linear precoders are optimized for the multicell throughput maximization. Compared to the sequential update by the ILA algorithm, the WMMSE algorithm may converge faster due to its distributive and simultaneous updating procedure [58].

The main contribution of this chapter is the development of low-complexity algorithms to solve the nonconvex WSR maximization problem in the multicell MIMO-MAC. The two approaches, namely ILA and WMMSE, are considered in order to maximize the network WSR. In addition, this chapter presents the distributed implementation of each algorithm, which allows certain operations in the algorithm to be performed in a distributed manner among the coordinated BSs and MSs.

When applying the ILA algorithm to the multicell MIMO-MAC, the approximation and decomposition step converts the nonconvex problem into a sequence of multiple MAC sum-rate maximization problems with *interference-penalty* terms, where each problem corresponds to the MAC in each cell. We then show that each decomposed problem is a convex program, which facilitates the finding of its optimal solution at its corresponding BS. However, due to the consideration of multiple MSs per cell, a closed-form optimal solution to the decomposed problem is not readily available. Instead, by exploring the inherently decoupled constraints for the transmit covariance matrix of each MS, we derive an equivalent optimization problem that can be solved sequentially over each variable matrix by a fast-converging algorithm. Interestingly, the decomposition in the ILA algorithm then reveals the structure of the optimal uplink precoders. In addition, the ILA solution approach also reveals the message signaling mechanism to facilitate its distributed implementation among the coordinated cells.

When applying the WMMSE algorithm to the multicell MIMO-MAC, we show that the network WSR maximization problem can also be transformed into an equivalent WMMSE problem. Taking SIC into consideration, we then show how to optimally determine the

MMSE precoders at the MSs, the MMSE decoders, and the weight matrices at the BSs. In addition, we present the message passing mechanism among the BSs themselves and between the BS and its connected MSs in the multicell MIMO-MAC that facilitates the distributed implementation of the WMMSE algorithm. For both ILA and WMMSE algorithms, monotonic convergence to at least local optimal solutions is subsequently proven. Simulation results show that the proposed algorithms can significantly improve the network WSR, in comparison to the multicell system with no interference coordination among the BSs. The simulations also confirm the convergence analysis of the proposed algorithms.

5.2 System Model and Problem Formulation

We consider the multiuser uplink transmission of a multicell system with Q separate cells operating on the same frequency channel. In each cell, multiple MSs, each equipped with multiple transmit antennas, are sending independent data streams to its connected multiple-antenna BS. For simplicity of the presentation, it is assumed that the numbers of antennas at the BS and MS are M and N, respectively, and the number of MSs in each cell is K. Since the multicell system operates on the same frequency channel, the intended signal from a MS to a BS is now subject to the intra-cell interference from other MSs in the same cell, as well as the ICI from the MSs in other cells. In the coordinated design of this multicell system, the precoders at each MS are jointly optimized to fully manage both the inter-cell and intra-cell interferences.

Considering the MAC at a particular cell, say cell-q, the received signal $\mathbf{y}_q$ at its BS can be modeled as

$$\mathbf{y}_q = \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{x}_{q_i} + \sum_{r\neq q}^{Q}\sum_{i=1}^{K} \mathbf{H}_{qr_i}\mathbf{x}_{r_i} + \mathbf{z}_q, \tag{5.1}$$

where $\mathbf{x}_{r_i} \in \mathbb{C}^{N\times 1}$ is the transmitted vector from N-antenna MS-i in the rth cell, $\mathbf{H}_{qr_i}$ models the channel from MS-i of cell-r to the qth BS, and $\mathbf{z}_q$ is the zero-mean additive Gaussian noise vector with the covariance matrix $\mathbf{Z}_q$.

Assuming linear precoding at each MS, the transmitted signal from MS-i in cell-q can be expressed as

$$\mathbf{x}_{q_i} = \mathbf{V}_{q_i}\mathbf{s}_{q_i}, \tag{5.2}$$

where $\mathbf{s}_{q_i} \in \mathbb{C}^{D\times 1}$ represents the information signal vector for MS-i, $\mathbf{V}_{q_i} \in \mathbb{C}^{N\times D}$ is the precoder matrix for MS-i, and $D = \min(M, N)$ is the maximum number of spatial data streams that can be supported by MS-i. Without loss of generality, columns of the precoding matrix $\mathbf{V}_{q_i}$ may be set to zero if the corresponding streams are not active. While our formulation allows each MS to multiplex up to D independent spatial streams, it may not be clear *a priori* how many spatial streams are actually active (capable of carrying information data). Instead, after the optimization process to determine the optimal precoder $\mathbf{V}_{q_i}$ for MS-i, the actual number of data streams for MS-i can be determined easily by assessing the rank of $\mathbf{V}_{q_i}$. It is assumed that $\mathbb{E}\left[\mathbf{s}_{q_i}\mathbf{s}_{q_i}^H\right] = \mathbf{I}$. In addition, the precoder $\mathbf{V}_{q_i}$ is constrained by transmit power limit P_{q_i}, *i.e.*,

$$\mathrm{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\} \leq P_{q_i}. \tag{5.3}$$

Let $\mathbf{X}_{q_i} = \mathbb{E}\left[\mathbf{x}_{q_i}\mathbf{x}_{q_i}^H\right] = \mathbf{V}_{q_i}\mathbf{V}_{q_i}^H$ be the transmit covariance matrix of MS-i of cell-q. Since $\mathrm{rank}\{\mathbf{X}_{q_i}\} = \mathrm{rank}\{\mathbf{V}_{q_i}\}$, should the optimization be carried over $\mathbf{X}_{q_i}$, assessing the rank of $\mathbf{X}_{q_i}$ then reveals the number of active streams supported by MS-i of cell-q. Denote $\mathbf{X}_q = \{\mathbf{X}_{q_i}\}_{i=1}^K$ as the uplink precoder profile of the K users in cell-q. Likewise, denote $\mathbf{X}_{-q} = (\mathbf{X}_1, \ldots, \mathbf{X}_{q-1}, \mathbf{X}_{q+1}, \ldots, \mathbf{X}_Q)$ as the precoding profile of all cells except cell-q. Denote

$$\mathbf{z}_{-q} = \sum_{r\neq q}^{Q}\sum_{i=1}^{K}\mathbf{H}_{qr_i}\mathbf{x}_{r_i} + \mathbf{z}_q \tag{5.4}$$

as the total ICI plus noise (IPN) at BS-q, and

$$\mathbf{R}_q = \mathbb{E}\left[\mathbf{z}_{-q}\mathbf{z}_{-q}^H\right] = \sum_{r\neq q}^{Q}\sum_{i=1}^{K}\mathbf{H}_{qr_i}\mathbf{X}_{r_i}\mathbf{H}_{qr_i}^H + \mathbf{Z}_q \tag{5.5}$$

as the covariance matrix of the IPN at BS-q.

In the coordinated design being considered, each BS only attempts to decode the signals from its connected MSs using the capacity-achieving multiuser decoding technique, namely successively interference cancellation (SIC) [88]. For instance, BS-q employs SIC for the transmissions from the K MSs within cell-q. Assuming the decoding order from MS-1 (first) to MS-K (last), for a certain IPN covariance $\mathbf{R}_q$, the achievable rate of user-i in cell-q can be expressed as

$$R_{q_i}(\mathbf{X}_q, \mathbf{X}_{-q}) = \log \frac{\left|\mathbf{R}_q + \sum_{j=i}^{K} \mathbf{H}_{qq_j} \mathbf{X}_{q_j} \mathbf{H}_{qq_j}^H\right|}{\left|\mathbf{R}_q + \sum_{j>i}^{K} \mathbf{H}_{qq_j} \mathbf{X}_{q_j} \mathbf{H}_{qq_j}^H\right|}, \tag{5.6}$$

where the intra-cell interference from user-1 to user-$(i-1)$ has been suppressed. Collectively, the MAC sum-rate of all K users in cell-q is given by [23]

$$R_q\left(\mathbf{X}_q, \mathbf{X}_{-q}\right) = \sum_{i=1}^{K} R_{q_i} = \log \left|\mathbf{I} + \mathbf{R}_q^{-1}\left(\sum_{i=1}^{K} \mathbf{H}_{qq_i} \mathbf{X}_{q_i} \mathbf{H}_{qq_i}^H\right)\right|, \tag{5.7}$$

where the denominators inside the log function are sequentially eliminated. Note that this sum-rate is obtained when BS-q does not decode the transmissions from the users in other cells. The network WSR is then given by $\sum_{q=1}^{Q} \omega_q R_q(\mathbf{X}_q, \mathbf{X}_{-q})$, where ω_q denotes the non-negative weight of cell-q. To maximize the network WSR, let us consider the following optimization

$$\begin{aligned}
\underset{\mathbf{X}_1,\ldots,\mathbf{X}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \log \left|\mathbf{I} + \mathbf{R}_q^{-1}\left(\sum_{i=1}^{K} \mathbf{H}_{qq_i} \mathbf{X}_{q_i} \mathbf{H}_{qq_i}^H\right)\right| \\
\text{subject to} \quad & \mathrm{Tr}\{\mathbf{X}_{q_i}\} \leq P_{q_i}, \ \forall i, \forall q \\
& \mathbf{X}_{q_i} \succeq 0, \ \forall i, \forall q.
\end{aligned} \tag{5.8}$$

It is observed that problem (5.8) is clearly a nonconvex problem due to the presence of $\mathbf{X}_{q_i}$'s in the interference terms $\mathbf{R}_r$'s, $r \neq q$. Thus, obtaining the globally optimal solution to the problem is computationally complex and intractable for practical applications. It may also require a centralized solver unit to obtain such a solution. In this case, designing low-complexity algorithms with distributed implementation to compute local optimizers becomes a more attractive option. To this end, we examine two simple and fast converging algorithms with the following goals: (i) to obtain at least locally optimal solutions to the problem, and (ii) to provide distributed implementation which requires neither a centralized unit nor full CSI across the coordinated cells.

5.3 The ILA Solution Approach for the Multicell MIMO-MAC

5.3.1 The ILA Algorithm for the Multicell MIMO-MAC

This section presents a solution approach to the original nonconvex problem (5.8) by reformlating it as a DC program [80, 85, 86]. Specifically, by iteratively isolating and approximating the nonconvex part of the objective function, the DC program will be decomposed into multiple convex optimization problems, which can be solved distributively at each MS with low complexity. In addition, the iterative procedure allows the MSs to continuously refine and improve their uplink precoders, which eventually yields a local optimal solution of the original problem.

Denote $f_q(\mathbf{X}_q, \mathbf{X}_{-q}) = \sum_{r\neq q}^{Q} \omega_r R_r(\mathbf{X}_q, \mathbf{X}_{-q})$ as the WSR of all cells except cell-q so that the network WSR can be expressed as $\omega_q R_q(\mathbf{X}_q, \mathbf{X}_{-q}) + f_q(\mathbf{X}_q, \mathbf{X}_{-q})$. Since $f_q(\mathbf{X}_q, \mathbf{X}_{-q})$ is not a concave function in $\mathbf{X}_{q_i}$, we take the approximation to this term. At a given value $(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q})$, after taking the Taylor expansion of f_q around $\bar{\mathbf{X}}_{q_i}$, $i = 1, \ldots, K$, and retaining the first linear term, we obtain

$$f_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q}) \approx f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\left\{\mathbf{A}_{q_i}\left(\mathbf{X}_{q_i} - \bar{\mathbf{X}}_{q_i}\right)\right\}, \tag{5.9}$$

where $\mathbf{A}_{q_i}$ is the negative partial derivative of f_q with respect to $\mathbf{X}_{q_i}$, evaluated at $\mathbf{X}_{q_i} = \bar{\mathbf{X}}_{q_i}$

$$\begin{aligned}
\mathbf{A}_{q_i} &= -\left.\frac{\partial f_q}{\partial \mathbf{X}_{q_i}}\right|_{\mathbf{X}_{q_i}=\bar{\mathbf{X}}_{q_i}} \\
&= -\sum_{r\neq q}^{Q} \omega_r \left.\frac{\partial R_r}{\partial \mathbf{X}_{q_i}}\right|_{\mathbf{X}_{q_i}=\bar{\mathbf{X}}_{q_i}} \\
&= \sum_{r\neq q}^{Q} \omega_r \mathbf{H}_{rq_i}^H \left[\mathbf{R}_r^{-1} - \left(\mathbf{R}_r + \sum_{j=1}^{K} \mathbf{H}_{rr_j}\mathbf{X}_{r_j}\mathbf{H}_{rr_j}^H\right)^{-1}\right] \mathbf{H}_{rq_i}\Bigg|_{\mathbf{X}_{q_i}=\bar{\mathbf{X}}_{q_i}}.
\end{aligned} \tag{5.10}$$

Using (5.9), the network WSR around $\bar{\mathbf{X}}_q$, $\omega_q R_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q}) + f_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q})$, can be approximated as $\omega_q R_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_{q_i}\mathbf{X}_{q_i}\} + \left[f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) + \sum_{i=1}^{K} \mathrm{Tr}\left\{\mathbf{A}_{q_i}\bar{\mathbf{X}}_{q_i}\right\}\right]$. Since the term $f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) + \sum_{i=1}^{K} \mathrm{Tr}\left\{\mathbf{A}_{q_i}\bar{\mathbf{X}}_{q_i}\right\}$ is now fixed, it does not affect the maximization of the network WSR. As a result, it can be omitted in the objective function, *i.e.*, maximizing $\omega_q R_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q}) + f_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q})$ is equivalent to maximizing $\omega_q R_q(\mathbf{X}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_{q_i}\mathbf{X}_{q_i}\}$, and the nonconvex problem (5.8) can be approximated as

$$\begin{aligned} \underset{\mathbf{X}_{q_1},\ldots,\mathbf{X}_{q_K}}{\text{maximize}} \ & \omega_q \log \left| \mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H \right| - \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_{q_i}\mathbf{X}_{q_i}\} \\ \text{subject to} \ & \mathrm{Tr}\{\mathbf{X}_{q_i}\} \le P_{q_i}, \ \forall i \\ & \mathbf{X}_{q_i} \succeq \mathbf{0}. \end{aligned} \tag{5.11}$$

which can be solved solely at cell-q. In other words, the optimization problem (5.8) can be approximately solved as Q per-cell separate optimization problems (5.11).

It is observed that the approximated problem (5.11) is similar to the MAC sum-rate maximization problem, studied in [23]. The difference here is the presence of the penalty term $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_{q_i}\mathbf{X}_{q_i}\}$, which encourages cell-$q$ to adopt a more cooperative precoding strategy by limiting the ICI to other cells. Should this term be absent, the multicell system is said to be in competition mode where each cell would selfishly maximize the sum-rate for its connected users only. This results in a noncooperative game among the cells, similar to the game studied in [12] for the case of one MS per cell. We shall present some numerical results for this noncooperative design in comparison to the considered coordinated design.

Note that the decomposed problem (5.11), corresponding to the precoder design at cell-q, is now a convex program, unlike the original problem (5.8). Thus, it can be readily solved by any efficient convex optimization technique [82]. However, these direct solution approaches may require a centralized solver unit at the BS, and hence are not suitable for distributed implementation at the MSs for the MAC. Fortunately, it is observed that the constraints for each transmit covariance matrix $\mathbf{X}_{q_i}$ are inherently decoupled in problem (5.11). By exploring this decoupled structure, the optimization (5.11) can be solved sequentially over each variable matrix, like the MAC sum-rate maximization problem in [23]. More importantly, the optimization process over each variable can be performed at the corresponding MS in a fully distributed manner. We elaborate these observations in the following theorem and later propose a fast-converging and distributed algorithm to solve problem (5.11).

Theorem 5.1. *For the K-user problem* (5.11), $\{\mathbf{X}_{q_i}\}_{i=1}^{K}$ *is an optimal solution if and only* *if* $\mathbf{X}_{q_i}$ *is the solution of the following optimization problem*

$$\begin{aligned} &\underset{\mathbf{X}_{q_i}}{\text{maximize}}\ \omega_q \log \left| \mathbf{I} + \mathbf{R}_{q_i}^{-1} \mathbf{H}_{qq_i} \mathbf{X}_{q_i} \mathbf{H}_{qq_i}^H \right| - \mathrm{Tr}\{\mathbf{A}_{q_i} \mathbf{X}_{q_i}\} \\ &\text{subject to } \mathrm{Tr}\{\mathbf{X}_{q_i}\} \le P_{q_i}, \mathbf{X}_{q_i} \succeq \mathbf{0}, \end{aligned} \tag{5.12}$$

where

$$\mathbf{R}_{q_i} = \mathbf{R}_q + \sum_{j \neq i}^{K} \mathbf{H}_{qq_j} \mathbf{X}_{q_j} \mathbf{H}_{qq_j}^H \tag{5.13}$$

is considered as noise.

Proof. The proof for this theorem follows the approach used in [23] for the sum-rate maximization in the MAC without the penalty components $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_{q_i} \mathbf{X}_{q_i}\}$. Before proceeding to the main part of the proof, we briefly revisit the solution of problem (5.12), which was previously given in [57].

Given μ_{q_i} as the Lagrangian multiplier associated with the power constraint $\mathrm{Tr}\{\mathbf{X}_{q_i}\} \le P_{q_i}$, it was shown in [57] that the optimal solution $\mathbf{X}_{q_i}^{\star}$ must be in the form of

$$\mathbf{X}_{q_i}^{\star} = \mathbf{G}_{q_i} \mathbf{P}_{q_i} \mathbf{G}_{q_i}^H, \tag{5.14}$$

where $\mathbf{G}_{q_i}$ is the (normalized) generalized eigen-matrix of the pair of matrices of $\mathbf{H}_{qq_i}^H \mathbf{R}_{q_i}^{-1} \mathbf{H}_{qq_i}$ and $(\mathbf{A}_{q_i} + \mu_{q_i} \mathbf{I})$. The matrix $\mathbf{P}_{q_i}$ is a non-negative diagonal matrix, obtained from the following WF solution

$$\mathbf{P}_{q_i} = \left[\omega_q \mathbf{\Sigma}_{q_i}^{(2)^{-1}} - \mathbf{\Sigma}_{q_i}^{(1)^{-1}} \right]^+, \tag{5.15}$$

where $\mathbf{\Sigma}_{q_i}^{(1)}$ and $\mathbf{\Sigma}_{q_i}^{(2)}$ are diagonal matrices given by

$$\begin{aligned} \mathbf{\Sigma}_{q_i}^{(1)} &= \mathbf{G}_{q_i}^H \mathbf{H}_{qq_i}^H \mathbf{R}_{q_i}^{-1} \mathbf{H}_{qq_i} \mathbf{G}_{q_i} \\ \mathbf{\Sigma}_{q_i}^{(2)} &= \mathbf{G}_{q_i}^H (\mathbf{A}_{q_i} + \mu_{q_i} \mathbf{I}) \mathbf{G}_{q_i}. \end{aligned}$$

In this solution, the dual variable μ_{q_i}, behaving as the water-level in the WF process, is adjusted to enforce the power constraint. Utilizing this optimal solution, we now show that the optimal solution to the multiuser problem (5.11) is indeed a collection of solutions to individual single-user problems (5.12).

If part: Let $\{\bar{\mathbf{X}}_{q_i}\}_{i=1}^{K}$ be the optimal solution to the original K-user problem (5.11). Suppose that $\bar{\mathbf{X}}_{q_i}$ is not the optimal solution of the corresponding problem (5.12) while treating $\mathbf{R}_{q_i} = \mathbf{R}_q + \sum_{j\neq i}^{K} \mathbf{H}_{qq_j}\bar{\mathbf{X}}_{q_j}\mathbf{H}_{qq_j}^H$ as noise. Then fixing all other covariance matrices $\bar{\mathbf{X}}_{q_j}, \forall j \neq i$, solving problem (5.12) obtains the optimal solution $\mathbf{X}_{q_i}^\star$. Clearly, $\mathbf{X}_{q_i}^\star$ strictly increases the objective function of the original problem (5.11). Thus, this contradicts with assumption on the optimality of $\{\bar{\mathbf{X}}_{q_i}\}_{i=1}^{K}$.

