LETTER

High-Rate Space-Time Block Coding Schemes

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SUMMARY A rate-6/4 full-diversity orthogonal space-time block code (STBC) is constructed for QPSK and 2 transmit antennas by applying constellation scaling and rotation to the set of quaternions used in Alamouti code. Also given is a rate-9/8 full-diversity quasi-orthogonal space-time block code (QOSTBC) for 4 transmit antennas. Lastly, a rate-10/8 code is presented for 4 transmit antennas. Simulation results indicate that these high-rate codes achieve better throughputs in the high signal-to-noise ratio region.

key words: *STBC, QOSTBC, diversity, coding gain, high-rate code*

1. Introduction

Space-time block codes (STBCs) from orthogonal designs proposed by Alamouti [1] and Tarokh, et al. [2] are attractive modulation schemes for wireless systems equipped with multiple transmit antennas. Two key features of Alamouti's *orthogonal* STBC (OSTBC) are their simple symbol-wise maximum likelihood (ML) decoding and full diversity order. However, in order to achieve the full diversity, the maximum rates attained with the orthogonal designs have been shown to be 1 symbol per time slot for systems with 2 transmit antennas and less than 1 for systems equipped with more than 2 transmit antennas.

In order to achieve a higher rate, Jafarkhani proposed in [3] a quasi-orthogonal space-time block code (QOSTBC) for systems with 4 transmit antennas, where the diversity order is relaxed to be 2 to maintain the full transmission rate of 1 symbol per time slot. It was then recognized in [4] and [5] that the QOSTBCs can be made to achieve the full diversity if constellation rotation is applied to the subsets of symbols. In summary, QOSTBC designs possess two important properties of Alamouti's OSTBC, namely full diversity and full rate. However, these properties are obtained at the expense of more complicated ML decoding.

More recently, a new class of full-diversity high-rate (rate > 1) space-time block codes was proposed in [6], where the authors exploit the inherent algebraic structure in the existing orthogonal designs for 2 transmit antennas and quasi-orthogonal designs for 4 transmit antennas. More specifically, the authors showed how to increase the rate of OSTBC designs for 2 transmit antennas and QOSTBC designs for 4 transmit antennas by accommodating 1 additional bit per space-time codeword. This paper extends the framework in [6] to attain even higher-rate full-diversity STBCs, namely, rate-6/4 STBC for 2 transmit antennas and rate-10/8 QOSTBC for 4 transmit antennas, and when QPSK constellation is used. At the high signal-to-noise ratio (SNR) region, the code designs in [6] achieve throughputs of 2.5 bits per time slot for 2 transmit antennas and 2.25 bits per time slot for 4 transmit antennas. Compared to the codes in [6], the codes presented here improve the system throughputs to 3 bits per time slot for 2 transmit antennas and 2.5 bits per time slot for 4 transmit antennas.

Notations: Superscripts $(\cdot)^{\mathcal{T}}$, $(\cdot)^*$, and $(\cdot)^{\mathcal{H}}$ stand for transpose, complex conjugate, and complex conjugate transpose operations, respectively.

2. High-Rate Codes for 2 Transmit Antennas

Start with the full class of complex orthogonal designs for 2 transmit antennas proposed in [7], pick the following 2×2 coding scheme, which is also the well-known Alamouti code [1]

$$\mathcal{Q}(x_1, x_2) \rightarrow \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \quad (1)$$

where x_1 and x_2 are the information symbols belonging to some constellation (such as QPSK or M-QAM). The rows correspond to time slots, while the columns corresponds to the transmit antennas. This design achieves the full rate of 1 symbol per time slot with a full diversity of 2. Moreover, it enjoys a low-complexity symbol-wise ML decoding.

Consider another 2×2 orthogonal design as follows

$$\tilde{\mathcal{Q}}(x_1, x_2) \rightarrow \begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \mathcal{Q}(x_1, x_2). \quad (2)$$

Then the expanded set $\mathcal{G} = \mathcal{Q} \cup \tilde{\mathcal{Q}}$ can be used to construct a new high-rate (> 1) full-diversity space-time block code with low-complexity ML decoding and optimized coding gain as described in [6] and reviewed in the following.