Only if part: Consider the partial Lagrangian of problem (5.11)

$$\mathcal{L}(\mathbf{X}_{q_i}, \boldsymbol{\mu}_q) = \sum_{i=1}^{K} \mu_{q_i} P_{q_i} + \omega_q \log \left| \mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H \right| - \sum_{i=1}^{K} \mathrm{Tr}\left\{(\mathbf{A}_{q_i} + \mu_{q_i}\mathbf{I})\mathbf{X}_{q_i}\right\}, \quad (5.16)$$

where $\boldsymbol{\mu}_q = [\mu_{q_1}, \ldots, \mu_{q_K}]^T$ are the Lagrangian dual variables associated with the power constraints. For optimality, the solution of problem (5.11) must satisfy the following Karush-Kuhn-Tucker (KKT) conditions

$$\begin{aligned} \omega_q \mathbf{H}_{qq_i}^H \left(\mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H \right)^{-1} \mathbf{H}_{qq_i} &= \mathbf{A}_{q_i} + \mu_{q_i}\mathbf{I}, \forall i \\ \mu_{q_i}\left(\mathrm{Tr}\left\{\mathbf{X}_{q_i}\right\} - P_{q_i}\right) &= 0, \forall i \\ \mu_{q_i} &\geq 0, \forall i. \end{aligned}$$

For the case of $K = 1$, it is straightforward to verify that the optimal solution of problem (5.12), $\mathbf{X}_{q_i}^\star = \mathbf{G}_{q_i}\mathbf{P}_{q_i}\mathbf{G}_{q_i}^H$, with $\mathbf{P}_{q_i}$ given in (5.15), satisfies the above KKT conditions. However, the KKT conditions for the single-user case are different from that of the multiuser case by the additional noise term $\sum_{j\neq i}^{K} \mathbf{H}_{qq_j}\mathbf{X}_{q_i}\mathbf{H}_{qq_j}^H$. Thus, if each $\mathbf{X}_{q_i}^\star$ satisfies the single-user condition while treating the signals of other MSs as noise, then collectively, the set of $\{\mathbf{X}_{q_i}^\star\}_{i=1}^{K}$ must satisfy the above KKT conditions for the multiuser case. Then, $\{\mathbf{X}_{q_i}^\star\}_{i=1}^{K}$ must be the optimal solution to the original problem (5.11). $\square$

Since the structure of the optimal solution to problem (5.11) has been revealed in Theorem 5.1, one can easily obtain its optimal solution by sequentially solving problem (5.12) for each user, *i.e.*, MS-1 to MS-K in cell-q, until convergence. We note that each problem (5.12) can be effectively solved by the WF process, as presented in [57]. This sequential optimization at cell-q accounts for the *inner-loop* iterative precoder updates of the K MSs in cell-q.

Algorithm 5.1: ILA Algorithm for Multicell MIMO-MAC

Initialize $\{\mathbf{X}_{q_i}\}_{\forall q, \forall i}$, such that $\mathrm{Tr}\{\mathbf{X}_{q_i}\} = P_{q_i}$;
repeat
  $\bar{\mathbf{X}}_{q_i} \leftarrow \mathbf{X}_{q_i}$;
  for $q = 1, 2, \ldots, Q$ **do**
    Compute $\mathbf{R}_q$ with $\bar{\mathbf{X}}_{q_i}$ at BS-q and exchange among the BSs;
    At BS-q, update the pricing matrix $\mathbf{A}_{q_i}$ at MS-i and perform;
    repeat
      for $i = 1, 2, \ldots, K$ **do**
        Compute $\mathbf{R}_{q_i}$ at the BS and pass it to MS-i;
        Perform $\underset{\mathbf{X}_{q_i}}{\text{maximize}}\ \omega_q \log \left| \mathbf{I} + \mathbf{R}_{q_i}^{-1} \mathbf{H}_{qq_i} \mathbf{X}_{q_i} \mathbf{H}_{qq_i}^H \right| - \mathrm{Tr}\left\{\mathbf{A}_{q_i} \mathbf{X}_{q_i}\right\}$, with $\mathrm{Tr}\{\mathbf{X}_{q_i}\} \leq P_{q_i}$ at MS-i;
      end
    until *convergence*;
  end
until *convergence*;

For the problem of Q cells (5.8), the proposed ILA algorithm requires each cell-q, $q = 1, \ldots, Q$ to continuously update the parameters $\{\mathbf{A}_{q_i}\}_{i=1}^K$ and to take turns to solve its corresponding optimization (5.11). This sequential procedure accounts for the *outer-loop* iterative updates across the Q cells. We summarize the ILA algorithm for the multicell MIMO-MAC in Algorithm 5.1. The convergence of the proposed ILA algorithm is given in the following theorem.

Theorem 5.2. *The Gauss-Seidel (sequential) iterative update always improves network WSR and is guaranteed to converge to at least a local maximum.*

Proof. Similar to the approach in [55, 57], the proof for this theorem is established by showing that the network sum-rate is strictly nondecreasing after an update at any given cell. Suppose that $\mathbf{X}_q = \bar{\mathbf{X}}_q = \left\{\bar{\mathbf{X}}_{q_i}\right\}_{i=1}^K, \forall q$ from the previous outer-loop iteration, and $\mathbf{X}_q^\star = \left\{\mathbf{X}_{q_i}^\star\right\}_{i=1}^K$ as the optimal solution obtained at cell-q after the current outer-loop iteration.

Similar to the technique applied in [57, 81], it can be easily shown that $f_q(\mathbf{X}_q, \mathbf{X}_{-q})$ is a convex function with respect to $\mathbf{X}_{q_i} \in \mathcal{S}_{q_i} \triangleq \{\mathbf{X}_{q_i} | \mathbf{X}_{q_i} \succeq \mathbf{0}, \mathrm{Tr}\{\mathbf{X}_{q_i}\} \leq P_{q_i}\}$. Thus, by the first-order condition for the convex function f_q [82], one has

$$f_q(\mathbf{X}_q^\star, \bar{\mathbf{X}}_{-q}) \geq f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\big\{\mathbf{A}_{q_i}(\mathbf{X}_{q_i}^\star - \bar{\mathbf{X}}_{q_i})\big\} \tag{5.17}$$

with $\mathbf{A}_{q_i}$ being defined in (5.10) at $\bar{\mathbf{X}}_{q_i}$.

After one Gauss-Seidel iteration, the network weighted sum-rate is updated such that

$$\begin{aligned}\sum_{q=1}^{Q} \omega_q R_q(\mathbf{X}_q^\star, \bar{\mathbf{X}}_{-q}) &= \omega_q R_q(\mathbf{X}_q^\star, \bar{\mathbf{X}}_{-q}) + f_q(\mathbf{X}_q^\star, \bar{\mathbf{X}}_{-q}) \\ &\geq \omega_q R_q(\mathbf{X}_q^\star, \bar{\mathbf{X}}_{-q}) + f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\big\{\mathbf{A}_{q_i}(\mathbf{X}_{q_i}^\star - \bar{\mathbf{X}}_{q_i})\big\} \\ &\geq \omega_q R_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) + f_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}) - \sum_{i=1}^{K} \mathrm{Tr}\big\{\mathbf{A}_{q_i}(\bar{\mathbf{X}}_{q_i} - \bar{\mathbf{X}}_{q_i})\big\} \\ &= \sum_{q=1}^{Q} \omega_q R_q(\bar{\mathbf{X}}_q, \bar{\mathbf{X}}_{-q}),\end{aligned}$$

where the first inequality is due to the one in (5.17), and the second inequality is due to the fact that $\mathbf{X}_q^\star$ is the optimal solution of problem (5.11). Since the network sum-rate is upper-bounded and nondecreasing after each update, the sequential optimization (5.11) generates a Cauchy sequence that must converge to one of the local maxima. □

It is worth mentioning that the proposed ILA algorithm can be executed by a central controller, which then passes the local optimal precoders to the corresponding MSs. In this case, the central controller must possess the full CSI knowledge of all channels in the network. On the other hand, it is possible to implement the proposed ILA algorithm in a distributed manner by assigning certain optimization steps in the algorithm to be performed each coordinated BS and MS. To this end, we next present the interpretation to the ILA algorithm that allows its distributed implementation. The complexity in implementing the algorithm will be then discussed in Section 5.5.

5.3.2 Distributed Implementation of the Proposed ILA Algorithm

In order to realize the distributed implementation of the proposed ILA algorithm, we make the following assumptions:

- *Assumption 1:* Each MS, say MS-i of cell-q, knows the channel matrices $\mathbf{H}_{rq_i}$'s to all the BS-r's in the network. This assumption allows the MS to control its induced ICI to other cells.

- *Assumption 2:* The coordinated BSs have reliable backhaul channels to exchange control information among themselves.

- *Assumption 3:* The channels are in block-fading or vary sufficiently slow such that they can be considered fixed during the optimization being performed.

It is to be noted that Algorithm 5.1 involves two levels of computations. At the inner-loop level, assuming $\mathbf{R}_q$ is known at BS-q and $\mathbf{A}_{q_i}$ is known at MS-i, cell-q performs the corresponding optimization (5.11) autonomously using the result from Theorem 5.1. The role of BS-q is to measure the total signaling plus noise $\mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H$, and then pass this value to its connected MSs. MS-i in cell-q, knowing its channel to its BS $\mathbf{H}_{qq_i}$, can compute the noise component $\mathbf{R}_{q_i}$. MS-i is then required to update its uplink covariance matrix by solving the optimization (5.12). This process, which corresponds to the inner-loop iterations, is performed until convergence in cell-q.

At the outer-loop level, each BS needs to exchange the data to compute the parameters $\{\mathbf{A}_{q_i}\}_{i=1}^{K}$ for the next update. It is observed from equation (5.10) that MS-i in cell-q needs to know the channels $\mathbf{H}_{rq_i}$'s to all the BSs (per Assumption 1), as well as the pricing matrix

$$\mathbf{B}_r = \mathbf{R}_r^{-1} - \left(\mathbf{R}_r + \sum_{j=1}^{K} \mathbf{H}_{rr_j}\mathbf{X}_{r_j}\mathbf{H}_{rr_j}^H\right)^{-1}, \tag{5.18}$$

in order to compute $\mathbf{A}_{q_i}$. Thus, it is required that each BS computes its corresponding price $\mathbf{B}_q, q = 1, \ldots, Q$, using local measurements on the IPN $\mathbf{R}_q$ and the total signal plus IPN $\mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H$. These factors are then exchanged among the BSs. Using the messages received from other cells, BS-q then can easily pass $\{\mathbf{B}_r\}_{r\neq q}$ to its connected MSs before the inner-loop iterative procedure. The outer-loop iteration is performed until the WSR reaches to a local maximum, as stated in Theorem 5.2.

Remark 5.1: It is shown in Theorem 5.2 that the proposed ILA algorithm allows the uplink precoders to be refined and improved after each Gauss-Seidel update, which ultimately converges to a local maximum. However, this update mechanism requires all the BSs to compute the pricing matrices $\mathbf{B}_q$'s and exchange them within the network after one cell updates its precoding matrices. To reduce the amount of information exchanges among the coordinated cells, the proposed algorithm can be also implemented using the Jacobi (simultaneous) iterative update. In particular, after the exchange of the pricing matrices, all the cells *simultaneously* update their precoding matrices. Specifically, the inner-loop iterations can be performed independently and concurrently in each cell. Although the convergence of the Jacobi update is not analytically proved, numerical simulations confirm its rapid convergence rate. Thus, in our simulations for the ILA algorithm, we utilize the Jacobi update instead of the Gauss-Seidel to reduce the computational time.

Remark 5.2: When the multicell MIMO-MAC system operates under the *competition* mode, the update of the precoders across the Q cells also involves two levels of iterations. In an outer-loop iteration, each BS, say BS-q, needs to measure its IPN covariance matrix $\mathbf{R}_q$. In an inner-loop iteration, BS-q needs to continuously measure and pass its total signal plus IPN $\mathbf{R}_q + \sum_{i=1}^{K} \mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}^H$ to its K connected MSs, while the MSs at cell-q take turns to selfishly maximize the MAC sum-rate of cell-q by the IWF procedure [23]. Compared to this *competition* mode, the ILA algorithm requires the inter-BS signaling in each outer-loop iteration for exchanging the pricing matrices $\mathbf{B}_q$'s. However, the pricing matrices $\mathbf{B}_r, r \neq q$ are required to be passed from BS-q to its K connected MSs only once before the inner-loop iterations. Thus, the ILA algorithm does require a similar amount of intra-cell BS-MS signaling as the *competition* mode.

5.4 The WMMSE Solution Approach for the Multicell MIMO-MAC

5.4.1 The WMMSE Algorithm for the Multicell MIMO-MAC

In Section 5.3, we have examined a linear convex approximation technique to solve the nonconvex optimization problem (5.8) by successively improving the uplink covariance matrices at the MSs. In this section, we examine a different approach to solve this optimization problem by relating it to a matrix-weighted sum-MSE minimization problem. In particular, the new problem of interest is to alternatively find the transmit beamformers $\mathbf{V}_{q_i}$'s and their corresponding receive beamformers $\mathbf{U}_{q_i}$'s, which shall be characterized shortly.

With $\mathbf{V}_{q_i}$'s as the variables to be optimized, the optimization problem (5.8) can be restated as

$$\begin{aligned} \underset{\{\mathbf{V}_{q_i}\}_{\forall i, \forall q}}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} R_{q_i} \\ \text{subject to} \quad & \text{Tr}\{\mathbf{V}_{q_i} \mathbf{V}_{q_i}^H\} \leq P_{q_i}, \ \forall i, \forall q, \end{aligned} \tag{5.19}$$

where the achievable rate R_{q_i}, given in (5.6), can be rewritten as

$$R_{q_i} = \log \frac{\left| \mathbf{Z}_q + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{qr_j} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{qr_j}^H + \sum_{j=i}^{K} \mathbf{H}_{qq_j} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_j}^H \right|}{\left| \mathbf{Z}_q + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{qr_j} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{qr_j}^H + \sum_{j>i}^{K} \mathbf{H}_{qq_j} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_j}^H \right|}. \tag{5.20}$$

It is to be noted that this achievable rate can be stated as a function of the error covariance matrix after the MMSE receive filtering. Since SIC is applied at each BS to its connected MSs, the signal from user-i is not corrupted by the intra-cell interference from user-i to user-$(i-1)$. Thus, while treating the interference as noise, the estimated signal for user-i in BS-q is then given by

$$\hat{\mathbf{s}}_{q_i} = \mathbf{U}_{q_i}^H \left(\sum_{j=i}^{K} \mathbf{H}_{qq_j} \mathbf{V}_{q_j} \mathbf{s}_{q_j} + \mathbf{z}_q \right), \tag{5.21}$$

where $\mathbf{U}_{q_i}^H$ is the linear receive beamformer for user-i. This receive beamformer is designed such that the MSE for the data streams from user-i of cell-q is minimized. As the MSE matrix $\mathbf{E}_{q_i}$ is obtained from

$$\begin{aligned} \mathbf{E}_{q_i} &= \mathbb{E}\left[(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})^H \right] \\ &= \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \right) \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \right)^H + \sum_{j>i}^{K} \mathbf{U}_{q_i}^H \mathbf{H}_{qq_j} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_j}^H \mathbf{U}_{q_i} \\ &\quad + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{U}_{q_i}^H \mathbf{H}_{qr_j} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{qr_j}^H \mathbf{U}_{q_i} + \mathbf{U}_{q_i}^H \mathbf{Z}_q \mathbf{U}_{q_i}, \end{aligned} \tag{5.22}$$

the optimal receive beamformer is indeed the well-known MMSE filter

$$\begin{aligned}\mathbf{U}_{q_i} &= \arg\min_{\mathbf{U}_{q_i}} \mathrm{Tr}\left\{\mathbf{E}_{q_i}\right\} \\ &= \left(\sum_{j=i}^{K} \mathbf{H}_{qq_j}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_j}^H + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{qr_j}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{qr_j}^H + \mathbf{Z}_q\right)^{-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_i}.\end{aligned} \tag{5.23}$$

Consequently, the MMSE matrix for user-i in cell-q is given by

$$\begin{aligned}\mathbf{E}_{q_i}^{\mathrm{MMSE}} &= \mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i} \\ &= \Bigg[\mathbf{I} + \mathbf{V}_{q_i}^H\mathbf{H}_{qq_i}^H\Bigg(\sum_{j>i}^{K} \mathbf{H}_{qq_j}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_j}^H \\ &\qquad + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{qr_j}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{qr_j}^H + \mathbf{Z}_q\Bigg)^{-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_i}\Bigg]^{-1}.\end{aligned} \tag{5.24}$$

Given (5.20) and (5.24), the relationship between the data rate and the MMSE matrix can be stated as

$$R_{q_i} = \log\left|\left(\mathbf{E}_{q_i}^{\mathrm{MMSE}}\right)^{-1}\right|. \tag{5.25}$$

Utilizing this relationship, we establish the equivalence between the WSR maximization problem in the multicell MIMO-MAC and the matrix-weighted sum-MSE minimization in the following theorem.

Theorem 5.3. *The multicell MIMO-MAC WSR maximization problem* (5.19) *is equivalent to the following matrix weighted sum-MSE minimization*

$$\begin{aligned}&\underset{\mathbf{W}_{q_i},\mathbf{V}_{q_i},\mathbf{U}_{q_i}}{\text{minimize}} && \sum_{q=1}^{Q}\omega_q\sum_{i=1}^{K}\left[\mathrm{Tr}\left\{\mathbf{W}_{q_i}\mathbf{E}_{q_i}\right\} - \log\left|\mathbf{W}_{q_i}\right|\right] \\ &\text{subject to} && \mathrm{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\} \leq P_{q_i}, \forall q, \forall i,\end{aligned} \tag{5.26}$$

where $\mathbf{W}_{q_i} \succeq \mathbf{0}$ *is the weight matrix for MS-i at cell-q. In particular, the globally optimal solutions* $\{\mathbf{V}\}_{\forall q,\forall i}$ *are identical for the two problems.*

Proof. The proof for this theorem is in the same spirit as the proof in [33, 58] for the case of downlink transmission. First, it is noticed that there are no constraints to the weight matrices $\mathbf{W}_{q_i}$'s and the receive beamformers $\mathbf{U}_{q_i}$'s in problem (5.26). Fixing all other variables, the optimal weight matrices $\mathbf{W}_{q_i}$'s are given by

$$\mathbf{W}_{q_i} = \mathbf{E}_{q_i}^{-1} = \mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}. \tag{5.27}$$

On the other hand, the optimal receive beamformers $\mathbf{U}_{q_i}$'s to problem (5.26) are the MMSE receivers, given in (5.23). Thus, with the optimal $\mathbf{W}_{q_i}$'s and $\mathbf{U}_{q_i}$'s, the following optimization problem is equivalent to problem (5.26)

$$\begin{aligned} \underset{\{\mathbf{V}_{q_i}\}_{\forall i, \forall q}}{\text{minimize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \log \left| \mathbf{E}_{q_i}^{\text{MMSE}} \right| \\ \text{subject to} \quad & \text{Tr}\left\{ \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \right\} \leq P_{q_i}, \forall q, \forall i. \end{aligned} \tag{5.28}$$

This problem is indeed the WSR maximization problem for the multicell MIMO-MAC (5.19), due to the connection between the MMSE matrix $\mathbf{E}_{q_i}^{\text{MMSE}}$ and the rate $\mathbf{R}_{q_i}$ given in (5.25). □

Having established the equivalence between the two optimization problems (5.19) and (5.26), we now proceed to solve the latter problem. Note that the objective function in (5.26) is convex in each of the optimization variables $\mathbf{U}_{q_i}, \mathbf{V}_{q_i}, \mathbf{W}_{q_i}$. Thus, it is possible to solve problem (5.26) by alternately optimizing one of the variables while fixing the other two until convergence. First, with fixed transmit beamformers $\mathbf{V}_{q_i}$'s, the receive beamformers $\mathbf{U}_{q_i}$'s are optimally designed as in (5.23). Second, fixing the transmit and receive beamformers $\mathbf{V}_{q_i}$'s and $\mathbf{U}_{q_i}$'s, the weighted-matrices $\mathbf{W}_{q_i}$'s are updated in a closed form solution, given in (5.27). Finally, by decomposing the objective function in (5.26), the update of the transmit beamformers $\mathbf{V}_{q_i}$'s are carried by solving decoupled optimization problems across the MSs

$$\begin{aligned} \underset{\mathbf{V}_{q_i}}{\text{minimize}} \quad & \omega_q \text{Tr}\left\{ \mathbf{W}_{q_i} \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \right) \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \right)^H \right\} \\ & + \omega_q \sum_{j=1}^{i-1} \text{Tr}\left\{ \mathbf{W}_{q_j} \mathbf{U}_{q_j}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \mathbf{H}_{qq_i}^H \mathbf{U}_{q_j} \right\} \\ & + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \omega_r \text{Tr}\left\{ \mathbf{W}_{r_j} \mathbf{U}_{r_j}^H \mathbf{H}_{rq_i} \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \mathbf{H}_{rq_i}^H \mathbf{U}_{r_j} \right\} \\ \text{subject to} \quad & \text{Tr}\left\{ \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \right\} \leq P_{q_i}. \end{aligned} \tag{5.29}$$

It is to be noted that this optimization can be carried independently and simultaneously across the MSs. As stated in [33,58], this problem is a convex quadratic program, which can be optimally solved by standard optimization techniques. Using the Lagrangian duality, the optimal solution to (5.29) can be derived as

$$\mathbf{V}_{q_i} = \omega_q \left(\sum_{r \neq q}^{Q} \sum_{j=1}^{K} \omega_r \mathbf{H}_{rq_i}^H \mathbf{U}_{r_j} \mathbf{W}_{r_j} \mathbf{U}_{r_j}^H \mathbf{H}_{rq_i} + \sum_{j=1}^{i} \omega_q \mathbf{H}_{qq_i}^H \mathbf{U}_{q_j} \mathbf{W}_{q_j} \mathbf{U}_{q_j}^H \mathbf{H}_{qq_i} + \mu_{q_i}^{\star} \mathbf{I} \right)^{-1} \mathbf{H}_{qq_i}^H \mathbf{U}_{q_i} \mathbf{W}_{q_i}, \tag{5.30}$$

where $\mu_{q_i}^{\star}$ is the optimal Lagrangian multiplier associated with the power constraint at the MS. It is noticed that in case of $\mu_{q_i}^{\star} = 0$ resulting in $\mathrm{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\} < P_{q_i}$, the corresponding MS effectively does not transmit at full power. Otherwise, $\mu_{q_i}^{\star}$ can be easily obtained by the bisection method such that the power constraint is met with equality.

Algorithm 5.2: WMMSE Algorithm for Multicell MIMO-MAC

Initialize $\{\mathbf{V}_{q_i}\}_{\forall q, \forall i}$, such that $\mathrm{Tr}\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\} = P_{q_i}$.
repeat
  Set $\bar{\mathbf{V}}_{q_i} \leftarrow \mathbf{V}_{q_i}, \ \forall q, \forall i$.
  Simultaneously update across Q cells
  for $q = 1, \ldots, Q$ **do**
    At the BS, sequentially update the receive beamformers and weight matrices;
    for $i = 1, \ldots, K$ **do**
      $\mathbf{U}_{q_i} \leftarrow \left(\sum_{j=i}^{K} \mathbf{H}_{qq_j} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_j}^H + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{qr_j} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{qr_j}^H + \mathbf{Z}_q \right)^{-1} \mathbf{H}_{qq_i} \mathbf{V}_{q_i}$;
      $\mathbf{W}_{q_i} \leftarrow \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{q_i} \mathbf{V}_{q_i}\right)^{-1}$;
    end
    At the K MSs, simultaneously update the transmit matrices;
    $\mathbf{V}_{q_i} \leftarrow$
    $\omega_q \left(\sum_{j=1}^{i} \omega_q \mathbf{H}_{qq_i}^H \mathbf{U}_{q_j} \mathbf{W}_{q_j} \mathbf{U}_{q_j}^H \mathbf{H}_{qq_i} + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \omega_r \mathbf{H}_{rq_i}^H \mathbf{U}_{r_j} \mathbf{W}_{r_j} \mathbf{U}_{r_j}^H \mathbf{H}_{rq_i} + \mu_{q_i}^{\star} \mathbf{I} \right)^{-1} \mathbf{H}_{qq_i}^H \mathbf{U}_{q_i} \mathbf{W}_{q_i}, \forall i$;
  end
until *convergence*;

We summarize the proposed WMMSE algorithm for the multicell MIMO-MAC as in Algorithm 5.2. In this algorithm, at each outer-loop iteration, the updates can be performed simultaneously across the Q coordinated cells. At *each* cell-q, the corresponding BS-q *sequentially* updates the receive beamformers $\mathbf{U}_{q_i}$ and the weight matrices $\mathbf{W}_{q_i}$ for its MS-i for i from 1 to K. After that, the transmit beamformers $\mathbf{V}_{q_i}$'s can be updated *simultaneously* by its MS-i. The iterative process is performed until each variable converges to a locally optimal solution.

We summarize the convergence and the optimality of the proposed WMMSE algorithm in the following theorem.

Theorem 5.4. *The alternating minimization process in the proposed WMMSE algorithm results in a monotonic decrease of the objective function in problem* (5.26). *For any limit point* $(\mathbf{W}^\star_{q_i}, \mathbf{U}^\star_{q_i}, \mathbf{V}^\star_{q_i})$ *that is the minimizer of problem* (5.26), $\mathbf{V}^\star_{q_i}$ *is also a local minimizer of the original problem* (5.19).

Proof. Denote

$$f(\{\mathbf{V}_{q_i}\}) = \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \log \left| \mathbf{E}^{\text{MMSE}}_{q_i} \right| \tag{5.31}$$

and

$$\begin{aligned} & g(\{\mathbf{U}_{q_i}\}, \{\mathbf{W}_{q_i}\}, \{\mathbf{V}_{q_i}\}) \\ & \quad = \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \left[\text{Tr}\left\{ \mathbf{W}_{q_i} \mathbf{E}_{q_i} \right\} - \log |\mathbf{W}_{q_i}| \right] \end{aligned} \tag{5.32}$$

as the cost functions of the original problem (5.19) and the restated WMMSE problem (5.26).

Since the constraint set of problem (5.26) is decoupled for the variables $\mathbf{U}_{q_i}, \mathbf{W}_{q_i}, \mathbf{V}_{q_i}$, applying the block coordinate descent method by alternatively minimizing over $\mathbf{U}_{q_i}, \mathbf{W}_{q_i}, \mathbf{V}_{q_i}$ must decrease its cost function monotonically [89]. In addition, if the power constraint on $\mathbf{V}_{q_i}$ is upper-bounded, the cost function (5.32) is lower-bounded. Thus, the proposed WMMSE must monotonically converge to at least a local minimum of the cost function (5.32). Note that the cost function of the original sum-rate maximization problem (5.19) does not necessarily improve after each iteration. However, given $(\mathbf{U}^\star_{q_i}, \mathbf{W}^\star_{q_i}, \mathbf{V}^\star_{q_i})$ as a local minimizer of problem (5.26) obtained from the WMMSE algorithm, it can be proved that $\mathbf{V}^\star_{q_i}$ is also a local minimizer of the original problem (5.19) as follows.

First, we need to show that the gradients of $f(\cdot)$ and $g(\cdot)$ with respect to $\mathbf{V}_{q_i}$ are the same at $\mathbf{V}^\star_{q_i}$. Similar to the approach in [33, 58], evaluating the gradients of $f(\cdot)$ and $g(\cdot)$ at the (m,n)-element of $\mathbf{V}_{q_i}$, one has

$$\begin{aligned}\left.\frac{\partial f(\{\mathbf{V}_{q_i}\})}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i}} &= \sum_{r=1}^{Q}\omega_r\sum_{j=1}^{K}\left.\frac{\partial\log\left|\mathbf{E}^{\mathrm{MMSE}}_{r_j}(\mathbf{V}_{q_i})\right|}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i}}\\ &= \sum_{r=1}^{Q}\omega_r\sum_{j=1}^{K}\mathrm{Tr}\left\{\left(\mathbf{E}^{\mathrm{MMSE}}_{r_j}(\mathbf{V}^\star_{q_i})\right)^{-1}\times\left.\frac{\partial\mathbf{E}^{\mathrm{MMSE}}_{r_j}(\mathbf{V}_{q_i})}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i}}\right\},\end{aligned} \tag{5.33}$$

and

$$\begin{aligned}\left.\frac{\partial g(\{\mathbf{U}_{q_i}\},\{\mathbf{W}_{q_i}\},\{\mathbf{V}_{q_i}\})}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i},\mathbf{U}_{q_i}=\mathbf{U}^\star_{q_i}} &= \sum_{r=1}^{Q}\omega_r\sum_{j=1}^{K}\mathbf{W}_{r_j}\left.\frac{\partial\mathbf{E}_{r_j}(\mathbf{U}_{q_i},\mathbf{V}_{q_i})}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i},\mathbf{U}_{q_i}=\mathbf{U}^\star_{q_i}}\\ &= \sum_{r=1}^{Q}\omega_r\sum_{j=1}^{K}\mathbf{W}^\star_{r_j}\left.\frac{\partial\mathbf{E}_{r_j}(\mathbf{U^\star}_{q_i},\mathbf{V}_{q_i})}{\partial[\mathbf{V}_{q_i}]_{m,n}}\right|_{\mathbf{V}_{q_i}=\mathbf{V}^\star_{q_i}}.\end{aligned} \tag{5.34}$$

Since $\mathbf{W}^\star_{r_j} = \left(\mathbf{E}^{\mathrm{MMSE}}_{r_j}(\mathbf{V}^\star_{q_i})\right)^{-1} = \left(\mathbf{E}_{r_j}(\mathbf{U}^\star_{q_i},\mathbf{V}^\star_{q_i})\right)^{-1}$, this yields the equivalence between (5.33) and (5.34). In addition, because $(\mathbf{U}^\star_{q_i},\mathbf{W}^\star_{q_i},\mathbf{V}^\star_{q_i})$ is a local minimizer of problem (5.26), it must satisfy the stationarity condition:

$$\mathrm{Tr}\left\{\nabla_{\mathbf{V}_{q_i}} g(\mathbf{U}^\star_{q_i},\mathbf{W}^\star_{q_i},\mathbf{V}^\star_{q_i})^H(\mathbf{V}_{q_i}-\mathbf{V}^\star_{q_i})\right\}\le 0, \forall\mathbf{V}_{q_i}. \tag{5.35}$$

Conversely, due to the equivalence between (5.33) and (5.34), $\mathbf{V}^\star_{q_i}$ also satisfies the stationarity condition:

$$\mathrm{Tr}\left\{\nabla_{\mathbf{V}_{q_i}} f(\mathbf{V}^\star_{q_i})^H(\mathbf{V}_{q_i}-\mathbf{V}^\star_{q_i})\right\}\le 0, \forall\mathbf{V}_{q_i}. \tag{5.36}$$

Thus, $\mathbf{V}_{q_i}$ must be also a local minimizer to the original problem (5.19). □

Like the ILA algorithm, the WMMSE algorithm can be implemented either at a central controller or distributively across the BSs and MSs, as discussed in the following section. The complexity in implementing the WMMSE algorithm will be addressed in Section 5.5.

5.4.2 Distributed Implementation of the Proposed WMMSE Algorithm

Note that the proposed WMMSE algorithm is executed by alternately computing the receive beamformers and the weight matrices at the BSs and computing the transmit beamformers at the MSs. Under the same assumptions stated in Section 5.3.2, the WMMSE algorithm can be implemented in a distributed manner as follows.

At the receiving end, each BS, say BS-q, needs to locally measure its IPN covariance matrix $\mathbf{R}_q$ and estimate the transmit beamformers $\mathbf{V}_{q_i}$ from its connected MSs. Knowing the decoding order, BS-q updates its receive beamformers $\mathbf{U}_{q_i}$'s and the weight matrices $\mathbf{W}_{q_i}$'s with local information as stated in (5.23) and (5.27). BS-q then computes the matrix $\omega_q \sum_{i=1}^{K} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H$ and exchanges it to the other coordinated BSs in the network. Subsequently, BS-q passes the updated matrices $\omega_r \sum_{j=1}^{K} \mathbf{U}_{r_j}\mathbf{W}_{r_j}\mathbf{U}_{r_j}^H$'s as well as the matrix $\omega_q \sum_{j=1}^{i} \mathbf{U}_{q_j}\mathbf{W}_{q_j}\mathbf{U}_{q_j}^H$ to its ith connected MS. At the transmitting end, MS-i then optimizes its transmit beamforming $\mathbf{V}_{q_i}$ within its power limit as stated in (5.29) and feeds back the updated $\mathbf{V}_{q_i}$ to BS-q.

Remark 5.3: In comparison to the sequential update in the ILA algorithm, the proposed WMMSE algorithm allows simultaneous updates across the coordinated BSs and across the MSs. This is due to the fact that the updating steps for the receive beamformers $\mathbf{U}_{q_i}$'s and the weight matrices $\mathbf{W}_{q_i}$'s are decoupled among the BSs, whereas the updating steps for the transmit beamformers $\mathbf{V}_{q_i}$'s are decoupled among the MSs. Between the two periods of simultaneous updates at BSs and at MSs, the BSs need to exchange the matrices $\omega_q \sum_{i=1}^{K} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H$'s.

5.5 Complexity of the Proposed Algorithms

In this section, we analyze the complexity in implementing the proposed ILA and WMMSE algorithms in a multicell system. Similar to the approach used in [58], the complexity of each algorithm is analyzed in each outer-loop iteration. Note that an outer-loop iteration can be also defined as an instance of signaling exchange among the BSs. To simplify the complexity analysis, let us denote $L = \max\{M, N\}$. In Table 5.1, we enumerate the complexity in undertaking the operations (by computing the listed variables) at each iteration as well as the total complexity of each algorithm. In addition, if an algorithm is to be implemented in a distributed manner, Column 3 "Message Type" in Table 5.1 classifies

Table 5.1 Complexity of the Proposed Algorithms

Algorithm	Operation	Message Type	Complexity	Number of Operations	Total Complexity
ILA	$\mathbf{R}_q$ in (5.5)	BS-Local	$\mathcal{O}(KQL^3)$	Q	$\mathcal{O}(KQ^2L^3)$
	$\mathbf{B}_q$ in (5.18)	BS $\leftrightarrow$ BS BS $\leftrightarrow$ MS	$\mathcal{O}(KL^3)$	Q	$\mathcal{O}(KQL^3)$
	$\mathbf{R}_{q_i}$ in (5.13)	BS $\leftrightarrow$ MS	$\mathcal{O}(KL^3)$	K	$\mathcal{O}(K^2L^3)$
	$\mathbf{A}_{q_i}$ in (5.10)	MS-Local	$\mathcal{O}(QL^3)$	K	$\mathcal{O}(KQL^3)$
	$\mathbf{X}_{q_i}$ in (5.14)	MS-Local	$\mathcal{O}(L^3)$	K	$\mathcal{O}(KL^3)$
Gauss-Seidel					$\mathcal{O}(KQ^3L^3 + K^2QL^3)$
Jacobi					$\mathcal{O}(KQ^2L^3 + K^2QL^3)$
WMMSE	$\mathbf{R}_q$ in (5.5)	BS-Local	$\mathcal{O}(KQL^3)$	Q	$\mathcal{O}(KQ^2L^3)$
	$\mathbf{U}_{q_i}$ in (5.23)	BS-Local	$\mathcal{O}(L^3)$	KQ	$\mathcal{O}(KQL^3)$
	$\mathbf{W}_{q_i}$ in (5.27)	BS-Local	$\mathcal{O}(L^3)$	KQ	$\mathcal{O}(KQL^3)$
	$\omega_q \sum_{i=1}^{K} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H$	BS $\leftrightarrow$ BS BS $\leftrightarrow$ MS	$\mathcal{O}(KL^3)$	Q	$\mathcal{O}(KQL^3)$
	$\omega_q \sum_{j=1}^{i} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H$	BS $\leftrightarrow$ MS	$\mathcal{O}(L^3)$	KQ	$\mathcal{O}(KQL^3)$
	$\mathbf{V}_{q_i}$ in (5.30)	MS-Local	$\mathcal{O}(L^3)$	KQ	$\mathcal{O}(KQL^3)$
WMMSE					$\mathcal{O}(KQ^2L^3)$

whether an operation is taken at the BS, *i.e.*, BS-Local, or at the MS, *i.e.*, MS-Local. The column also classifies whether a variable obtained from its corresponding operation is passed as a signaling message among the BSs, *i.e.*, BS $\leftrightarrow$ BS, or between the BS and its connected MSs, *i.e.*, BS $\leftrightarrow$ MS. We elaborate the content of Table 1 in the following.

In the ILA algorithm, at each iteration, the covariance matrix of the ICI plus noise $\mathbf{R}_q$ must be computed at BS-q. As given in (5.5), $\mathbf{R}_q$ is approximately the summation of KQ components, where matrix multiplication in each component $\mathbf{H}_{qr_i}\mathbf{X}_{r_i}\mathbf{H}_{qr_i}$ yields the complexity of $\mathcal{O}(L^3)$. Thus, the complexity of computing $\mathbf{R}_q$ is $\mathcal{O}(KQL^3)$. Similarly, the calculation of each pricing matrix $\mathbf{B}_q$ in (5.18) involves two matrix inversions and a summation of K components $\mathbf{H}_{qq_i}\mathbf{X}_{q_i}\mathbf{H}_{qq_i}$ yields the complexity of $\mathcal{O}(KL^3)$. The same technique can be applied to the calculations of $\mathbf{R}_{q_i}$, $\mathbf{A}_{q_i}$, and $\mathbf{X}_{q_i}$. With the Gauss-Seidel update, only one cell at the time, say cell-q, updates its K matrices $\mathbf{A}_{q_i}, i = 1, \ldots, K$, which yields the complexity of $\mathcal{O}(KQL^3)$. The optimization at cell-q then involves the calculation of the noise matrices $\mathbf{R}_{q_i}$'s and their inverses. Thus, ignoring the few iterations by the bisection step in optimizing the transmit covariance at each MS, this optimization shall take the complexity of $\mathcal{O}(K^2L^3)$. Consequently, in order to sequentially update all the transmit covariances across all Q cells, one has the complexity of $\mathcal{O}(KQ^3L^3 + K^2QL^3)$.

On the contrary, with the Jacobi update, all the precoders can be updated simultaneously across the Q cells after each instance of calculating the IPN matrices $\mathbf{R}_q$'s and the pricing matrices $\mathbf{B}_q$'s. In this case, the complexity of the ILA algorithm with the Jacobi update is $\mathcal{O}(KQ^2L^3 + K^2QL^3)$, which is lower than that with the Gauss-Seidel update by a factor of Q.

Similar to the complexity analysis of the ILA algorithm, the complexity of the WMMSE algorithm can be found to be $\mathcal{O}(KQ^2L^3)$. First, calculating the IPN covariance matrices $\mathbf{R}_q, q = 1, \ldots, Q$, and the signaling messages $\omega_q \sum_{i=1}^{K} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H, q = 1, \ldots, Q$, yields the overall complexity of $\mathcal{O}(KQ^2L^3)$. Second, utilizing the calculated IPN matrix $\mathbf{R}_q$ at BS-q, the receive beamformers $\mathbf{U}_{q_i}$'s and weight matrices $\mathbf{U}_{q_i}$'s can be updated sequentially from MS-1 (first) to MS-K (last), which yields the overall complexity of $\mathcal{O}(KQL^3)$. Third, the complexity of updating KQ transmit beamformers $\mathbf{V}_{q_i}$'s is $\mathcal{O}(KQL^3)$. Thus, per outer-loop iteration, in order to update the receive beamformers, the weight matrices, and the transmit beamformers for Q cells in the network, the complexity of the WMMSE algorithm is $\mathcal{O}(KQ^2L^3)$, which is comparable to that of the ILA algorithm with the Jacobi update.

Remark 5.4: In terms of total computational complexity, the distributed implementation of each algorithm is roughly the same as the its centralized implementation. In terms of implementation, the distributed approach requires certain message passing among the coordinated BSs and MSs, as detailed in Table 5.1. In addition, Table 5.1 also quantifies the number of messages that need to be exchanged at each outer-loop iteration. In the distributed implementation structure, the computation load in the optimization process can be shared across the coordinated BSs and MSs.

5.6 Simulation Results

This section presents simulation results on the achievable sum-rate of a multicell system in the uplink transmission under various levels of coordination and on the convergence behaviors of the proposed ILA and WMMSE algorithms. We compare the results for three operating modes: (i) the *coordination* mode obtained from the proposed ILA and WMMSE algorithms (with equal weight for each cell), (ii) the *competition* mode where each cell selfishly maximizes the sum-rate for its connected MS only, and (iii) the *network MIMO* mode where the whole system is a single large MIMO MAC channel. Considered is a 3-cell system, where the distance between any two BSs is normalized to 2, as illustrated in Fig. 5.1. The number of MSs is set to 3 per cell, unless stated otherwise, and each MS is randomly located on a circle at distance d from its connected BS. The BS and MS are equipped with 4 and 2 antennas, respectively. The transmit power of each MS is limited to 1 W. The intra-cell and inter-cell channel coefficients are generated as products of two components: one accounts for the large-scale fading with a path loss exponent of 3 and one represents the small-scale fading using i.i.d. complex Gaussian random variables with zero mean and unit variance. The AWGN power spectral density σ^2 is set at 0.01 W/Hz.

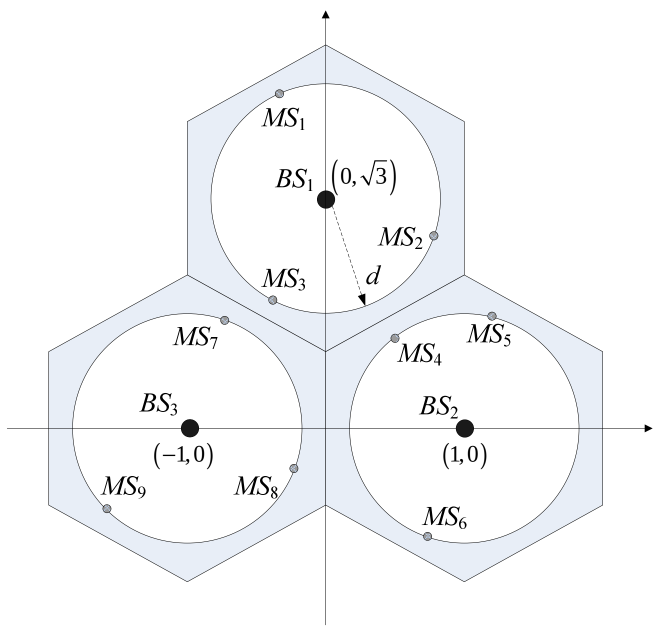

Fig. 5.1 A multiuser multicell system with 3 cells and 3 MSs per cell. Each MS is randomly located at a distance d from its connected BS.