Suppose that $\mathbf{G}_1$ and $\mathbf{G}_2 \in \mathcal{G}$. The sufficient condition for the STBC $\mathcal{G}$ to achieve the full diversity order of 2 is that the coding gain distance matrix $\mathbf{A} = (\mathbf{G}_1 - \mathbf{G}_2)(\mathbf{G}_1 - \mathbf{G}_2)^{\mathcal{H}}$

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is full rank for all $\mathbf{G}_1$, $\mathbf{G}_2 \in \mathcal{G}$ and $\mathbf{G}_1 \neq \mathbf{G}_2$ [2]. If both $\mathbf{G}_1$ and $\mathbf{G}_2$ belong to either $\mathcal{Q}$ or $\tilde{\mathcal{Q}}$, then $\mathbf{A}$ will be full rank. However, if $\mathbf{G}_1 \in \mathcal{Q}$ and $\mathbf{G}_2 \in \tilde{\mathcal{Q}}$, for instance, the matrix $\mathbf{A}$ loses its rank. Moreover, to maximize the coding gain, the determinant of $\mathbf{A}$ needs to be maximized.

A method to maintain the rank of matrix $\mathbf{A}$, proposed in [6], is to simply scale the constellation in one of the sets, $\mathcal{Q}$ or $\tilde{\mathcal{Q}}$. Specifically, if the orthogonal design $\tilde{\mathcal{Q}}$ is expanded by a factor of $k > 1$ compared to the orthogonal design $\mathcal{Q}$, the matrix $\mathbf{A}$ is guaranteed to be full rank. It was stated in [6] that $k = \sqrt{3}$ is the optimal power scaling factor for the case of 2 transmit antennas, in the sense that the coding gain is maximized and the peak-to-average ratio, a side effect of constellation scaling, is minimized.

A different way to maintain the rank of matrix $\mathbf{A}$ is to apply constellation rotation to one of the STBC designs, $\mathcal{Q}$ or $\tilde{\mathcal{Q}}$. For example, if $\mathcal{Q}$ is kept unchanged, $\tilde{\mathcal{Q}}$ is defined as

$$\tilde{\mathcal{Q}}(x_1 e^{j\theta}, x_2) \rightarrow \begin{bmatrix} x_1 e^{j\theta} & x_2 \\ x_2^* & -x_1^* e^{-j\theta} \end{bmatrix}. \tag{3}$$

It can be shown that the determinant of $\mathbf{A} = (\mathbf{G}_1 - \mathbf{G}_2)(\mathbf{G}_1 - \mathbf{G}_2)^{\mathcal{H}} > 0$, for all $\mathbf{G}_1$, $\mathbf{G}_2 \in \mathcal{G}$ and $\mathbf{G}_1 \neq \mathbf{G}_2$. In fact, the minimum determinant of $\mathbf{A}$ is maximized with $\theta = \pi/4$ [6].

2.1 Review of Rate-5/4 STBC Designs

Consider QPSK modulation. In the conventional Alamouti's scheme, 4 bits select 2 independent QPSK symbols, x_1 and x_2. These symbols are then space-time encoded as in (1) or (2). The space-time encoded symbols are sent via 2 transmit antennas and over 2 time slots. As suggested in [6], one additional bit can be accommodated by simply using it to select one of the encoding schemes in (1) and (2). To illustrate this, suppose that b_0 is the first bit coming to the space-time encoder. The QPSK symbols x_1 and x_2 that carry other 4 bits are encoded as follows

$$b_0 = 0 \Rightarrow \sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{4}$$

$$b_0 = 1 \Rightarrow k\sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix}. \tag{5}$$

As mentioned before, it is stated in [6] that $k = \sqrt{3}$ is the optimal scaling factor. The 2 constellation sets are scaled accordingly to maintain the same average transmitted power.