We first investigate the achievable network sum-rates versus the intra-cell MS-BS distance d of the various algorithms, which are run until convergence. As d is varied, $10,000$ channel realizations at each value of d are used to obtain the average network sum-rates plotted in Fig. 5.2. As shown in the figure, when the distance d becomes smaller, the network sum-rate increases in all 3 operating modes. This is due to the increase in strength of intra-cell channels and the reduction in the strength of inter-cell channels. Out of the 3 operating modes, *network MIMO* obtains the largest sum-rate, as it is the upper bound for any uplink multicell transmission scheme. It is also observed that by implementing the interference coordination among the cells using the proposed algorithms (ILA and WMMSE),

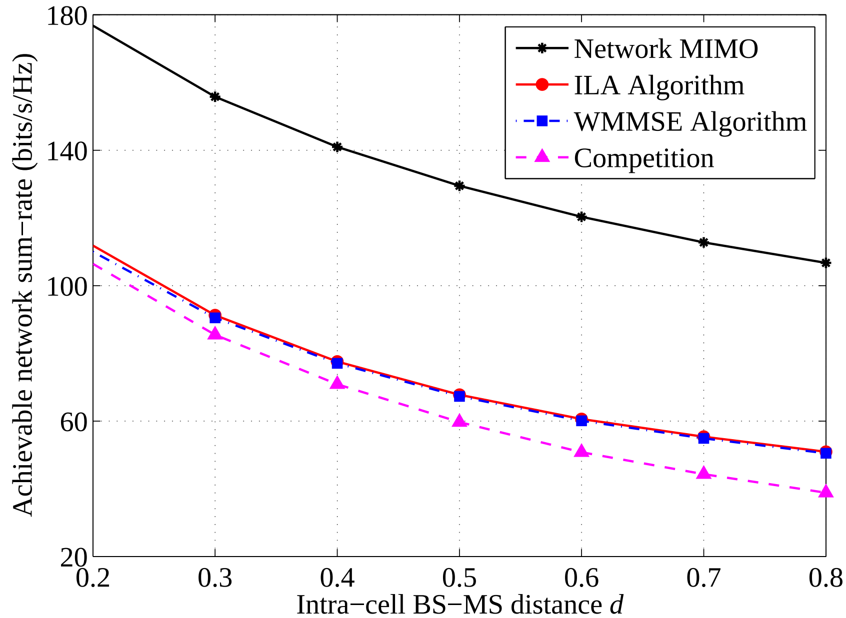

Fig. 5.2 Network sum-rates under the considered operating modes.

one can improve the network sum-rate by 5 to 15 b/s/Hz over the *competition* mode, especially in the high ICI region (large d). Note that the performances of the coordination mode are obtained from the same initialization with $\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H = \mathbf{X}_{q_i} = (P_{q_i}/N)\mathbf{I}$ for both ILA and WMMSE algorithms. Fig. 5.2 shows that there is literally no difference in the performances obtained from the ILA and WMMSE algorithms with the same identity matrix starting point. In other words, they both converge to the same local maximum.

As the proposed ILA and WMMSE algorithms do not guarantee a globally optimal performance, it is interesting to investigate the effects of the starting point on their achieved network sum-rate. For this, we run simulations for 10 different randomly generated starting points and record the best sum-rate result out of the 10 fully converged maxima for each algorithm. The resulting plots for the 2 proposed algorithms designated by *ILA-10_random*, and *WMMSE-10_random*, in Fig. 5.3 show a negligible performance difference between the two proposed algorithms, and a slightly increased network sum-rate as compared to the case with identity matrix starting point in Fig. 5.2. This close performance indicates that the identity matrix is a good and simple choice for the starting point. As previously discussed, both the proposed ILA and WMMSE algorithms exhibit the monotonic convergence, but

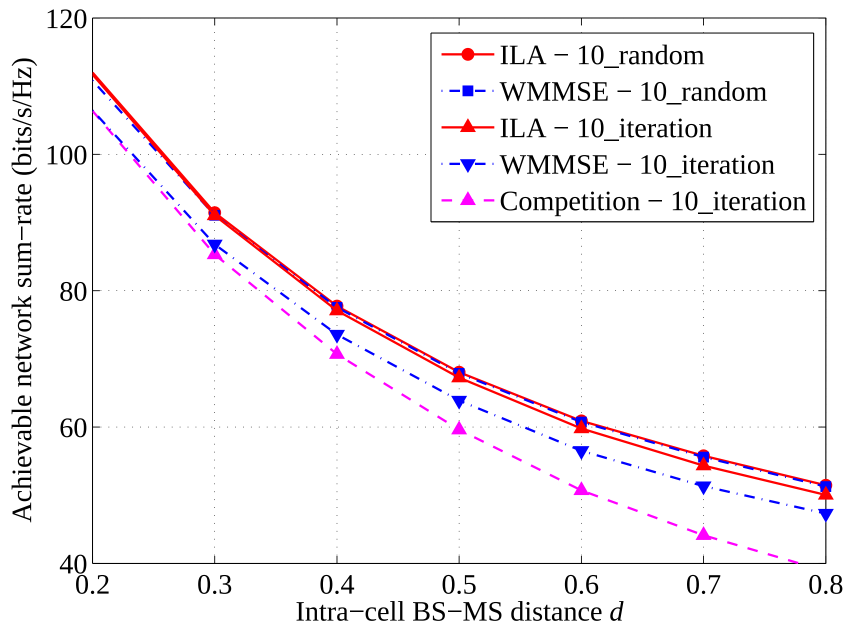

Fig. 5.3 Network sum-rates under the *coordination* mode, obtained from the ILA and WMMSE algorithms with 10 random starting points or with 10 outer-loop iterations.

require the coordinated cells to exchange signaling information after each outer-loop iteration. For practical implementation, it may be desirable to limit the number of iterations in order to reduce the amount of signaling exchange and it is interesting to understand the effect of number of iterations on their performance. For illustration, we include in Fig. 5.3 the plots *ILA-10_iteration*, *WMMSE-10_iteration*, and *Competition-10_iteration* representing the achieved network sum-rates after 10 outer-loop iterations of the ILA, WMMSE algorithms and competition mode, respectively. The simulation results in Fig. 5.3 indicate that the ILA algorithm is faster to converge than the WMMSE algorithm in terms of the number of outer-loop iterations. Both algorithms outperform the competition mode after just 10 iterations. Note that the ILA algorithm in this simulation is implemented with the Jacobi update. Thus, it requires a similar amount of inter-BS signaling as the WMMSE algorithm.

To investigate further their convergence behavior, we obtain simulation results for a specific channel realization with $d = 0.5$ and plot the network sum-rates achieved after each outer-loop iteration by ILA using Jacobi and Gauss-Seidel updates, the WMMSE, and

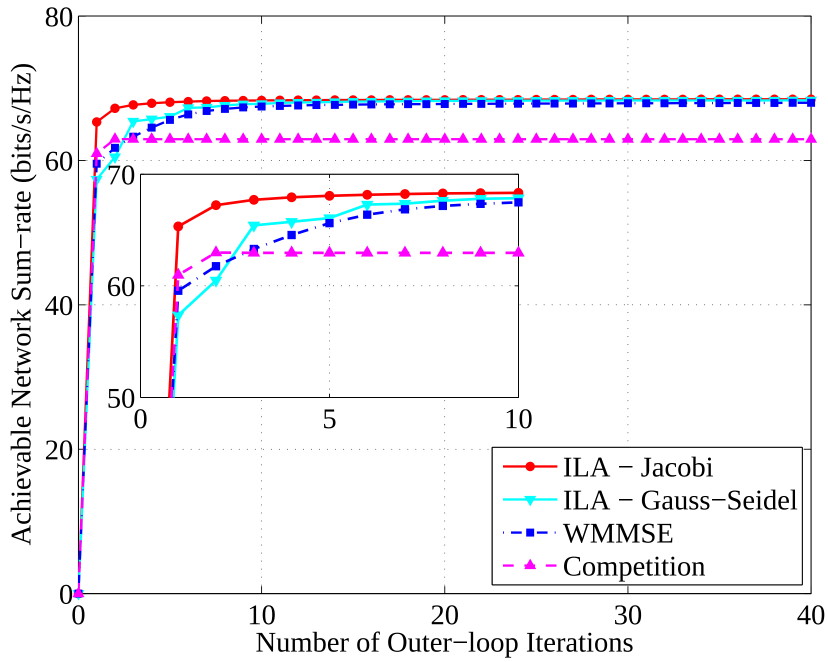

Fig. 5.4 Convergence of the proposed ILA and WMMSE algorithms to maximize the network sum-rate with the *coordination*.

competition (for comparison). As observed in Fig. 5.4, the coordination mode, obtained by the ILA and WMMSE algorithms, offers higher network sum-rate than the competition mode. It is also observed that the ILA algorithm does monotonically converge with both Jacobi and Gauss-Seidel updates. The network sum-rate is also improved monotonically by the WMMSE algorithm. These convergence behaviors of the ILA and WMMSE algorithms agree with our analysis. Interestingly, Fig. 5.4 confirms that the ILA algorithm has a faster convergence than the WMMSE algorithm in terms of number of outer-loop iterations. This observation explains why the ILA algorithm obtains better sum-rate performance than the WMMSE algorithm after 10 outer-loop iterations as shown in Fig. 5.3.

As previously discussed, the WMMSE algorithm has only outer-loop iterations while ILA has both inner- and outer-loop iterations. To investigate its inner-loop convergence, in Fig. 5.5, we plot the sum-rate achieved at cell-1 after each number of inner-loop iterations by the ILA, for the same channel realization used in Fig. 5.4, and at outer-loop iteration #2 (when $\mathbf{A}_{q_i}$'s are non-zero). Note that the inner-loop iterations in the ILA algorithm is to to maximize the MAC sum-rate with penalty terms (5.11). For comparison, the convergence

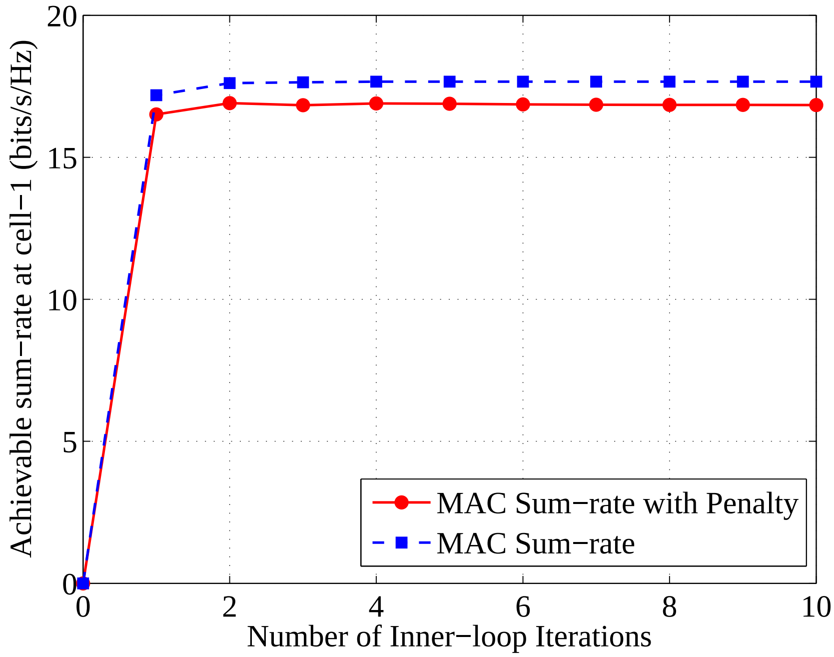

Fig. 5.5 Convergence of the proposed iterative algorithm to solve Problem (5.11).

of inner-loop iterations in the competition mode, *i.e.*, the IWF algorithm for MAC sum-rate maximization [23], is also illustrated. Fig. 5.5 indicates that, at the outer-loop iteration #2, due to the penalty terms, the ILA inner-loop algorithm achieves lower sum-rate in cell-1 than the competition mode (without the penalty terms). However, as the cell adopts a more cooperative strategy by limiting the ICI to other cells, the overall network sum-rate performance is indeed improved, as shown in Fig. 5.4.

Finally, Fig. 5.6 compares the CPU running time of the proposed ILA and WMMSE algorithms versus the number of MSs in each cell under the same termination criterion. Although the CPU running time is rather relative, all algorithms are programmed with the same code implementation and run on the same platform, which allows us to realize the relative trend in computational complexity of each algorithm. As shown in the figure, the ILA algorithm with the Jacobi update shall improve the running time over that with the Gauss-Seidel update roughly by a factor of $Q = 3$. Interestingly, while the WMMSE requires more outer-loop iterations, *i.e.*, more inter-BS signaling exchange, it has less running time than the ILA algorithm. Intuitively, the WMMSE does not require the inner-loop it-

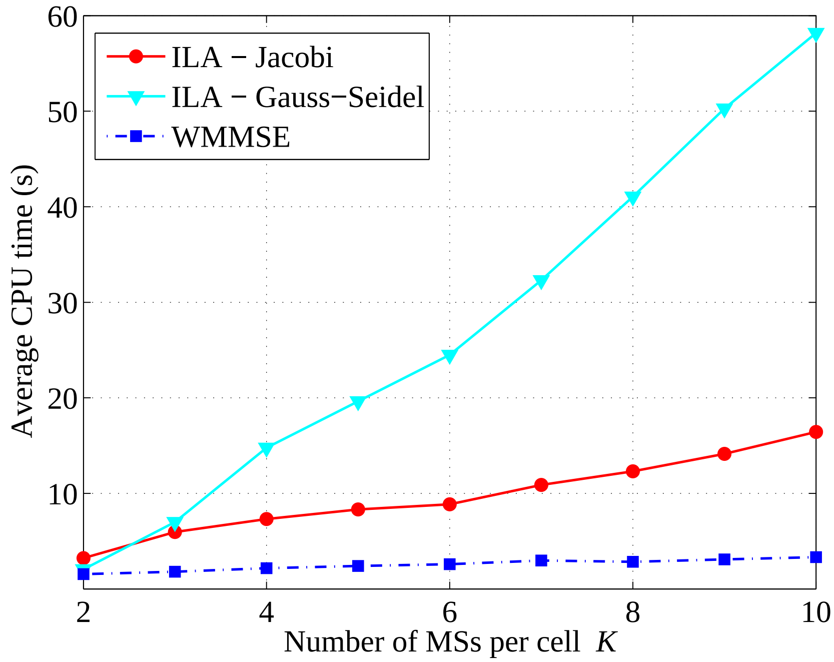

Fig. 5.6 Average CPU time versus the number of MSs per cell.

erations, while the ILA algorithm does for sequentially optimizing precoders at a particular cell. Thus, the WMMSE algorithm imposes less intra-cell BS-MS operation and signaling than the ILA algorithm. In fact, the running time of the WMMSE algorithm is almost linear with K, as our complexity analysis indicates.

5.7 Concluding Remarks

This chapter examined the problem of WSR maximization in the multicell MIMO MAC. Under the coordination mode among the multiple cells, the network WSR maximization problem was shown to be nonconvex. The chapter then proposed two solution approaches, namely ILA and WMMSE, to approximate and transform the original nonconvex problem into convex optimization ones. In the ILA approach, the nonconvex optimization problem is successively approximated and decomposed into a set of convex problems, which can be solved distributively at each MS. In the WMMSE algorithm, by transforming the original problem into a weighted MSE minimization problem, the network WSR is maximized by alternatively optimizing the weight matrices and MMSE decoders at each BS and the precoder at each MS. Simulations confirmed the convergence analysis of the proposed algorithm and showed a significant enhancement in the network sum-rate as compared to competitive design.

Chapter 6

Sum-rate Maximization in the Multicell MIMO Broadcast Channel with Interference Coordination

6.1 Introduction

Optimizing the precoding designs in an interference network is a challenging task due to the nonconcavity of the WSR function. Different numerical methods for designing the precoders that maximize the WSR have been investigated in the literature [55, 57, 81]. Specifically, the gradient projection method was applied in [81] to search for a locally optimal transmit strategy. Successive convex approximation was applied in [55, 57] to decompose the original nonconvex problem into a sequence of simpler convex problems, which can be solved separately at the transmitters. Note that these works only considered the network with one MS per cell. For a more general case of multiple MSs per cell, recent works in [9, 58, 92] studied the optimal linear precoding to maximize the WSR with per-BS constraints. Specifically, an iterative algorithm was proposed in [9] to solve the KKT conditions of the nonconvex WSR maximization problem. Another solution approach to the nonconvex WSR maximization problem is to transform it into a minimization of the weighted mean squared error (WMMSE) problem [33, 58, 87]. The WMMSE problem

The materials presented in Chapter 6 have been presented at the 2013 IEEE International Conference on Communications in Budapest, Hungary [90], and submitted to the IEEE Transactions on Signal Processing [91].

then can be solved by iteratively optimizing the weight matrices, the MMSE precoders, and the MMSE decoders [33]. Thus, by establishing the equivalence between the WSR maximization problem and the WMMSE minimization problem, a locally optimal solution to the former can be found from the solution of the latter.

In this chapter, we consider a coordinated multicell system in a general setting with multiple MSs per cell, where each BS or MS is equipped with multiple transmit antennas. In each cell, the BS concurrently transmits information signals to its connected MSs, which emulates a MIMO broadcast (MIMO-BC) system. The main focus of this chapter is to jointly optimize the precoding covariance matrices at the BSs in order to maximize the network-wide WSR under the IC mode. While most of the works considered linear precoding at each BS for the multicell MIMO-BC system [9, 10, 55, 57, 58, 61, 73, 81, 92], our focus in this chapter is on nonlinear precoding design. Specifically, in the BC with multiple MSs per cell, each BS utilizes dirty-paper coding (DPC) to encode the data for the MSs within its cell. It is well-known that DPC is the capacity-achieving multiuser precoding technique for a single-cell system [25–28]. In this chapter, we extend the study of DPC onto the multicell system with interference coordination. This consideration potentially allows the multicell network to realize extra performance from the nonlinear precoding over the linear precoding. Since the maximization of WSR in a multicell MIMO-BC with DPC is a nonconvex problem, finding its globally optimal solution is computationally complex. To address this concern, we consider two low-complexity solution approaches, namely ILA and WMMSE, to numerically search for at least locally optimal solutions of the problem. Note that the two aforementioned solution approaches were applied to effectively solve the WSR maximization problem in the multicell MIMO-MAC in Chapter 5. In this chapter, we show how each approach can be adopted to locally maximize the WSR in the multicell MIMO-BC.

In the ILA solution approach, the sum-rate function at all cells except a particular cell under consideration is approximated into a linear interference penalty. Thus, maximizing the network WSR is equivalent to maximizing the BC sum-rate with DPC at the given cell while minimizing a penalty term on the ICI generated by its corresponding BS. Although this per-cell BC problem is yet to be convex, we show that the problem is equivalent to the one in the multiple-access channel (MAC) via the so-called BC-MAC duality. Since the MAC problem is convex and thus optimally solved, the optimal solution to the BC problem is also obtained by the MAC-BC transformation [28]. Interestingly, it will be proved that the network WSR is always improved by optimizing the DPC precoders at any given BS. With the ILA algorithm, each BS is required to iteratively take turn and refine its precoders. We then prove the monotonic convergence of the ILA algorithm to at least a local maximum. In addition, we develop a message exchange mechanism that allows the proposed algorithm to be implemented in a fully distributed manner.

In the WMMSE solution approach, we devise a version of the WMMSE algorithm for the multicell MIMO-BC with DPC precoding. To avoid the confusion with the original WMMSE algorithm for the case of linear precoding in [58], the newly devised algorithm will be referred to as the DPC-WMMSE algorithm. Similar to the original WMMSE algorithm, we show that the DPC-WMMSE can obtain a locally optimal solution to the multicell MIMO-BC WSR maximization problem. In addition, the DPC-WMMSE can be implemented distributively via a message exchange mechanism among the coordinated BSs. Simulation results confirm the convergence analysis of the ILA and DPC-WMMSE algorithm, and show that the proposed algorithm significantly improves the network WSR, in comparison with linear precoding or with no IC between the BSs.

6.2 System Model and Problem Formulation

The system model considered in this chapter is the same as in Chapter 4. Consider the downlink transmission of a multicell system with Q separate cells operating on the same frequency. In each cell, a multiple-antenna BS concurrently sends independent data streams to multiple MSs, each equipped with multiple transmit antennas. For simplicity in presentation, it is assumed that the number of antennas at each BS and MS are M and N, respectively, and the number of MSs per cell is K. At a particular cell, say cell-q, the downlink transmission to MS-i can be modeled as

$$\mathbf{y}_{q_i} = \mathbf{H}_{qq_i} \sum_{j=1}^{K} \mathbf{x}_{q_j} + \sum_{r \neq q}^{Q} \mathbf{H}_{rq_i} \sum_{j=1}^{K} \mathbf{x}_{r_j} + \mathbf{z}_{q_i}, \tag{6.1}$$

where $\mathbf{x}_{r_j} \in \mathbb{C}^{M\times 1}$ is the transmitted vector from the BS-r intended for its connected MS-j, $\mathbf{H}_{rq_i}$ models the channel from BS-r to MS-i of cell-q, and $\mathbf{z}_{q_i}$ is the zero-mean additive Gaussian noise vector with the covariance matrix $\mathbf{Z}_{q_i}$. Since the multicell system operates on the same frequency channel, the intended signal from BS-q to its MS-i is now subject to the intra-cell interference from the signals intended for other co-located MSs in $\mathbf{H}_{qq_i}\sum_{j\neq i}^{K}\mathbf{x}_{q_j}$, as well as the ICI from other cells in $\sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\big(\sum_{j=1}^{K}\mathbf{x}_{r_j}\big)$.