Similarly, one can design a rate-5/4 STBC by applying constellation rotation as follows

$$b_0 = 0 \Rightarrow \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{6}$$

$$b_0 = 1 \Rightarrow \begin{bmatrix} x_1 e^{j\pi/4} & x_2 \\ x_2^* & -x_1^* e^{-j\pi/4} \end{bmatrix}. \tag{7}$$

Of the two methods to design rate-5/4 STBCs described above, it was shown in [6] that the method based on constellation scaling outperforms the one based on constellation rotation. This is also illustrated later in Sect. 4.

2.2 Rate-6/4 STBC Design

This section presents a simple way to accommodate 2 additional information bits to increase the transmission rate to 6/4. This is accomplished by combining the 2 design methods that achieve rate-5/4 described above. To this end, the first bit b_0 is used to choose the constellation scaling factor, the second bit b_1 is used to perform constellation rotation. It is simple to show that this design still guarantees the full-diversity of the code. The design is as follows

$$b_0 b_1 = 00 \Rightarrow \sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{8}$$

$$b_0 b_1 = 01 \Rightarrow \sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 e^{j\pi/4} & x_2 \\ -x_2^* & x_1^* e^{-j\pi/4} \end{bmatrix} \tag{9}$$

$$b_0 b_1 = 10 \Rightarrow k\sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{bmatrix} \tag{10}$$

$$b_0 b_1 = 11 \Rightarrow k\sqrt{\frac{2}{k^2+1}} \begin{bmatrix} x_1 e^{j\pi/4} & x_2 \\ x_2^* & -x_1^* e^{-j\pi/4} \end{bmatrix}. \tag{11}$$

where $k = \sqrt{3}$ is used as the scaling factor.

2.3 Decoding

This section describes the decoding process at the receiver. For simplicity but without loss of generality, assume that there is only one receive antenna. Let $\mathbf{S}_i$ be the transmitted codeword matrix. The received signal can be expressed as

$$\mathbf{R} = \mathbf{S}_i \mathbf{H} + \mathbf{W} \tag{12}$$

where $\mathbf{R} = [r_1, r_2]^{\mathcal{T}}$ is the received vector, $\mathbf{H} = [h_1, h_2]^{\mathcal{T}}$ is the channel gain vector, and $\mathbf{W} = [w_1, w_2]^{\mathcal{T}}$ is the noise vector. Since a quasi-static Rayleigh fading channel is considered, h_1 and h_2 are identically independent (i.i.d.) circularly symmetric Gaussian random variables of unit variance. The noise variables w_1 and w_2 are also i.i.d. circularly symmetric Gaussian random variables with variance $N_0/2$ per dimension.

Coherent detection with full channel state information is assumed at the receiver. Simple maximal-ratio combining (MRC) and ML decoding [1] are performed to generate different candidate solutions, each corresponds to one orthogonal design. Then these candidate solutions are compared to yield the final decision. For the rate-5/4 code, let $\mathbf{S}_j$ and $\mathbf{S}_k$ be the two candidate solutions, these two candidate solutions are then compared using the metrics $||\mathbf{R} - \mathbf{S}_j\mathbf{H}||^2$ and $||\mathbf{R} - \mathbf{S}_k\mathbf{H}||^2$, which favors the one that has a lower metric. The uncoded bit b_0 is determined based on the decision between $\mathbf{S}_j$ and $\mathbf{S}_k$, while the other 4 bits follow from the winning solution. The decoding process for rate-6/4 code works exactly the same. However, there would be 4 candidate solutions to consider. The decision on these candidates would determine the 2 additional uncoded bits. Obviously, the drawback of accommodating more uncoded bits in the system is the induction of higher decoding complexity at the receiver.

3. High-Rate Codes for 4 Transmit Antennas

3.1 Review of Rate-9/8 Code

Similar to the method used to achieve rate-5/4 full-diversity STBC, we can utilize the various designs of rate-1 full-diversity QOSTBC to create a new rate-9/8 code.