Let $D = \min(M, N)$ be the number of data sub-streams for each MS. The transmit signal vector $\mathbf{x}_{q_i}$ for MS-i can be expressed as

$$\mathbf{x}_{q_i} = \mathbf{V}_{q_i}\mathbf{s}_{q_i}, \tag{6.2}$$

where $\mathbf{V}_{q_i} \in \mathbb{C}^{M\times D}$ is the precoding matrix and $\mathbf{s}_{q_i} \in \mathbb{C}^{D\times 1}$ represents the information signal vectors. Without loss of generality, it is assumed that $\mathbb{E}[\mathbf{s}_{q_i}\mathbf{s}_{q_i}^H] = \mathbf{I}$. Denote $\mathbf{Q}_{q_i} = \mathbb{E}\left[\mathbf{x}_{q_i}\mathbf{x}_{q_i}^H\right] = \mathbf{V}_{q_i}\mathbf{V}_{q_i}^H$ as the transmit covariance matrix intended for MS-i of cell-q. Let $\mathbf{Q}_q = \{\mathbf{Q}_{q_i}\}_{i=1}^K$ be the downlink precoder profile of the K users at cell-q. Likewise, let $\mathbf{Q}_{-q} = (\mathbf{Q}_1, \ldots, \mathbf{Q}_{q-1}, \mathbf{Q}_{q+1}, \ldots, \mathbf{Q}_Q)$ denote the precoding profile of all cells except cell-q. Denote $\mathbf{z}_{-q_i} = \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K}\mathbf{x}_{r_j} + \mathbf{z}_{q_i}$ as the total ICI plus additive Gaussian noise at MS-i of cell-q, whose covariance $\mathbf{R}_{q_i}$ is defined as

$$\mathbf{R}_{q_i} = \mathbb{E}\left[\mathbf{z}_{-q_i}\mathbf{z}_{-q_i}^H\right] = \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\left(\sum_{j=1}^{K}\mathbf{Q}_{r_j}\right)\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i}. \tag{6.3}$$

As the multicell system operates in IC mode, each BS only attempts to encode and transmit information signals to the MSs within its cell. Unlike the BD precoding designs in Chapter 4, this chapter considers the capacity-achieving multiuser encoding technique, namely dirty-paper coding (DPC) [25–27], for the downlink transmissions from a BS to its connected MSs. At cell-q, assuming the encoding order from user-K to user-1, DPC is utilized such that the intended codeword for user-i does not see the intra-cell interference from user-$(i+1)$ to user-K. As previously mentioned in Chapter 2, the achievable data rate at user-i of cell-q with DPC is given by

$$R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\frac{\left|\mathbf{R}_{q_i} + \mathbf{H}_{qq_i}\left(\sum_{j=1}^{i}\mathbf{Q}_{q_j}\right)\mathbf{H}_{qq_i}^H\right|}{\left|\mathbf{R}_{q_i} + \mathbf{H}_{qq_i}\left(\sum_{j=1}^{i-1}\mathbf{Q}_{q_j}\right)\mathbf{H}_{qq_i}^H\right|}. \tag{6.4}$$

Let $R_q^{\mathrm{BC}} = \sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}$ be the sum-rate at cell-q for its K connected MSs. Collectively, the network WSR is given by $\sum_{q=1}^{Q}\omega_q\sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q})$, where ω_q denotes the non-negative weight of cell-q. Given P_q as the maximum transmit power at BS-q, the network WSR is maximized by the following optimization

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} R_{q_i}^{\text{BC}} \\ \text{subject to} \quad & \sum_{i=1}^{K} \text{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q, \ \forall q \\ & \mathbf{Q}_{q_i} \succeq \mathbf{0}, \ \forall i, \forall q. \end{aligned} \tag{6.5}$$

Note that problem (6.5) is nonconvex because of the presence of $\mathbf{Q}_{q_i}$'s in the ICI terms $\mathbf{R}_{r_j}$'s with $r \neq q$, as well as the intra-cell interference term in $R_{q_j}^{\text{BC}}$ with $j < i$. To this end, we consider the two solution approaches, namely ILA and WMMSE, which have been applied in Chapter 5 to the WSR maximization problem in the multicell MIMO-BC (6.5).

6.3 The ILA Solution Approach for the Multicell MIMO-BC

6.3.1 The ILA Algorithm for the Multicell MIMO-BC

This section investigates the ILA solution approach to obtain at least a locally optimal solution to the nonconvex problem (6.5). Let $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{r\neq q}^{Q} \omega_r R_r(\mathbf{Q}_q, \mathbf{Q}_{-q})$ denote the WSR of all cells except cell-q. As $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is not concave in $\mathbf{Q}_{q_i}$, we shall take an approximation of f_q into a linear term. At a given value of $\bar{\mathbf{Q}}_{q_i}$, taking the Taylor expansion of f_q around $\bar{\mathbf{Q}}_{q_i}$ and retaining the first linear term, one has

$$f_q(\mathbf{Q}_q, \bar{\mathbf{Q}}_{-q}) \approx f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^{K} \text{Tr}\big\{\mathbf{A}_q\big(\mathbf{Q}_{q_i} - \bar{\mathbf{Q}}_{q_i}\big)\big\}\,, \tag{6.6}$$

where $\mathbf{A}_q$ is the negative partial derivative of f_q with respect to $\mathbf{Q}_{q_i}$, evaluated at $\mathbf{Q}_{q_i} = \bar{\mathbf{Q}}_{q_i}$, given by

$$\begin{aligned}\mathbf{A}_q = -\left.\frac{\partial f_q}{\partial \mathbf{Q}_{q_i}}\right|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}} &= -\sum_{r\neq q}^{Q}\omega_r\sum_{j=1}^{K}\left.\frac{\partial R_{r_j}}{\partial \mathbf{Q}_{q_i}}\right|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}}\\
&= \sum_{r\neq q}^{Q}\omega_r\sum_{j=1}^{K}\mathbf{H}_{qr_j}^{H}\left[\left(\mathbf{R}_{r_j}+\sum_{k=1}^{j-1}\mathbf{H}_{rr_j}\mathbf{Q}_{r_k}\mathbf{H}_{rr_j}^{H}\right)^{-1}\right.\\
&\qquad \left.\left. - \left(\mathbf{R}_{r_j}+\sum_{k=1}^{j}\mathbf{H}_{rr_j}\mathbf{Q}_{r_k}\mathbf{H}_{rr_j}^{H}\right)^{-1}\right]\mathbf{H}_{qr_j}\right|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}}. \qquad (6.7)\end{aligned}$$

Note that this partial derivative has the same form with respect to each $\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}$, and is positive semi-definite, *i.e.*, $\mathbf{A}_q \succeq \mathbf{0}$. Then, one can approximate problem (6.5) into a set of Q per-cell problems, where the optimization performed at cell-q is equivalent to

$$\begin{aligned}\underset{\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}}{\text{maximize}}\quad & \omega_q\sum_{i=1}^{K}R_{q_i}^{\text{BC}} - \sum_{i=1}^{K}\text{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\} \qquad (6.8)\\
\text{subject to}\quad & \sum_{i=1}^{K}\text{Tr}\{\mathbf{Q}_{q_i}\}\le P_q\\
& \mathbf{Q}_{q_i}\succeq\mathbf{0},\forall i.\end{aligned}$$

Some remarks regarding the optimization problem (6.8) are provided below:

Remark 6.1: Problem (6.8) is similar to the sum-rate maximization problem in the BC with DPC, studied in [27, 28, 30], albeit the presence of the penalty term $\sum_{i=1}^{K}\text{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}$ charged on ICI generated by BS-q. The penalty term encourages the BS to design its precoders in a coordinated manner by controlling its induced ICI to other cells. Should the penalty term be omitted, the BS would only maximize the downlink capacity for its connected users. As a result, the precoding design in this multicell system is a noncooperative game between the BSs, where each BS acts as a rational and selfish player. This multicell precoding game is similar to the game studied in [12] for the case of 1 user per cell. It is to be noted that the study of the multicell precoding game with DPC for the case of multiple users per cell is beyond the scope of this work. Nonetheless, we shall present some numerical results for this noncooperative design in comparison to the considered coordinated design.

Remark 6.2: It is worth mentioning that [93, 94] studied the BC sum-rate maximization with strict constraints on the induced ICI $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_q \mathbf{Q}_{q_i}\}$. The considered problem (6.8) is different from the studies in [93, 94], since we attempt to minimize the ICI penalty term with a sum-power constraint on the transmit covariances.

Remark 6.3: Although problem (6.8) is not a convex optimization problem, its resemblance to the BC's sum-rate maximization problem enables its transformation into a dual MAC maximization problem via the so-called BC-MAC duality. In a conventional multiuser MIMO system with the objective of maximizing the system sum-rate, BC-MAC duality was proved for the case of a single sum-power constraint [27–29], a set of linear power constraints [21, 27], and multiple general transmit covariance constraints [93]. In the following, it will be shown that the BC-MAC duality also holds for the multiuser MIMO system with the objective of maximizing the system sum-rate while minimizing the penalty term imposed on the transmit covariances. As a result, the nonconvex problem (6.8) can be optimally solved via the convex MAC problem by utilizing this BC-MAC duality.

For simplicity in presentation, the subscript representing the BS is dropped without loss of generality. We first consider the optimization (6.8) without the sum power constraint $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_i\} \leq P$, which can be stated as

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}} \quad & \omega \sum_{i=1}^{K} R_i^{\mathrm{BC}} - \sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i\} \\ \text{subject to} \quad & \mathbf{Q}_i \succeq \mathbf{0},\ \forall i. \end{aligned} \tag{6.9}$$

In the following, we are interested in obtaining the globally optimal solution $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ to problem (6.9). Should $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ meet the sum power constraint in (6.8), *i.e.*, $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_i^\star\} \leq P$, they must be the global maximizer of problem (6.8) as well.

By changing the variables $\tilde{\mathbf{Q}}_i = \mathbf{A}^{1/2}\mathbf{Q}_i\mathbf{A}^{1/2}$ and denoting $\tilde{\mathbf{H}}_i = \mathbf{R}_i^{-1/2}\mathbf{H}_i\mathbf{A}^{-1/2}$, the data-rate to user-$i$ can be rewritten as

$$R_i^{\mathrm{BC}} = \log \frac{\left|\mathbf{I} + \tilde{\mathbf{H}}_i \left(\sum_{j=1}^{i} \tilde{\mathbf{Q}}_i\right) \tilde{\mathbf{H}}_i^H\right|}{\left|\mathbf{I} + \tilde{\mathbf{H}}_i \left(\sum_{j=1}^{i-1} \tilde{\mathbf{Q}}_i\right) \tilde{\mathbf{H}}_i^H\right|}. \tag{6.10}$$

Thus, problem (6.9) is equivalent to

$$\begin{aligned} \underset{\tilde{\mathbf{Q}}_1,\ldots,\tilde{\mathbf{Q}}_K}{\text{maximize}} \quad & \omega \sum_{i=1}^{K} R_i^{\mathrm{BC}} - \sum_{i=1}^{K} \mathrm{Tr}\{\tilde{\mathbf{Q}}_i\} \\ \text{subject to} \quad & \tilde{\mathbf{Q}}_i \succeq \mathbf{0},\ \forall i. \end{aligned} \tag{6.11}$$

In order to solve the nonconvex problem (6.11), we utilize the known BC-MAC duality property as follows. Consider a dual MAC with K N-antenna MSs transmitting to an M-antenna BS, where the uplink channel from user-i to the BS is assumed to be $\tilde{\mathbf{H}}_i^H$ and background noise at the BS is AWGN with unit variance. It is assumed that the BS employs SIC to decode the signals from the K MSs. With the decoding order from user-1 to user-K, SIC ensures that the received signal from user-i is not interfered by the signals from user-1 to user-$(i-1)$. Denoting $\mathbf{X}_i$ as the uplink precoding covariance matrix at user-i, the achievable data-rate for user-i in the MAC is thus given by

$$R_i^{\mathrm{MAC}} = \log \frac{\left|\mathbf{I} + \sum_{j=i}^{K} \tilde{\mathbf{H}}_j^H \mathbf{X}_j \tilde{\mathbf{H}}_j\right|}{\left|\mathbf{I} + \sum_{j>i}^{K} \tilde{\mathbf{H}}_j^H \mathbf{X}_j \tilde{\mathbf{H}}_j\right|}. \tag{6.12}$$

The key relationship between the BC and its dual MAC is presented in the following theorem.[1]

Theorem 6.1. *[28] For a given set of downlink covariance matrices $\tilde{\mathbf{Q}}_1,\ldots,\tilde{\mathbf{Q}}_K$ in the BC, it is always possible to find a set of uplink covariance matrices $\mathbf{X}_1,\ldots,\mathbf{X}_K$ such that $R_i^{\mathrm{MAC}} = R_i^{\mathrm{BC}}$ and $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{X}_i\} = \sum_{i=1}^{K}\mathrm{Tr}\{\tilde{\mathbf{Q}}_i\}$ through the BC-MAC transformation. Vice versa, for a given set of uplink covariance matrices $\mathbf{X}_1,\ldots,\mathbf{X}_K$, it is always possible to find a set of downlink covariances $\tilde{\mathbf{Q}}_1,\ldots,\tilde{\mathbf{Q}}_K$ such that $R_i^{\mathrm{BC}} = R_i^{\mathrm{MAC}}$ and $\sum_{i=1}^{K}\mathrm{Tr}\{\tilde{\mathbf{Q}}_i\} = \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{X}_i\}$ through the MAC-BC transformation.*

From Theorem 6.1, instead of solving the nonconvex problem (6.8), one may consider

[1] The duality between the BC with a general linear constraint on $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i\}$ and the MAC was established in [93] using a technique called SINR duality. In this work, we apply a simple change of variables to show this duality.

the following optimization problem

$$\begin{aligned} \underset{\mathbf{X}_1,\dots,\mathbf{X}_K}{\text{maximize}} \;\; & \omega \sum_{i=1}^{K} R_i^{\text{MAC}} - \sum_{i=1}^{K} \text{Tr}\{\mathbf{X}_i\} \\ \text{subject to} \;\; & \mathbf{X}_i \succeq \mathbf{0},\ \forall i, \end{aligned} \tag{6.13}$$

which can be interpreted as a MAC sum-rate maximization with a penalty term on the transmit power at the MSs. Certainly, if the set $\mathbf{X}_1^\star, \dots, \mathbf{X}_K^\star$ is optimal in (6.13), it is possible to find the set $\tilde{\mathbf{Q}}_1^\star, \dots, \tilde{\mathbf{Q}}_K^\star$ that is optimal in (6.11) with the same maximum value. By contradiction, if $\tilde{\mathbf{Q}}_1^\star, \dots, \tilde{\mathbf{Q}}_K^\star$ were not optimal, the BC-MAC transformation would ensure that $\mathbf{X}_1^\star, \dots, \mathbf{X}_K^\star$ would not be optimal. Thus, the BC-MAC duality also holds for the considered problem with the objective of maximizing the sum-rate while minimizing the penalty term imposed on the transmit covariances. Consequently, by finding the optimal solution of problem (6.13), one also obtains the optimal solution of problem (6.11).

Note that the objective function in (6.13) can be simplified as

$$\omega \sum_{i=1}^{K} R_i^{\text{MAC}} - \sum_{i=1}^{K} \text{Tr}\{\mathbf{X}_i\} = \omega \log \left| \mathbf{I} + \sum_{i=1}^{K} \tilde{\mathbf{H}}_i^H \mathbf{X}_i \tilde{\mathbf{H}}_i \right| - \sum_{i=1}^{K} \text{Tr}\{\mathbf{X}_i\}, \tag{6.14}$$

which is concave in $\mathbf{X}_1, \dots, \mathbf{X}_K$. Consequently, problem (6.13) is convex. In addition, the inherently decoupled constraints for each variable matrix $\mathbf{X}_i$ allows the sequential maximization of the objective function over each variable matrix [83]. More specifically, MS-i optimizes its covariance matrix $\mathbf{X}_i$ by performing

$$\underset{\mathbf{X}_i \succeq \mathbf{0}}{\text{maximize}} \;\; \omega \log \left| \mathbf{I} + \left(\mathbf{I} + \sum_{j \neq i}^{K} \tilde{\mathbf{H}}_j^H \mathbf{X}_j \tilde{\mathbf{H}}_j \right)^{-1} \tilde{\mathbf{H}}_i^H \mathbf{X}_i \tilde{\mathbf{H}}_i \right| - \text{Tr}\{\mathbf{X}_i\}, \tag{6.15}$$

while treating the signal from other MSs as noise. Using the eigen-decomposion $\tilde{\mathbf{H}}_i \big(\mathbf{I} + \sum_{j \neq i}^{K} \tilde{\mathbf{H}}_j^H \mathbf{X}_j \tilde{\mathbf{H}}_j \big)^{-1} \tilde{\mathbf{H}}_i^H = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^H$, the optimal solution can be obtained in closed-form as $\mathbf{X}_i = \mathbf{U}_i \big[\omega \mathbf{I} - \mathbf{\Sigma}_i^{-1} \big]^+ \mathbf{U}_i^H$. Each MS-$i$ can iteratively update its covariance matrix while keeping other covariance matrices fixed [83]. Note that this procedure always improves the objective function (6.14).

Since the objective function (6.14) is a subtraction of a log function of $\mathbf{X}_1, \dots, \mathbf{X}_K$ to a linear function of $\mathbf{X}_1, \dots, \mathbf{X}_K$, it must have an upper bound. As a result, the sequential optimization of (6.15) over $\mathbf{X}_1, \ldots, \mathbf{X}_K$ is guaranteed to monotonically converge to the optimal solution $\mathbf{X}_1^\star, \ldots, \mathbf{X}_K^\star$ of problem (6.13). Consequently, one can obtain the optimal solution $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$ to problem (6.11) from $\mathbf{X}_1^\star, \ldots, \mathbf{X}_K^\star$ by the MAC-BC transformation [28]. The optimal solution of (6.9) is then given by $\mathbf{Q}_i^\star = \mathbf{A}^{-1/2}\tilde{\mathbf{Q}}_i^\star\mathbf{A}^{-1/2}$. As $\mathbf{X}_1^\star, \ldots, \mathbf{X}_K^\star$ is the globally optimal solution to the MAC problem (6.13), $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ must be the globally optimal solution to the BC problem (6.9). It is then straightforward to verify whether $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ meet the sum-power constraint $\sum_{i=1}^K \operatorname{Tr}\{\mathbf{Q}_i^\star\} \leq P$. If the constraint is not satisfied, one may consider the Lagrangian of original BC problem (6.8), which can be stated as

$$\mathcal{L}(\mathbf{Q}_1, \ldots, \mathbf{Q}_K, \lambda) = \omega \sum_{i=1}^{K} R_i^{\mathrm{BC}} - \sum_{i=1}^{K} \operatorname{Tr}\{(\mathbf{A} + \lambda\mathbf{I})\mathbf{Q}_i\} + \lambda P, \tag{6.16}$$

where $\lambda \geq 0$ is the Lagrangian multiplier associated with the power constraint $\sum_{i=1}^K \operatorname{Tr}\{\mathbf{Q}_i\} \leq P$. The Lagrangian dual function is then given by

$$g(\lambda) = \sup_{\mathbf{Q}_1 \succeq \mathbf{0}, \ldots, \mathbf{Q}_K \succeq \mathbf{0}} \mathcal{L}(\mathbf{Q}_1, \ldots, \mathbf{Q}_K, \lambda) \tag{6.17}$$

and the dual problem is defined as

$$\begin{aligned} \underset{\lambda}{\text{minimize}} \quad & g(\lambda) \\ \text{subject to} \quad & \lambda \geq 0. \end{aligned} \tag{6.18}$$

We first focus on the maximization of the Lagrangian dual function for a given λ, which can be stated as

$$\begin{aligned} \underset{\mathbf{Q}_1, \ldots, \mathbf{Q}_K}{\text{maximize}} \quad & \omega \sum_{i=1}^{K} R_i^{\mathrm{BC}} - \sum_{i=1}^{K} \operatorname{Tr}\{(\mathbf{A} + \lambda\mathbf{I})\mathbf{Q}_i\} \\ \text{subject to} \quad & \mathbf{Q}_i \succeq \mathbf{0}, \ \forall i. \end{aligned} \tag{6.19}$$

Clearly, problem (6.19) is similar to problem (6.9). Thus, one can obtain the globally optimal solution to problem (6.19) by adopting the approach in solving problem (6.9). The difference is in the change of variables where $\tilde{\mathbf{Q}}_i = (\mathbf{A} + \lambda\mathbf{I})^{1/2}\mathbf{Q}_i(\mathbf{A} + \lambda\mathbf{I})^{1/2}$ and $\tilde{\mathbf{H}}_i = \mathbf{R}_i^{-1/2}\mathbf{H}_i(\mathbf{A}+\lambda\mathbf{I})^{-1/2}$. Then, by solving the dual MAC problem (6.13) and performing the MAC-BC transformation one can obtain the globally optimal solution $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$. Subsequently, the optimal solution to the Lagrangian dual problem (6.19) is given by $\mathbf{Q}_i^\star =$

$(\mathbf{A}+\lambda\mathbf{I})^{-1/2}\tilde{\mathbf{Q}}_i^\star(\mathbf{A}+\lambda\mathbf{I})^{-1/2}$.

It remains to minimize $g(\lambda)$ subject to the constraint $\lambda \geq 0$ in (6.18). By the Lagrangian duality theory, $g(\lambda)$ is convex in λ [82]. However, $g(\lambda)$ may not be differentiable. Fortunately, it is possible to find the subgradient of $g(\lambda)$. Suppose that at λ, $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ is the optimal solution of (6.19). For any given $\lambda' > 0$, one has

$$\begin{aligned} g(\lambda') &= \max_{\{\mathbf{Q}_i\}} \omega \sum_{i=1}^K R_i^{\mathrm{BC}}\big(\{\mathbf{Q}_i\}\big) - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A}+\lambda'\mathbf{I})\mathbf{Q}_i\} + \lambda' P \\ &\geq \omega \sum_{i=1}^K R_i^{\mathrm{BC}}\big(\{\mathbf{Q}_i^\star\}\big) - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A}+\lambda'\mathbf{I})\mathbf{Q}_i^\star\} + \lambda' P \\ &= g(\lambda) + \left(P - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\}\right)(\lambda' - \lambda). \end{aligned} \tag{6.20}$$

Thus, $P - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\}$ can be chosen as the subgradient of $g(\lambda)$. The subgradient search direction suggests to increase λ if $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} \geq P$ or decrease λ otherwise. Since λ is searched in a one-dimensional space, the bisection method can be efficiently applied to find the optimal $\lambda^\star$. We summarize the proposed algorithm to solve the nonconvex problem (6.8) in Algorithm 6.1.

The optimality of the proposed algorithm is proved in the following theorem.

Theorem 6.2. *The proposed Algorithm 6.1 achieves the globally optimal solution to problem* (6.8).

Proof. Per the proposed Algorithm 6.1, if the obtained solution set $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ for the case $\lambda = 0$ meets the power constraint $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} \leq P$, then $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ is the globally optimal solution to problem (6.8). This is due to the equivalence between the BC problem (6.11) and the MAC problem (6.13).

We now focus on the case $\lambda^\star > 0$. Suppose that the obtained solution $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ from the proposed algorithm is not globally optimal, and there is another solution set $\hat{\mathbf{Q}}_1, \ldots, \hat{\mathbf{Q}}_K$ satisfying the conditions:

(i) $\sum_{i=1}^K \mathrm{Tr}\{\hat{\mathbf{Q}}_i\} \leq P$

(ii) $\omega \sum_{i=1}^K R_i^{\mathrm{BC}}(\hat{\mathbf{Q}}) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}\hat{\mathbf{Q}}_i\} > \omega \sum_{i=1}^K R_i^{\mathrm{BC}}(\mathbf{Q}^\star) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i^\star\}$.

Algorithm 6.1: Iterative Algorithm for the MIMO-BC Sum-rate Maximization with a Penalty Term

1 **for** *a given* $\lambda \geq 0$ **do**
2 Change the variables as $\tilde{\mathbf{Q}}_i = (\mathbf{A} + \lambda \mathbf{I})^{1/2} \mathbf{Q}_i (\mathbf{A} + \lambda \mathbf{I})^{1/2}$ and $\tilde{\mathbf{H}}_i = \mathbf{R}_i^{-1/2} \mathbf{H}_i (\mathbf{A} + \lambda \mathbf{I})^{-1/2}$;
3 Solve the equivalent uplink MAC problem $\underset{\mathbf{X}_1,\ldots,\mathbf{X}_K}{\text{maximize}}\ \omega \log \left| \mathbf{I} + \sum_{i=1}^K \tilde{\mathbf{H}}_i^H \mathbf{X}_i \tilde{\mathbf{H}}_i \right| - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{X}_i\}$ by;
4 **repeat**
5 **for** $i = 1, 2, \ldots, K$ **do**
6 Perform the eigen-decomposition $\tilde{\mathbf{H}}_i \big(\mathbf{I} + \sum_{j \neq i}^K \tilde{\mathbf{H}}_j^H \mathbf{X}_j \tilde{\mathbf{H}}_j \big)^{-1} \tilde{\mathbf{H}}_i^H = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^H$;
7 Update $\mathbf{X}_i = \mathbf{U}_i \big[\omega \mathbf{I} - \mathbf{\Sigma}_i^{-1} \big]^+ \mathbf{U}_i$;
8 **end**
9 **until** *convergence to* $\mathbf{X}_1^\star, \ldots, \mathbf{X}_K^\star$;
10 Compute $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ from $\mathbf{X}_1^\star, \ldots, \mathbf{X}_K^\star$ by the MAC-BC transformation;
11 Compute $\mathbf{Q}_i^\star = (\mathbf{A} + \lambda \mathbf{I})^{-1/2} \tilde{\mathbf{Q}}_i^\star (\mathbf{A} + \lambda \mathbf{I})^{-1/2}, i = 1, \ldots, K$;
12 **end**
13 **case** $\lambda = 0$
14 Follow step 1 to 12 to obtain $\mathbf{Q}_i^\star$;
15 **if** $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} \leq P$ **then** stop the algorithm;
16 **case** $\lambda > 0$
17 Set $\lambda_{\min} = 0$ and $\lambda_{\max}$ large;
18 **repeat**
19 $\lambda = (\lambda_{\min} + \lambda_{\max})/2$;
20 Follow step 1 to 12 to obtain $\mathbf{Q}_i^\star$;
21 **if** $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} > P$ **then** set $\lambda_{\min} = \lambda$; **otherwise**, set $\lambda_{\max} = \lambda$
22 **until** $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} = P$ *or* $(\lambda_{\max} - \lambda_{\min})$ *is small enough*;

Since $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ globally maximizes the Lagrangian as given in problem (6.19), one has

$$\omega \sum_{i=1}^K R_i^{\mathrm{BC}}(\mathbf{Q}^\star) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i^\star + \lambda^\star \mathbf{Q}_i^\star\} \geq \omega \sum_{i=1}^K R_i^{\mathrm{BC}}(\hat{\mathbf{Q}}) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}\hat{\mathbf{Q}}_i + \lambda^\star \hat{\mathbf{Q}}_i\}. \tag{6.21}$$

Thus, condition (ii) then guarantees

$$\lambda^\star \sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} < \lambda^\star \sum_{i=1}^K \mathrm{Tr}\{\hat{\mathbf{Q}}_i\}. \tag{6.22}$$

Since Algorithm 6.1 guarantees $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\} = P$ for the case $\lambda^\star > 0$, one has

$$P = \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\} < \sum_{i=1}^{K}\mathrm{Tr}\{\hat{\mathbf{Q}}_i\}, \tag{6.23}$$

which contradicts the condition (i). Thus the proof for this theorem follows by contradiction. □

As proved in Theorem 6.1, the optimization (6.8) carried at cell-q can be effectively and optimally solved. For the network WSR maximization problem (6.5), the proposed ILA algorithm requires each cell-q, $q = 1, \ldots, Q$ to iteratively update the matrix $\mathbf{A}_q$ and solve its approximated optimization problem (6.8).

Theorem 6.3. *The optimization* (6.8) *performed at any given BS-q always improves the network WSR $\sum_{q=1}^{Q}\omega_q R_q^{\mathrm{BC}}$. Thus, the Gauss-Seidel iterative update is guaranteed to coverge to at least a local maximum.*

Proof. Suppose that $\mathbf{Q}_q = \bar{\mathbf{Q}}_q = \left\{\bar{\mathbf{Q}}_{q_i}\right\}_{i=1}^{K}, \forall q$ was obtained from the previous iteration, and $\mathbf{Q}_q^\star = \left\{\mathbf{Q}_{q_i}^\star\right\}_{i=1}^{K}, \forall q$ is the optimal solution after the current iteration. Note that $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is a convex function with respect to $\mathbf{Q}_q \in \mathcal{S}_q \triangleq \{\{\mathbf{Q}_{q_i}\} | \mathbf{Q}_{q_i} \succeq \mathbf{0}, \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q\}$ [57,81]. Thus, the first-order condition of the convex function f_q [82] dictates that

$$f_q(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \geq f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\left\{\mathbf{A}_q(\mathbf{Q}_{q_i}^\star - \bar{\mathbf{Q}}_{q_i})\right\} \tag{6.24}$$

with $\mathbf{A}_q$ being defined in (6.7) at $\bar{\mathbf{Q}}_{q_i}$. After one Gauss-Seidel iteration, the network WSR is updated as

$$\begin{aligned}
\sum_{q=1}^{Q}\omega_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) &= \omega_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) + f_q(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \\
&\geq \omega_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) + f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\left\{\mathbf{A}_q(\mathbf{Q}_{q_i}^\star - \bar{\mathbf{Q}}_{q_i})\right\} \\
&\geq \omega_q R_q^{\mathrm{BC}}(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) + f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\left\{\mathbf{A}_q(\bar{\mathbf{Q}}_{q_i} - \bar{\mathbf{Q}}_{q_i})\right\} \\
&= \sum_{q=1}^{Q}\omega_q R_q^{\mathrm{BC}}(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}),
\end{aligned}$$

where the first inequality is due to the one in (6.24), and the second inequality is due to $\mathbf{Q}_q^\star$ being the optimal solution of problem (6.8). Hence, the network WSR is strictly nondecreasing after an update of the covariance matrices at any given BS. With the Gauss-Seidel (sequential) iterative update, each BS takes turns to refine its precoders and improve the network WSR. Since the network WSR is upper-bounded, the Gauss-Seidel iterative update is guaranteed to converge to at least a local maximum. □

Algorithm 6.2: ILA Algorithm for the Multicell MIMO-BC with DPC

```
1 Initialize {Q_{q_i}}_{∀q,∀i}, such that ∑_{i=1}^{K} Tr{Q_{q_i}} = P_q.;
2 repeat
3     Q̄_{q_i} ← Q_{q_i};
4     for q = 1, 2, ..., Q do
5         At the BS, update the matrix A_q as given in (6.7);
6         Update Q_{q_i}, i = 1, ..., K by executing Algorithm 6.1;
7     end
8 until convergence;
```

The ILA algorithm for the multicell MIMO-BC is summarized in Algorithm 6.2. In Algorithm 6.2, we refer to the iterative procedure 2-8 as an outer-loop iteration and refer to the update at a particular BS using the iterative Algorithm 6.1 in step 6 as an inner-loop iteration. It is worth mentioning that certain optimization steps in Algorithm 6.2 can be assigned and executed in a distributed manner across the coordinated BSs. We address the distributed implementation of the ILA algorithm in Section 6.3.2.

6.3.2 Distributed Implementation of the Proposed ILA Algorithm

In order to implement the proposed ILA algorithm distributively, the following assumptions are taken in consideration:

- *Assumption 1:* Each BS, say BS-q, knows the channel matrices $\mathbf{H}_{qr_i}$'s to all the MSs in the network. This assumption allows the BS to control its induced ICI to other cells.
- *Assumption 2:* The coordinated BSs have reliable backhaul links to exchange control information among themselves.

- *Assumption 3:* The channels are in block-fading or vary sufficiently slow such that they can be considered fixed during the optimization process.

It is to be noted that the optimization (6.8) can be performed distributively at the corresponding BS with local information. Thus, it remains to show that the factors $\mathbf{A}_q$'s can also be computed in a distributed manner through a message exchange mechanism among the BSs. It is observed from equation (6.7) that in order to compute $\mathbf{A}_q$, BS-q has to possess the channels $\mathbf{H}_{qr_j}$'s to all the MSs in the other cells, as stated in Assumption 1. In addition, BS-q needs to acquire the pricing matrix

$$\mathbf{B}_{r_j} = \omega_r \left[\mathbf{C}_{r_j}^{-1} - \left(\mathbf{C}_{r_j} + \mathbf{H}_{rr_j} \mathbf{Q}_{r_j} \mathbf{H}_{rr_j}^H \right)^{-1} \right] \tag{6.25}$$

from other cells, where $\mathbf{C}_{r_j} = \mathbf{R}_{r_j} + \sum_{k=1}^{j-1} \mathbf{H}_{rr_j} \mathbf{Q}_{r_k} \mathbf{H}_{rr_j}^H$. Thus, it is required that MS-j at cell-r computes its corresponding factor $\mathbf{B}_{r_j}$ using only local measurements. In fact, $\mathbf{C}_{r_j}$ is the total interference plus noise and $\mathbf{C}_{r_j} + \mathbf{H}_{rr_j} \mathbf{Q}_{r_j} \mathbf{H}_{rr_j}^H$ is the total signal and interference plus noise, pertaining to MS-i of cell-q. After computing the pricing matrix $\mathbf{B}_{r_j}$, the MS can feed back this parameter to its connected BS. These factors $\mathbf{B}_{r_j}$'s are then exchanged among the BSs to compute $\mathbf{A}_q$'s.

Remark 6.4: It is proved in Theorem 6.3 that the optimization carried at a given BS always improves the network WSR, which leads to the convergence of the ILA algorithm with the Gauss-Seidel update. However, before each update, all the MSs are required to compute their pricing matrices $\mathbf{B}_{r_j}$'s and exchange them within the whole network. As a result, the Gauss-Seidel update may demand a lot of computation at the MSs and message exchanges in the network. To reduce the amount of computation and message exchanges, the proposed ILA algorithm can be also implemented by the Jacobi (simultaneous) iterative update. Specifically, after each instance of pricing update and message exchange, all the BSs simultaneously update their covariance matrices. Although the convergence of the ILA algorithm with the Jacobi update is not analytically proved, we observe in simulations that the Jacobi update converges much faster than the Gauss-Seidel update in terms of number of iterations.

6.4 The WMMSE Solution Approach for the Multicell MIMO-BC

6.4.1 The DPC-WMMSE Algorithm for the Multicell MIMO-BC

In Section 6.3, we examined a linear convex approximation technique to solve the nonconvex optimization problem (6.5) by successively improving the downlink covariance matrices at the BSs. In this section, we examine the second approach to solve this nonconvex problem by transforming it into a matrix-weighted sum-MSE minimization problem. Following the approach proposed in [33, 58], we develop a version of the WMMSE algorithm for the multicell MIMO-BC with DPC precoding. The newly developed algorithm shall be referred to as the DPC-WMMSE algorithm, in order to avoid the confusion with the original WMMSE algorithm for the case of linear precoding in [58]. Unlike the ILA algorithm, which tries to optimize over the transmit covariances $\mathbf{Q}_{q_i}$'s, the optimization in the DPC-WMMSE algorithm is carried over the transmit beamforming matrix $\mathbf{V}_{q_i}$.

With $\mathbf{V}_{q_i}$'s being the variables, the optimization problem (6.5) can be restated as

$$\begin{aligned} \underset{\{\mathbf{V}_{q_i}\}_{\forall i, \forall q}}{\text{maximize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} R_{q_i}^{\text{BC}} \\ \text{subject to} \quad & \sum_{i=1}^{K} \text{Tr}\{\mathbf{V}_{q_i} \mathbf{V}_{q_i}^H\} \le P_{q_i}, \ \forall q, \end{aligned} \tag{6.26}$$

where the achievable rate R_{q_i}, given in (6.4), can be restated as

$$R_{q_i}^{\text{BC}} = \log \frac{\left| \mathbf{Z}_{q_i} + \sum\limits_{r \neq q}^{Q} \sum\limits_{j=1}^{K} \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{rq_i}^H + \sum\limits_{j=1}^{i} \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_i}^H \right|}{\left| \mathbf{Z}_{q_i} + \sum\limits_{r \neq q}^{Q} \sum\limits_{j=1}^{K} \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{rq_i}^H + \sum\limits_{j=1}^{i-1} \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_i}^H \right|}. \tag{6.27}$$

Since DPC with the encoding order from MS-K to MS-1 is applied at each BS, MS-i receives the signal from BS-q as if there was no intra-cell interference from user-$(i+1)$ to user-K. Thus, while treating the ICI as noise, the estimated signal for user-i in BS-q is given by

$$\hat{\mathbf{s}}_{q_i} = \mathbf{U}_{q_i}^H \left(\sum_{j=1}^{i} \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{s}_{q_j} + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{s}_{r_j} + \mathbf{z}_{q_i} \right), \tag{6.28}$$

where $\mathbf{U}_{q_i}^H$ is the receive beamformer at MS-i. Given the transmit beamforming matrices $\mathbf{V}_{q_i}$'s, the receive beamformer matrix $\mathbf{U}_{q_i}$ is designed to minimize the MSE for the data streams intended for MS-i of cell-q. Let $\mathbf{E}_{q_i}$ be the expected MSE matrix of MS-i, which is defined as

$$\begin{aligned}
\mathbf{E}_{q_i} &= \mathbb{E}\left[(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})^H\right] \\
&= \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}\right)\left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}\right)^H + \sum_{j=1}^{i-1} \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_i}^H \mathbf{U}_{q_i} \\
&\quad + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{U}_{q_i}^H \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{rq_i}^H \mathbf{U}_{q_i} + \mathbf{U}_{q_i}^H \mathbf{Z}_{q_i} \mathbf{U}_{q_i}.
\end{aligned} \tag{6.29}$$

To minimize the sum MSE, the optimal receive beamformer $\mathbf{U}_{q_i}$ can be derived straightforwardly as the Wiener filter, *i.e.*, MMSE receiver

$$\begin{aligned}
\mathbf{U}_{q_i} &= \arg\min_{\mathbf{U}_{q_i}} \mathrm{Tr}\left\{\mathbf{E}_{q_i}\right\} \\
&= \left(\sum_{j=1}^{i} \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_i}^H + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i} \right)^{-1} \mathbf{H}_{qq_i} \mathbf{V}_{q_i}.
\end{aligned} \tag{6.30}$$

The resultant MMSE matrix for user-i in cell-q is then given by

$$\begin{aligned}
\mathbf{E}_{q_i}^{\mathrm{MMSE}} &= \mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \qquad (6.31) \\
&= \left[\mathbf{I} + \mathbf{V}_{q_i}^H \mathbf{H}_{qq_i}^H \left(\sum_{j=1}^{i-1} \mathbf{H}_{qq_i} \mathbf{V}_{q_j} \mathbf{V}_{q_j}^H \mathbf{H}_{qq_i}^H + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{rq_i} \mathbf{V}_{r_j} \mathbf{V}_{r_j}^H \mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i} \right)^{-1} \mathbf{H}_{qq_i} \mathbf{V}_{q_i} \right]^{-1}.
\end{aligned}$$

Similar to the case of linear precoding, the relationship between the data rate and the MMSE matrix with DPC can be expressed as

$$R_{q_i} = \log \left| \left(\mathbf{E}_{q_i}^{\mathrm{MMSE}}\right)^{-1} \right|. \tag{6.32}$$

Due to this relationship, the equivalence between the WSR maximization problem in the multicell MIMO-BC and the matrix-weighted sum-MSE minimization can be established in the following theorem.

Theorem 6.4. *The multicell MIMO-BC WSR maximization problem* (6.26) *is equivalent to the following matrix weighted sum-MSE minimization*

$$\begin{aligned} \underset{\mathbf{W}_{q_i},\mathbf{V}_{q_i},\mathbf{U}_{q_i}}{\text{minimize}} \quad & \sum_{q=1}^{Q} \omega_q \sum_{i=1}^{K} \left[\mathrm{Tr}\left\{ \mathbf{W}_{q_i} \mathbf{E}_{q_i} \right\} - \log \left| \mathbf{W}_{q_i} \right| \right] \\ \text{subject to} \quad & \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \right\} \leq P_q, \forall q, \end{aligned} \tag{6.33}$$

where $\mathbf{W}_{q_i} \succeq \mathbf{0}$ *is the weight matrix for MS-i at cell-q. In particular, the globally optimal solutions* $\{\mathbf{V}\}_{\forall q, \forall i}$ *are identical for the two problems.*

Proof. The proof for this theorem is similar to that in [33,58] for the case of linear precoding. Thus, we omit the details for brevity. □

Since solving problem (6.26) is equivalent to solving problem (6.33), we now proceed to numerically obtain the solution to the latter problem. Although the objective function in (6.33) is not jointly convex over the whole set of variables, it is convex in each set of the variables $\mathbf{U}_{q_i}, \mathbf{V}_{q_i}, \mathbf{W}_{q_i}$. Thus, it is possible to find a locally optimal solution to problem (6.33) by alternately optimizing one set of the variables while fixing the other two until convergence. First, with fixed transmit beamformers $\mathbf{V}_{q_i}$'s, the receive beamformers $\mathbf{U}_{q_i}$'s are given as in (6.30). Second, fixing the transmit and receive beamformers $\mathbf{V}_{q_i}$'s and $\mathbf{U}_{q_i}$'s, the weighted-matrices $\mathbf{W}_{q_i}$'s are updated in a closed form solution

$$\mathbf{W}_{q_i} = \mathbf{E}_{q_i}^{-1} = \mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}. \tag{6.34}$$

Finally, by decomposing the objective function in (6.33), the transmit beamformers $\mathbf{V}_{q_i}$'s are updated by solving decoupled optimization problems across the BSs

$$\begin{aligned}\underset{\mathbf{V}_{q_1},\ldots,\mathbf{V}_{q_K}}{\text{minimize}}\quad & \sum_{i=1}^{K}\Bigg[\omega_q \mathrm{Tr}\left\{\mathbf{W}_{q_i}\left(\mathbf{I}-\mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i}\right)\left(\mathbf{I}-\mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i}\right)^H\right\}\\ &\qquad +\omega_q\sum_{j>i}^{K}\mathrm{Tr}\left\{\mathbf{W}_{q_j}\mathbf{U}_{q_j}^H\mathbf{H}_{qq_j}\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\mathbf{H}_{qq_j}^H\mathbf{U}_{q_j}\right\}\\ &\qquad +\sum_{r\neq q}^{Q}\sum_{j=1}^{K}\omega_r \mathrm{Tr}\left\{\mathbf{W}_{r_j}\mathbf{U}_{r_j}^H\mathbf{H}_{qr_j}\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\mathbf{H}_{qr_j}^H\mathbf{U}_{r_j}\right\}\Bigg] \qquad (6.35)\\ \text{subject to}\quad & \sum_{i=1}^{K}\mathrm{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\}\le P_q.\end{aligned}$$

This optimization then can be carried simultaneously and separately at BS-q. Since problem (6.35) is a convex quadratic program, its optimal solution can be derived as

$$\mathbf{V}_{q_i}=\omega_q\left(\sum_{j=i}^{K}\omega_q\mathbf{H}_{qq_j}^H\mathbf{U}_{q_j}\mathbf{W}_{q_j}\mathbf{U}_{q_j}^H\mathbf{H}_{qq_j}+\sum_{r\neq q}^{Q}\sum_{j=1}^{K}\omega_r\mathbf{H}_{qr_j}^H\mathbf{U}_{r_j}\mathbf{W}_{r_j}\mathbf{U}_{r_j}^H\mathbf{H}_{qr_j}+\mu_q^\star\mathbf{I}\right)^{-1}\mathbf{H}_{qq_i}^H\mathbf{U}_{q_i}\mathbf{W}_{q_i}, \qquad (6.36)$$

where $\mu_q^\star$ is the optimal Lagrangian multiplier associated with the power constraint. In case of $\mu_q^\star=0$, $\sum_{i=1}^{K}\mathrm{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\}<P_q$, then the BS-$q$ does not utilize its full power. Otherwise, $\mu_q^\star$ can be easily obtained by the bisection method until the power constraint is met with equality.