Consider the following example of a rate-1 full-diversity complex quasi-orthogonal STBC with rotation [5]

$$\begin{aligned} &C_1(x_1, x_2, x_3, x_4) \\ &\equiv \begin{bmatrix} Q(x_1, x_2) & Q(x_3 e^{j\theta}, x_4 e^{j\theta}) \\ -Q^*(x_3 e^{j\theta}, x_4 e^{j\theta}) & Q^*(x_1, x_2) \end{bmatrix} \\ &= \begin{bmatrix} x_1 & x_2 & x_3 e^{j\theta} & x_4 e^{j\theta} \\ -x_2^* & x_1^* & -x_4^* e^{-j\theta} & x_3^* e^{-j\theta} \\ -x_3^* e^{-j\theta} & -x_4^* e^{-j\theta} & x_1^* & x_2^* \\ x_4^* e^{j\theta} & -x_3^* e^{j\theta} & -x_2^* & x_1^* \end{bmatrix} \end{aligned} \quad (13)$$

where x_1, x_2, x_3, x_4 are QPSK symbols. According to [5], $\theta = \pi/4$ is the optimal rotation angle for QPSK constellation to achieve the maximum diversity order and coding gain.

Some of other QOSTBC designs are as follows

$$\begin{aligned} &C_2(x_1, x_2, x_3, x_4) \\ &\equiv \begin{bmatrix} \tilde{Q}(x_1, x_2) & \tilde{Q}(x_3 e^{j\theta}, x_4 e^{j\theta}) \\ \tilde{Q}^*(x_3 e^{j\theta}, x_4 e^{j\theta}) & -\tilde{Q}^*(x_1, x_2) \end{bmatrix} \end{aligned} \quad (14)$$

$$\begin{aligned} &C_3(x_1, x_2, x_3, x_4) \\ &\equiv \begin{bmatrix} Q(x_1, x_2) & Q(x_3 e^{j\theta}, x_4 e^{j\theta}) \\ Q^*(x_3 e^{j\theta}, x_4 e^{j\theta}) & -Q^*(x_1, x_2) \end{bmatrix} \end{aligned} \quad (15)$$

$$\begin{aligned} &C_4(x_1, x_2, x_3, x_4) \\ &\equiv \begin{bmatrix} \tilde{Q}(x_1, x_2) & \tilde{Q}(x_3 e^{j\theta}, x_4 e^{j\theta}) \\ -\tilde{Q}^*(x_3 e^{j\theta}, x_4 e^{j\theta}) & \tilde{Q}^*(x_1, x_2) \end{bmatrix}. \end{aligned} \quad (16)$$

As suggested in [6], a rate-9/8 code can be designed as follows. From each stream of nine information bits at the encoder, the first information bit selects between 2 QOSTBC designs (C_1 and C_2 for example), the next eight bits are then QPSK modulated and space-time encoded based on the corresponding QOSTBC design. Similar to the case of the rate-5/4 code for 2 transmit antennas, in order to maintain the full-diversity property, the constellation sets of the 2 QOSTBC designs have to be scaled relatively to each other. In [6], $k = \sqrt{2}$ is used as the scaling factor for rate-9/8 code.

3.2 Rate-10/8 Code

One can apply the same technique used in obtaining the rate-6/4 code to design a new rate-10/8 code. By utilizing all 4 QOSTBC designs listed in (13)–(16), 2 additional bits can be accommodated per space-time codeword (i.e., 4 time slots). While the first bit could be used to select the power level of the constellation sets, the second bit could be used to determine one of the 2 QOSTBC designs on the same power level. Note that the coding gain distance matrix between 2 distinguished QOSTBC mappings at the same power level may not be full rank. This means that the full diversity order of 4 will not be guaranteed for the proposed rate-10/8 code. However, our simulation indicates a better throughput is achieved with the rate-10/8 code compared to the rate-9/8 code at high SNR.

For both rate-9/8 and rate-10/8 codes, the decoding process is similar to the process applied for the case of 2 transmit antennas. All candidate solutions, $\mathbf{S}_i$, are generated, then the metrics $\|\mathbf{R} - \mathbf{S}_i\mathbf{H}\|^2$ are compared, where $\mathbf{R}$ and $\mathbf{H}$ are the receive and channel gain vectors, respectively. The smallest metric is used to determine the best solution, then the decision on additional uncoded bit(s) is made accordingly.

4. Results

Consider a system with QPSK modulation and 1 receive antenna. The receiver performs coherent ML decoding with full channel information at the receiver. Figure 1 compares the effective throughput of the Alamouti scheme, the rate-5/4 code in [6] and the proposed rate-6/4 code in terms of the average SNR at the receiver. More precisely, SNR $= E_s/N_0$, where E_s is the average symbol energy of QPSK constellation and N_0 is one-sided power spectral density of white Gaussian noise. Similar to [6], the effective throughput η is defined as $\eta = (1 - \text{FER}) \cdot R \cdot \log_2(M)$, where R is the code rate, M is the constellation size (which is 4 for QPSK modulation), and FER is the frame error rate. Each frame is set to contain 20 codewords. Observe that, while the Alamouti scheme can only achieve the throughput of 2 bits per time slot, the rate-5/4 code can increase the throughput to 2.5 bits per time slot. The proposed code even increases the throughput to 3 bits per time slot at high SNR.

As can be seen from Fig. 1, at high SNR the high-rate codes start to pick up the higher throughput compared to the Alamouti scheme. Starting at the SNR of about 16 dB, the throughput of the rate-5/4 code in [6] surpasses the through-

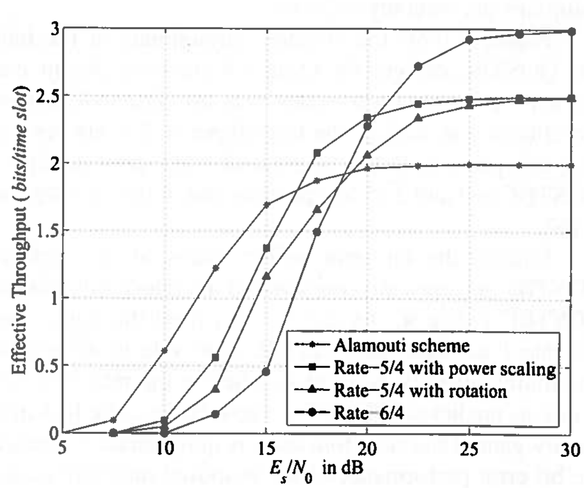

Fig. 1 Effective throughputs of high-rate full-diversity STBC designs for 2 transmit antennas.

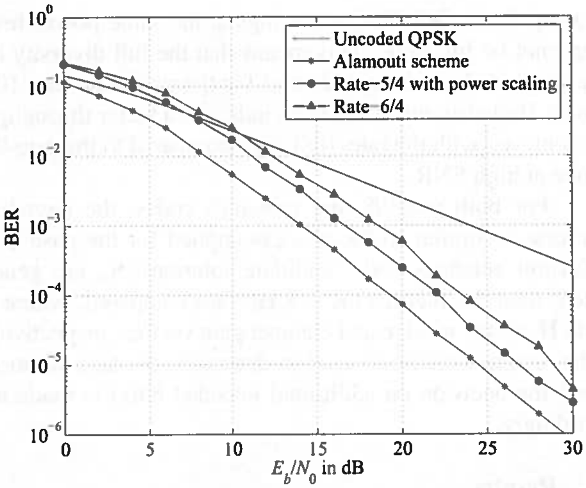

Fig. 2 Bit error rates of high-rate STBC designs for 2 transmit antennas.

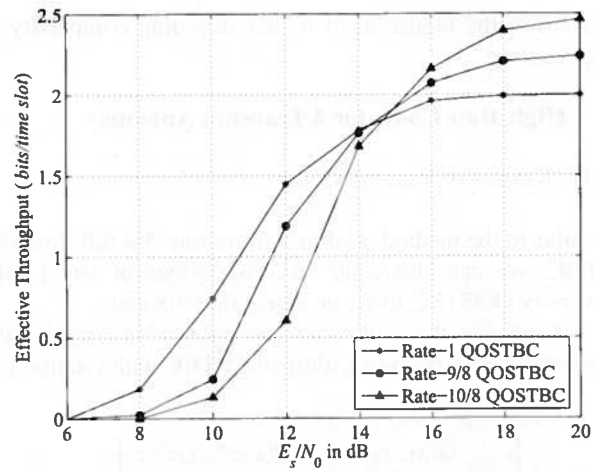

Fig. 3 Effective throughputs of high-rate QOSTBC designs for 4 transmit antennas.