We summarize the DPC-WMMSE algorithm for the multicell MIMO-BC as in Algorithm 6.3. In this algorithm, in each outer-loop iteration, each set of variables (*i.e.*, the receive beamforming matrices $\mathbf{U}_{q_i}$'s, the weight matrices $\mathbf{W}_{q_i}$'s, and the transmit beamforming matrices $\mathbf{V}_{q_i}$'s) can be updated simultaneously across the Q cells. Compared to the ILA algorithm, the DPC-WMMSE algorithm does not require any inner-loop iteration or the BC-MAC transformations because of the direct update of the variables $\mathbf{U}_{q_i}$, $\mathbf{W}_{q_i}$, and $\mathbf{V}_{q_i}$. Compared to the original WMMSE algorithm with linear precoding in [33, 58], the DPC-WMMSE algorithm requires some modifications to the transmit beamforming matrices $\mathbf{V}_{q_i}$'s and the receive beamforming matrices $\mathbf{U}_{q_i}$ to accommodate the DPC.

In the DPC-WMMSE algorithm, the iterative process is executed by alternatively optimizing over each set of variables in $\mathbf{U}_{q_i}$'s, $\mathbf{W}_{q_i}$'s, and $\mathbf{V}_{q_i}$'s. Since the constraint set of problem (6.33) is decoupled for each set of variables, the alternative optimization over $\mathbf{U}_{q_i}$'s, $\mathbf{W}_{q_i}$'s, and $\mathbf{V}_{q_i}$'s must decrease the objective function monotonically [89]. Moreover,

Algorithm 6.3: DPC-WMMSE Algorithm for the Multicell MIMO-BC with DPC

Initialize $\{\mathbf{V}_{q_i}\}_{\forall q, \forall i}$, such that $\sum_{j=1}^{K} \mathrm{Tr}\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\} = P_q$;
repeat
  Set $\bar{\mathbf{V}}_{q_i} \leftarrow \mathbf{V}_{q_i}, \ \forall q, \forall i.$;
  Simultaneously update across Q cells;
  for $q = 1, \ldots, Q$ **do**
    At the K MSs, update the receive beamformers and weight matrices;
    for $i = 1, \ldots, K$ **do**

$$\mathbf{U}_{q_i} \leftarrow \left(\sum_{j=1}^{i} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i} \right)^{-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_i};$$

$$\mathbf{W}_{q_i} \leftarrow \left(\mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{q_i}\mathbf{V}_{q_i}\right)^{-1};$$

    end
    At the BS, update the transmit matrices;

$$\mathbf{V}_{q_i} \leftarrow \omega_q \left(\sum_{j=i}^{K} \omega_q \mathbf{H}_{qq_j}^H\mathbf{U}_{q_j}\mathbf{W}_{q_j}\mathbf{U}_{q_j}^H\mathbf{H}_{qq_j} + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \omega_r \mathbf{H}_{qr_j}^H\mathbf{U}_{r_j}\mathbf{W}_{r_j}\mathbf{U}_{r_j}^H\mathbf{H}_{qr_j} + \mu_q^\star\mathbf{I} \right)^{-1} \mathbf{H}_{qq_i}^H\mathbf{U}_{q_i}\mathbf{W}_{q_i}, \forall i;$$

  end
until *convergence*;

the cost function in (6.33) is lower-bounded due to the power constraints on $\mathbf{V}_{q_i}$'s. Thus, the DPC-WMMSE must converge to at least a local minimum $(\mathbf{U}_{q_i}^\star, \mathbf{W}_{q_i}^\star, \mathbf{V}_{q_i}^\star)$ of the cost function (6.33). Note that the cost function of the original sum-rate maximization problem (6.26) does not necessarily improve after each iteration. However, given $(\mathbf{U}_{q_i}^\star, \mathbf{W}_{q_i}^\star, \mathbf{V}_{q_i}^\star)$ as a local minimizer of problem (6.33) obtained from the DPC-WMMSE algorithm, $\mathbf{V}_{q_i}^\star$ is also a local optimizer of the original problem (6.26). This observation was proved for the WMMSE algorithm in case of linear precoding [58]. The same proof can be applied here for the DPC-WMMSE algorithm with the DPC consideration.

6.4.2 Distributed Implementation of the DPC-WMMSE Algorithm

As shown in [58], the WMMSE algorithm can be implemented in a distributed manner. Under the same assumptions given in Section 6.3.2, distributed implementation to the DPC-WMMSE algorithm is also possible. At the receiving end, say MS-i in cell-q, the MS needs to locally measure its total signal plus interference and update its receive beamforming matrix $\mathbf{U}_{q_i}$ and weight matrix $\mathbf{W}_{q_i}$ as in (6.30) and (6.34) with local information. It then feeds back the updated $\mathbf{U}_{q_i}$ and $\mathbf{W}_{q_i}$ to its connected BS-q. BS-q then computes the matrix $\omega_q \sum_{i=1}^{K} \mathbf{U}_{q_i}\mathbf{W}_{q_i}\mathbf{U}_{q_i}^H$ and exchanges it to the other coordinated BSs in the network. At the transmitting end, BS-q, knowing the encoding order for its connected MSs, can update its transmit beamforming matrices $\mathbf{V}_{q_i}$'s within its power limit as stated in (6.35) and feeds back the updated $\mathbf{V}_{q_i}$ to its MS-i.

6.5 Simulation Results

This section presents some numerical evaluations on the achievable downlink sum-rate of a multicell system with different levels of coordination and on the convergence behavior of the proposed algorithms. We compare the sum-rate between 3 schemes: (i) the coordination mode with DPC obtained from the ILA and DPC-WMMSE algorithms (with equal weights $\omega_1 = \ldots = \omega_Q$), (ii) the competition mode where each BS selfishly maximizes the sum-rate for its connected MSs using DPC, and (iii) the coordination mode with linear precoding obtained from the WMMSE algorithm in [58]. Unless stated otherwise, the ILA scheme is implemented with the Gauss-Seidel update due to its guaranteed convergence. We consider a generic 3-cell system with 3 MSs per cell, as illustrated in Fig. 6.1. The numbers of antennas at each BS and each MS are assumed to be 4 and 2, respectively. The BSs are located at a normalized distance of 2 and the MSs are randomly located on a circle at distance d from its connected BS. The channel coefficients are generated by the path loss model with a path loss exponent of 3. The additive Gaussian noise at each MS is assumed to be white with the covariance matrix $\mathbf{Z}_{q_i} = \sigma^2\mathbf{I}$ and σ^2 is set at 0.01. The transmit power P_q at each BS is constrained at 1 W, *i.e.*, $P_q/\sigma^2 = 20$ dB, unless stated otherwise.

Fig. 6.2 illustrates the total network sum-rate versus the intracell MS-BS distance d obtained from the 3 schemes. As d is varied, $10,000$ channel realizations at each value of d are used used to obtain the average network sum-rate. Note that the effect of ICI is more apparent with increasing d due to the decreasing gain of intra-cell channels and the increasing gain of inter-cell channels. Thus, the network sum-rate is reduced with increasing d, as observed in the figure for all the schemes. Out of the 3 schemes, scheme (i) always show a superior sum-rate performance, since it takes advantages of both the nonlinear precoding in DPC and the interference coordination. At the low ICI region, i.e, small d, scheme (i) significantly outperforms scheme (iii) due to the use of DPC over linear precoding. On the other hand, at the high ICI region, by implementing the IC

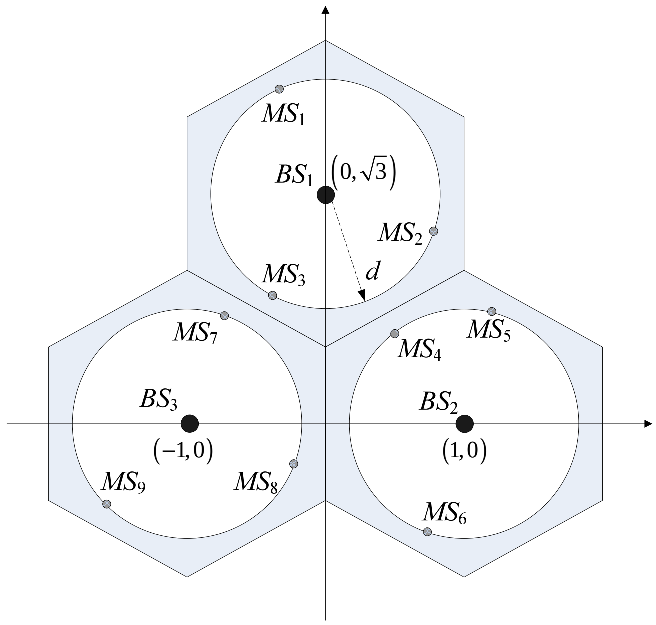

Fig. 6.1 A multicell system with 3 cells and 3 MSs per cell. Each MS is randomly located at a distance d from its connected BS.

with the proposed algorithm, one can significantly improve the network sum-rate over the competitive design. In scheme (i), it can be observed that the sum-rates obtained by the ILA and DPC-WMMSE algorithms closely match over whole range of d. However, at low d, the ILA algorithm outperforms the DPC-WMMSE algorithm. This behavior is probably due to the reason that each BS focuses more on maximizing its own sum-rate than limiting the ICI at the low-ICI region. In this case, Algorithm 6.1 utilized in the ILA algorithm can obtain the optimal solution to the per-cell sum-rate maximization problems, whereas the DPC-WMMSE obtains the sum-rate from the transformed WMMSE problem.

Fig. 6.3 illustrates the network sum-rate versus the transmit power to AWGN ratio P/σ^2 (with same power P at each BS) for $d = 0.7$. It is observed from the figure that increasing the transmit power at each BS shall increase the network sum-rate for all 3 schemes. However, at the high P/σ^2 region the sum-rate obtained from scheme (ii) becomes saturated. This is due to the reason that the competitive design does not attempt to control ICI, and thus increases the ICI relatively with the transmit power. In this case, it is more appealing to implement the IC designs in schemes (i) and (iii). Similar to the observation in Fig. 6.2, the non-linear precoding design in scheme (i) can extract extra performance

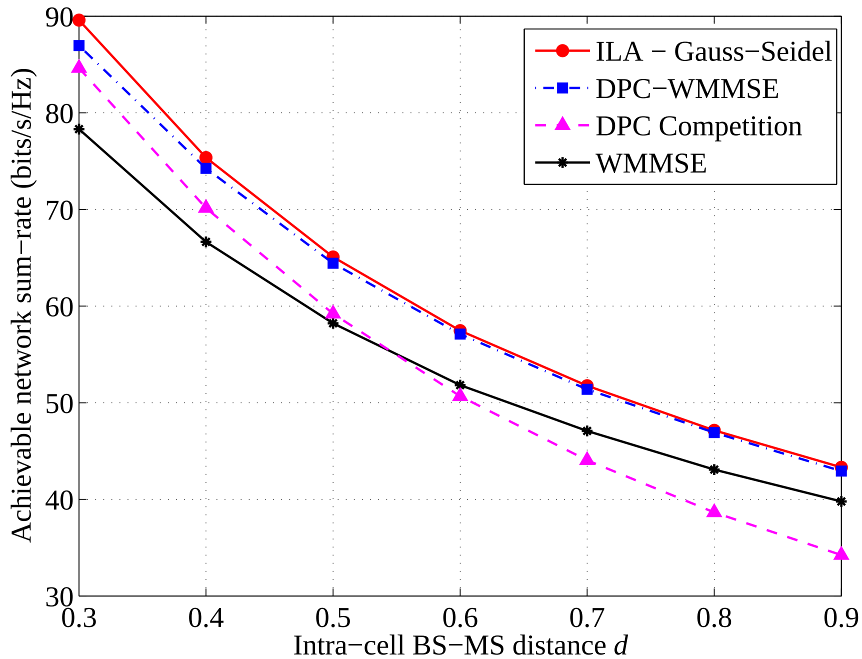

Fig. 6.2 Network sum-rates versus the intra-cell BS-MS distance d.

from the multicell network over the linear precoding design in scheme (iii). Interestingly, the ILA algorithm also provides a better sum-rate than the DPC-WMMSE algorithm at the high P/σ^2 region.

It is to be noted that the ILA, DPC-WMMSE, WMMSE algorithms, and the competitive design may require many iterations to fully converge. Herein, we define an iteration as an instance of message exchange among the BSs. As previously mentioned, to implement the interference coordination among the cells, each algorithm (ILA with Gauss-Seidel update, DPC-WMMSE, and WMMSE) requires the coordinated BSs to exchange signaling messages after each outer-loop iteration. For practical implementation, it may be desirable to set the number of iterations to reduce the amount of message exchanges. In Fig. 6.4, we compare the obtained sum-rates from each algorithm after 10 iterations versus the intra-cell BS-MS distance d. Fig. 6.4 shows that the ILA algorithm converges at a faster rate than the WMMSE algorithm in terms of the number of outer-loop iterations. Of the two types of updating in the ILA algorithm, the convergence of the Jacobi update is clearly faster than that of the Gauss-Seidel update.

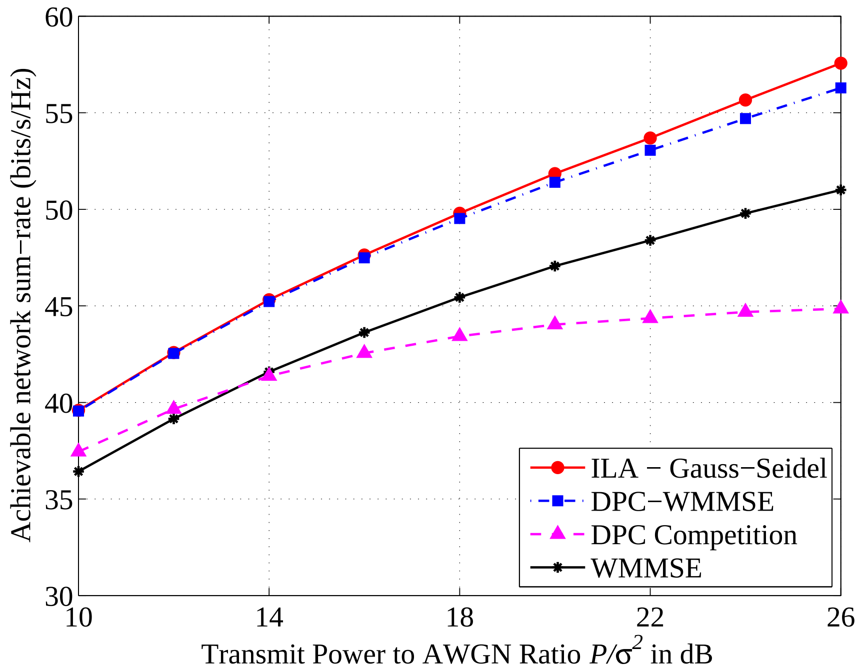

Fig. 6.3 Network sum-rates versus the transmit power to AWGN ratio for $d = 0.7$.

For a specific channel realization with $d = 0.7$, we illustrate the convergence behavior of the proposed ILA and DPC-WMMSE algorithms in Fig. 6.5. After each iteration, the network sum-rates obtained from the algorithms are plotted. In general, the ILA algorithm converges faster than the DPC-WMMSE algorithm, as Fig. 6.4 also indicates. As observed from Fig. 6.5, once a BS updates its covariance matrices, the overall network sum-rates are always improved by the ILA algorithm with the Gauss-Seidel update and the DPC-WMMSE algorithm. This behavior agrees with our analysis on the convergence of the algorithms. Interestingly, the ILA algorithm also experiences the monotonic convergence with the Jacobi update. Both the ILA and DPC-WMMSE algorithms eventually converge to a network sum-rate that is superior than the one obtained by the competitive design.

With the same sample channel realization as in Fig. 6.5, Fig. 6.6 displays the convergence of Algorithm 6.1 in maximizing the BC sum-rate with a penalty term, *i.e.*, problem (6.8). After each update of the dual variable λ, the evolutions of the sum-rate and the transmit power at BS-1 are plotted in the figure. As can be observed from the figure, the algorithm converges very fast in a few iterations. Due to the penalty term, BS-1 needs to

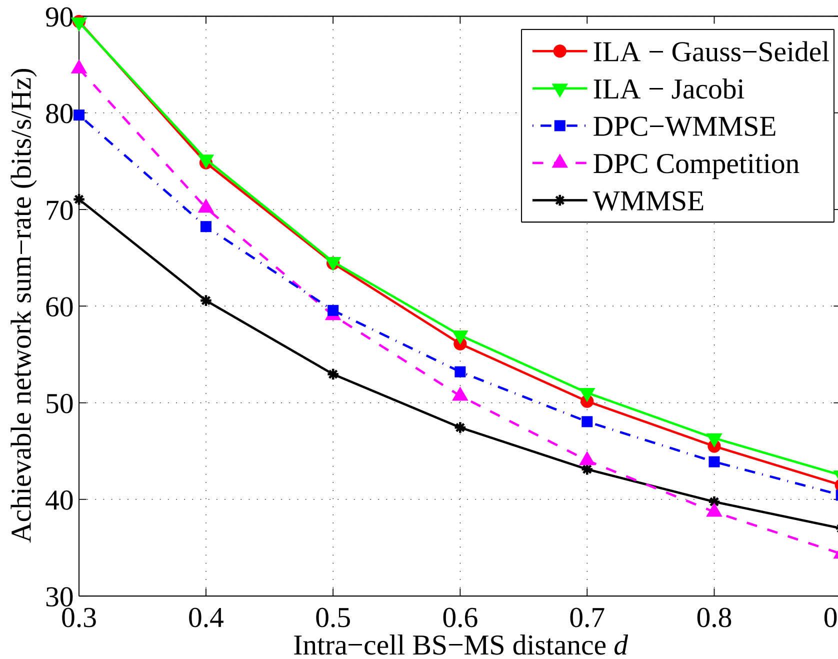

Fig. 6.4 Network sum-rates versus intra-cell BS-MS distance d, obtained from ILA, DPC-WMMSE, and WMMSE algorithms with 10 outer-loop iterations.

balance its achievable sum-rate with the ICI induced to cell-2 and -3. Thus, its sum-rate is undoubtedly reduced, compared to the one obtained in a conventional BC without the penalty term. Nonetheless, under the IC mode, each BS adopts a less selfish strategy to improve the overall network sum-rate, as shown in Fig. 6.5.

6.6 Concluding Remarks

This chapter examined the problem of network WSR maximization in the multicell MIMO-BC with DPC. Under the coordination mode, the network sum-rate maximization problem was shown to be nonconvex. This work then considered two low-complexity solution approaches, namely ILA and DPC-WMMSE to search for locally optimal solutions. In the first approach, successive convex approximation technique was utilized to transform the problem into multiple per-cell problems, which are then optimized distributively at each BS. In particular, each BS attempted to maximize the BC sum-rate to its connected BS

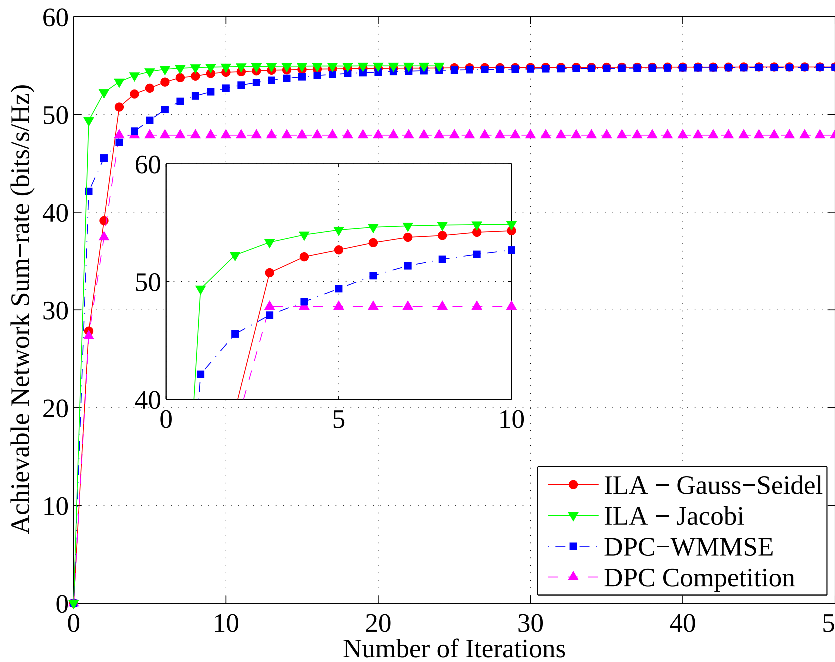

Fig. 6.5 Convergence of the proposed ILA and DPC-WMMSE algorithm to maximize the network sum-rate with interference coordination.

with a penalty term on its induced ICI to other cells. A distributed and fast converging algorithm was then proposed to efficiently find a locally optimal solution to the network WSR maximization problem. In the second approach, by establishing the equivalence between the maximization of the WSR and the minimization of the weighted MSE, the WSR problem was locally optimized by alternatively optimizing over the weight matrices and MMSE decoders at the MSs and the MMSE precoders at the BSs. As the proposed algorithms allow the multicell to take advantage of both DPC and coordinating the ICI, they show a significant improvement in the network sum-rate compared to competitive design and the linear precoding.

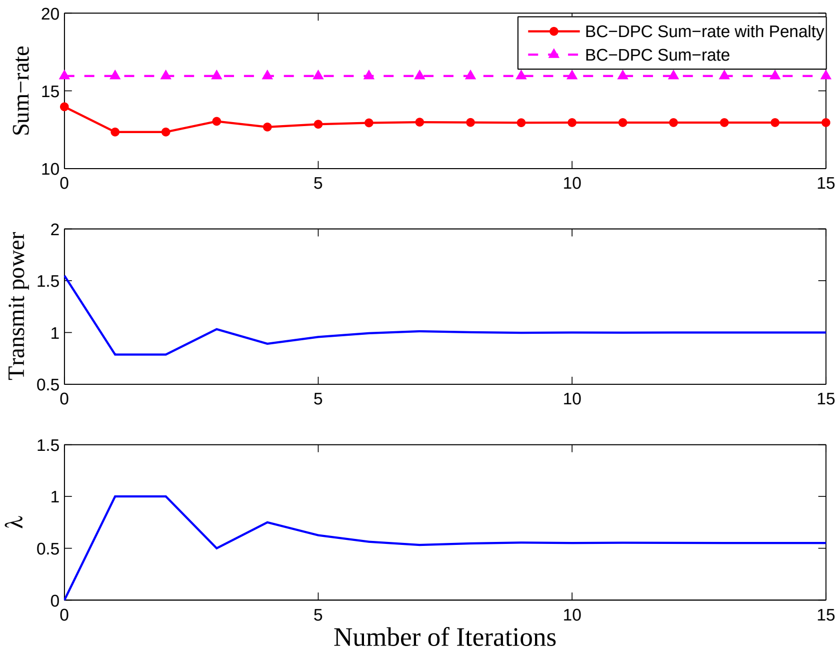

Fig. 6.6 The convergence of Algorithm 6.1 to solve Problem (6.8).

Chapter 7

Conclusion and Future Works

7.1 Summary

Inter-cell interference management has been a critical issue for current and future wireless cellular systems with universal frequency reuse. Regarded as a key technology of the LTE-A standard, CoMP champions the concept of interference-aware multicell coordination to actively deal with the ICI. By coordinating the transmissions from multiple cells, significant power reduction and rate enhancement can be realized. However, the standardization and successful implementation of CoMP to its full potential may face several technical challenges. Due to the large-scale and distributed nature of the multicell system, it is quite difficult to gather the CSI from the whole coordinated system to fully control the ICI. In addition, CoMP may induce a significant amount of signaling message exchanges, which become undesirable due to the limitations of the backhaul links.

This dissertation has been concerned with the precoding perspectives in multiuser CoMP systems under two operating modes: interference aware and interference coordination. Specifically, we have examined various precoding techniques and proposed low-complexity algorithms in devising optimal CoMP precoders. We have also presented the message exchange mechanism among the coordinated BSs and MSs to facilitate the distributed implementation of the proposed algorithms.

Chapter 3 has considered the game theoretical approach in multicell precoding designs with the objective of minimizing the transmit power at the BSs. Via the game theory framework, we have examined the conditions on the existence and uniqueness of a stable NE for the multicell system in the IA mode. To improve the efficiency of the NE, we have considered a more cooperative game through a pricing mechanism. A condition on the pricing factors has been presented to allow the new NE point approaching Pareto-optimal solutions obtained by the IC mode. Chapter 4 has studied multiuser precoding designs in a multicell system with the objective of maximizing the data-rate in the downlink transmission. We have examined the multicell system where BD is applied on a per-cell basis. In the IA mode, we have considered and characterized the SNG among the BSs. In the IC mode, we have developed a low-complexity algorithm to distributively design the BD precoders across the coordinated BSs. Extension to BD-DPC precoding has also been given in Chapter 4.

In Chapter 5, we have studied the joint WSR maximization in the multicell MIMO-MAC under the IC mode. Since this WSR maximization problem is nonconvex, obtaining its globally optimal solution is rather computationally complex. We have then proposed two iterative solution approaches: one is based on successive convex approximation (ILA algorithm) and the other is based on the transformation to the weighted MSE minimization problem (WMMSE algorithm). Both solution approaches have revealed the structure of the optimal uplink precoders and the message signaling mechanism to facilitate their distributed implementation among the coordinated cells. Chapter 6 has examined the precoding designs to maximize the WSR in a multicell MIMO-BC under the IC mode. The two approaches, namely ILA and WMMSE, have been exploited to search for local optimal solutions to this nonconvex problem. Distributed implementation to the two approaches has also been presented in the chapter. Interestingly, we have shown a connection between the MIMO-BC and MIMO-MAC problems to expedite the search for the former problem's solution. Generally, simulation results have shown a significant performance improvement in terms of transmit power and achievable sum-rate by the IC mode over the IA mode.

7.2 Potential Future Works

The research presented in this Ph.D. dissertation has examined only a small tip of iceberg on the precoding design perspective of CoMP. There are avenues for further research to refine and improve the precoders in practical implementation of CoMP.

1. *Precoding designs with limited feedback in the CoMP setting:* The current precoding designs for CoMP come with the requirements for perfect CSI knowledge across the

coordinated cells. In practice, it is generally challenging to acquire and exchange the CSI signaling accurately, especially in the presence of quantization effects, fast varying environment, and CSI feedback delays. Precoding designs with limited feedback in MIMO single-cell systems have attracted a lot of research interests due to their minor performance deficiency, compared to the designs with perfect CSI feedback [95]. Certainly, it will be interesting to investigate the performance of precoding designs with limited feedbacks in a CoMP system.

2. *Regularized ZF/BD precoding designs under the CoMP:* In Chapter 3, we have investigated the design of BD precoders in a CoMP system. While ZF/BD precoders are near optimal at high SNR, their performances are poor at the low SNR region. Regularized ZF/BD precoding has been shown as an effective and simple technique to tackle the limitation of plain ZF/BD precoding in a single-cell system [96–100]. Extension of the regularized ZF/BD precoding to the multicell setting is also an interesting research direction.

3. *Nonlinear precoding with vector perturbation*: While DPC is the optimal multiuser precoding technique, DPC is highly complex to implement. Practical nonlinear precoding techniques such as vector perturbation (VP) can significant improve the system performance over the linear precoding [101–104]. There is quite a limited number of works on VP precoding in a multicell system [105]. It will be interesting to further investigate the performance of VP precoding in conjunction with Regularized ZF/BD precoding in a CoMP system.

Appendix A

Dirty-Paper Coding

A.1 The Theory of Dirty-Paper Coding

The name "Dirty-paper coding" (DPC) comes from the title of the paper by Max Costa [25], who determined the capacity of a Gaussian channel having interference that is known to the transmitter. The system model of a Gaussian channel with an interference source is given by

$$y = x + i + z, \tag{A.1}$$

where x is the transmitted signal, $i \sim \mathcal{CN}(0, \sigma_i^2)$ is the Gaussian interference source, and $z \sim \mathcal{CN}(0, \sigma^2)$ is the Gaussian noise. The transmitted signal x is used for sending a codeword u with a power constraint $|x|^2 \leq P$. When the interference i is known at the receiver, it can be fully suppressed at the receiver such that the capacity of the system is

$$C = \log\left(1 + \frac{P}{\sigma^2}\right). \tag{A.2}$$

Now, what happens if the interference i is known at the transmitter only, but *not* at the receiver? In this case, the transmitter can simply *pre-cancel* the interference. However, part of the power P will be used for interference cancelation, which makes the capacity of the system less than C. Interestingly, it was shown in Costa's paper [25] that the capacity C is still achievable for the system (A.1) by the so-called "dirty-paper coding" technique. More specifically, depending on a *non-causal* random state of s, the transmitter shifts its designed alphabet set for the codeword u in the direction of s, instead of canceling s. This

DPC technique is analogous to writing adaptively over the dirt pattern on the dirty paper sheet instead of avoiding the dirt.

A.2 Dirty-Paper Coding in a Multiuser Downlink System

DPC has found is its great application in multiuser downlink transmissions by reducing the effect caused by inter-user interference. In fact, it was proved that DPC is the capacity-achieving multiuser coding technique for MISO systems [26] and for MIMO systems [27]. In a nutshell, for the multiuser DPC encoding scheme, the codewords for the multiple users are encoded in sequence, such that the codeword indeed for a particular user, say user-i, is designed by taking advantage of the non-causal knowledge of the interference caused by the codewords of the users encoded before user-i. With DPC, the achievable capacity for the transmission to user-i is the same as if there is no interference induced by the codewords of these users. A detailed expression on the achievable rate for each user in a multiuser MIMO downlink system is given in Section 2.1.3.

Appendix B

A Brief Overview of Game Theory

Game theory is a branch of mathematics studying the conflict and cooperation between rational decision-makers [106–108]. The aim of game theory is to model and understand the interaction between competing entities. Game theory has been mainly applied in economics, political science, and psychology. Recently, the application of game theory has been successfully adopted to the study of various engineering fields, including signal processing, information and communication theories [109]. As communication networks become more intelligent, decentralized, and self-organized, many communication problems can be naturally formulated as a game between the rational network entities. This dissertation, aims to the study various precoding designs in multicell wireless networks, relies on game theory in modeling and studying the interaction between the multiple cells. The aim of this appendix is to present a brief overview of game theory and review the theory behind the multicell games studied in this dissertation.

Let $\Omega = \{1, \ldots, Q\}$ denote a finite set of Q players and $\mathcal{S}_q$ denote the set of admissible strategies of player-q. Let $s_q \in \mathcal{S}_q$ be a strategy of player-q and $\mathbf{s}_{-q}$ be a strategy profile of the other players, except player-q. Collectively, let $\mathbf{s} = (s_q, \mathbf{s}_{-q}) \in \mathcal{S} \triangleq \prod_{q=1}^{Q} \mathcal{S}_q$. The aim of player-$q$, given other players' strategies $\mathbf{s}_{-q}$, is to choose a strategy that maximizes his payoff function $u_q(s_q, \mathbf{s}_{-q})$, *i.e.*,

$$\begin{aligned} \underset{s_q}{\text{maximize}} \quad & u_q(s_q, \mathbf{s}_{-q}) \\ \text{subject to} \quad & s_q \in \mathcal{S}_q, \end{aligned} \tag{B.1}$$

Such optimal strategy is termed as the *best response* strategy.

Mathematically, a generic game $\mathcal{G}$ can be defined as

$$\mathcal{G} = \Big(\Omega, \{\mathcal{S}_q\}_{q\in\Omega}, \{u_q\}_{q\in\Omega}\Big). \tag{B.2}$$

Definition B.1. *A strategy profile $\mathbf{s}^\star = (s_q^\star, \mathbf{s}_{-q}^\star)$ constitutes a (pure) Nash Equilibrium of game $\mathcal{G}$ when*

$$u_q(s_q^\star, \mathbf{s}_{-q}^\star) \geq u_q(s_q, \mathbf{s}_{-q}^\star), \quad \forall s_q \in \mathcal{S}_q, \forall q \in \Omega. \tag{B.3}$$

NE is an important concept in game theory to analyze the outcome of the strategic interaction between the rational players. In particular, the analysis of NE characterizes the stable operating point of the game where each player has no incentive to unilaterally change its strategy, *i.e.*, given other players' strategies, one player cannot improve its own utility at a NE. Note that the definition of the NE can be also generalized to contain mixed strategies, *i.e.*, the possibility of choosing a set of pure strategies by each player. However, the discussion of this dissertation is limited to the case of pure strategies only. One other important concept in game theory is Pareto-optimality, which is defined as follows.

Definition B.2. *A strategy profile $\mathbf{s}^{\mathrm{PO}}$ is said to be Pareto-optimal if there is a nonempty subset of players $\Omega_{++} \in \Omega$ such that*

$$u_q(\mathbf{s}^{\mathrm{PO}}) > u_q(\mathbf{s}), \forall i \in \Omega_{++}, \forall \mathbf{s} \in \mathcal{S} \tag{B.4}$$

and

$$u_q(\mathbf{s}^{\mathrm{PO}}) \geq u_q(\mathbf{s}), \forall i \in \Omega \backslash \Omega_{++}, \forall \mathbf{s} \in \mathcal{S}. \tag{B.5}$$

In other words, $\mathbf{s}^{\mathrm{PO}}$ is Pareto-optimal if there exists no other profiles of strategies of which one of more players can improve their utility without reducing the utilities of the others. In general, a NE-achieving strategy is not Pareto-optimal [52].

As mentioned in Section 1.2, CoMP under the IA mode is effectively a SNG where each cell is a rational player and each BS tries to adopts a strategy to maximize a utility function only to its connected MSs. Thus, CoMP under the IA mode results in a NE strategy. In contrast, CoMP under the IC mode results in a Pareto-optimal strategy.

Definition B.3. *A strategy profile $\mathbf{s}$ is said to be more Pareto-efficient than a strategy profile $\mathbf{s}'$ if $u_q(\mathbf{s}) \geq u_q(\mathbf{s}')$ for each q and $u_q(\mathbf{s}) > u_q(\mathbf{s}')$ for some q.*

Appendix C

Vector Norms, Matrix Norms, and Contraction Mapping

C.1 Vector Norms and Matrix Norms

In this appendix, we present some definitions of the vector norms and matrix norms applied in this work. The weighted *maximum norm* of a vector $\mathbf{x} \in \mathbb{C}^Q$, induced by a positive vector $\mathbf{w} = [w_1, \ldots, w_Q]^T$, is defined as [66]

$$\|\mathbf{x}\|_{\infty,\text{vec}}^{\mathbf{w}} = \max_{q=1,\ldots,Q} \frac{|x_q|}{w_q}. \tag{C.1}$$

The *matrix norm* of a matrix $\mathbf{A} \in \mathbb{C}^{Q\times Q}$ induced by $\|\cdot\|_{\infty,\text{vec}}^{\mathbf{w}}$ is defined as [66]

$$\|\mathbf{A}\|_{\infty,\text{mat}}^{\mathbf{w}} = \max_{q=1,\ldots,Q} \frac{1}{w_q} \sum_{r=1}^{Q} [\mathbf{A}]_{q,r} w_r. \tag{C.2}$$

Given $\mathcal{X} = \prod_{q=1}^{Q} \mathcal{X}_q$, where each $\mathcal{X}_q$ is a non-empty subset of $\mathbb{C}^{N_q}$ and where $N_1 + \ldots + N_Q = N$, any vector $\mathbf{x} \in \mathcal{X}$ can be decomposed as $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_Q)$ with $\mathbf{x}_q \in \mathcal{X}_q$. Given a norm $\|\cdot\|_q$ on $\mathbb{C}^{N_q}$ for each q, the *block-maximum* norm induced by a positive vector $\mathbf{w}$ is defined as [67]

$$\|\mathbf{x}\|_{\text{block}}^{\mathbf{w}} = \max_{q=1,\ldots,Q} \frac{\|\mathbf{x}_q\|_q}{w_q}. \tag{C.3}$$

C.2 Contraction Mapping

C.2.1 Contraction Mapping

Many nonlinear problems are typically solved by iterative methods and the convergence is a critical consideration. Several iterative algorithms can be stated as

$$\mathbf{x}(t+1) = \mathbf{T}\big(\mathbf{x}(t)\big), \tag{C.4}$$

where $\mathbf{T}$ is a mapping from a subset $\mathcal{X} \subset \mathbb{C}^Q$ into itself that has the property

$$\left\|\mathbf{T}\big(\mathbf{x}^{(1)}\big) - \mathbf{T}\big(\mathbf{x}^{(2)}\big)\right\| \leq \alpha \left\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\right\|, \quad \forall \mathbf{x}^{(1)}, \mathbf{x}^{(2)} \in \mathcal{X}. \tag{C.5}$$

Here $\|\cdot\|$ is some norm and the constant $\alpha \in [0,1)$. Then, such a mapping $\mathbf{T}(\mathbf{x})$ is a called a *contraction mapping*, or simply a *contraction*, with modulus α and the iteration (C.4) is called a *contracting mapping* [67].

Theorem C.1. *[67, p. 182] Given that* $\mathbf{T} : \mathcal{X} \mapsto \mathcal{X}$ *is a contraction with modulus* $\alpha \in [0,1)$ *and* $\mathcal{X} \subset \mathbb{R}$. *Then*

(i) The mapping $\mathbf{T}$ *has a unique fixed point* $\mathbf{x}^\star \in \mathcal{X}$.

(ii) For every initial vector $\mathbf{x}(0) \in \mathcal{X}$, *the sequence* $\big\{\mathbf{x}(t)\big\}$ *generated by* $\mathbf{x}(t+1) = \mathbf{T}\big(\mathbf{x}(t)\big)$ *converges to* $\mathbf{x}^\star$ *geometrically.*

C.2.2 Contraction Over Cartesian Product Sets

Let $\mathbf{T}_q : \mathcal{X} \mapsto \mathcal{X}_q$ be the qth (block)-component of $\mathbf{T}$, that is

$$\mathbf{T}(\mathbf{x}) = \big(\mathbf{T}_1(\mathbf{x}), \ldots, \mathbf{T}_Q(\mathbf{x})\big). \tag{C.6}$$

Such a mapping will be called a *block-contraction* if

$$\left\|\mathbf{T}\big(\mathbf{x}^{(1)}\big) - \mathbf{T}\big(\mathbf{x}^{(2)}\big)\right\|_{\text{block}} \leq \alpha \left\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\right\|_{\text{block}}, \quad \forall \mathbf{x}^{(1)}, \mathbf{x}^{(2)} \in \mathcal{X}, \tag{C.7}$$

where the modulus $\alpha \in [0, 1)$. Notice that

$$\begin{aligned}\left\|\mathbf{T}_q(\mathbf{x}^{(1)}) - \mathbf{T}_q(\mathbf{x}^{(2)})\right\|_q &\leq \max_{r=1,\ldots,Q} \left\|\mathbf{T}_r(\mathbf{x}^{(1)}) - \mathbf{T}_r(\mathbf{x}^{(2)})\right\|_r \\ &= \left\|\mathbf{T}(\mathbf{x}^{(1)}) - \mathbf{T}(\mathbf{x}^{(2)})\right\|_{\text{block}} \\ &\leq \alpha \left\|\mathbf{x}^{(1)} - \mathbf{x}^{(2)}\right\|_{\text{block}}, \quad \forall \mathbf{x}^{(1)}, \mathbf{x}^{(2)} \in \mathcal{X}, \forall q. \end{aligned} \tag{C.8}$$

Given the mapping $\mathbf{x}_q(t+1) = \mathbf{T}_q\big(\mathbf{x}(t)\big)$, the block-mapping $\mathbf{T}(\mathbf{x})$ can be updated sequentially or simultaneously over the Q blocks. Under the Gauss-Seidel (sequential) implementation, only one block is updated at a time. For example, with the q-th block-component is being updated, the block-mapping is given by

$$\mathbf{T}\big(\mathbf{x}(t+1)\big) = \big(\mathbf{x}_1(t), \ldots, \mathbf{T}_q\big(\mathbf{x}(t)\big), \ldots, \mathbf{x}_Q(t)\big). \tag{C.9}$$

Under the Jacobi (simultaneous) implementation, of all block-components are updated at the same time. Thus,

$$\mathbf{T}\big(\mathbf{x}(t+1)\big) = \big(\mathbf{T}_1\big(\mathbf{x}(t)\big), \ldots, \mathbf{T}_q\big(\mathbf{x}(t)\big), \ldots, \mathbf{T}_Q\big(\mathbf{x}(t)\big)\big). \tag{C.10}$$

Theorem C.2. *[67, p. 186] If $\mathbf{T} : \mathcal{X} \mapsto \mathcal{X}$ satisfies the block-contraction property* (C.7) *and $\mathcal{X}$ is closed, the sequence of vectors from the Gauss-Seidel or the Jacobi update converges to a unique fixed point of $\mathbf{T}$ geometrically.*

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