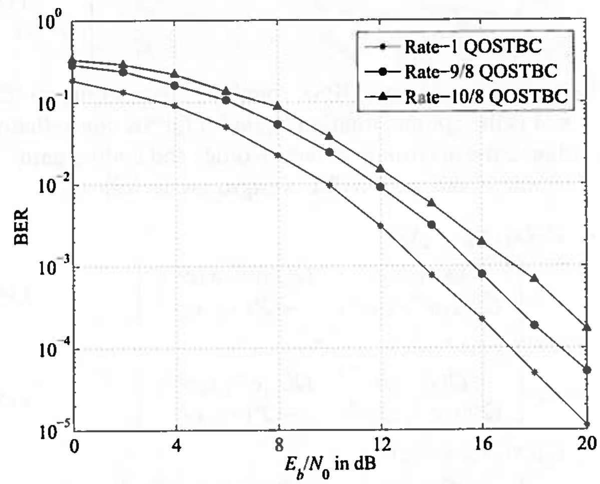

Fig. 4 Bit error rates of high-rate QOSTBC designs for 4 transmit antennas.

put of the Alamouti scheme. On the other hand, at SNR of 20 dB, the proposed rate-6/4 code achieves a higher throughput than the rate-5/4 code. Assuming that SNR is known at the transmitter (through a feedback channel, for example), one can then design a system with an adaptive encoding scheme to give the maximum achievable throughput of the system. At low SNR, the Alamouti scheme should be used. Then, at high SNR, the system can switch to rate-5/4 code or even rate-6/4 code. A strong error-detecting outer code, such as a cyclic code, could be used in conjunction to detect erroneous frames.

In order to make a fair comparison in terms of bit error performance, the plots in Fig. 2 are based on the signal-to-noise ratio per information bit at the receiver. As illustrated in the figure, a high-rate STBC design would trade a higher rate to a poorer bit-error-rate (BER) compared to the Alamouti scheme. While at the same BER, the rate-5/4 code is 2 dB worse than the Alamouti scheme, the rate-6/4 code is only 3 dB away from the Alamouti scheme. Figure 2, however, clearly indicates that the proposed rate-6/4 code still maintains the diversity order of 2.

Figure 3 plots the effective throughputs of the high-rate QOSTBC designs for 4 transmit antennas. Again, each frame is set to contain 20 codewords. At high SNR, the proposed code can achieve the throughput of 2.5 bits per time slot, compared to the throughput of 2 bits per time slot of QOSTBC and and 2.25 bits per time slot of the rate-9/8 code in [6].

Finally, the bit error performances of the high-rate QOSTBC schemes are compared to the rate-1 full diversity QOSTBC in Fig. 4. As can be seen from the figure, both the rate-1 and rate-9/8 QOSTBCs are able to achieve the maximum diversity order of 4, whereas the rate-10/8 code is not, as predicted in Sect. 3.2. Nevertheless, the loss in diversity gain of the rate-10/8 code is quite small. Moreover, the bit error performance of the proposed rate-10/8 code is only 2 dB away from the rate-9/8 one in [6] at the BER of 10^{-4}. This explains why the throughput of the rate-10/8 code starts surpassing the throughput of the rate-9/8 code at a fairly small SNR (about 15 dB) as observed in Fig. 3.

5. Conclusion

In this paper, a simple method was investigated to design high-rate space-time block codes with QPSK constellation. The proposed codes accommodate 2 additional bits per space-time codeword by combining both constellation scaling and rotation in the case of rate-6/4 code, and by using 4 different QOSTBC designs in the case of rate-10/8 code. At high SNR, it was shown that these codes are able to yield better throughputs than the codes in [6] and the conventional Alamouti scheme for 2 transmit antennas, and the QOSTBC for 4 transmit antennas. It is noted that a better throughput from the proposed designs requires a slightly higher decoding complexity.

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