Block-Diagonalization Precoding in a Multiuser Multicell MIMO System: Competition and Coordination

Duy H. N. Nguyen, *Student Member, IEEE*, Hung Nguyen-Le, and Tho Le-Ngoc, *Fellow, IEEE*

***Abstract*—This paper studies a multiuser multicell system where block-diagonalization (BD) precoding is utilized on a per-cell basis. We examine and compare the multicell system under two operating modes: competition and coordination. In the *competition* mode, the paper considers a strategic non-cooperative game (SNG), where each base-station (BS) greedily determines its BD precoding strategy in a distributed manner, based on the knowledge of the inter-cell interference at its connected mobile-stations (MS). Via the game-theory framework, the existence and uniqueness of a Nash equilibrium in this SNG are subsequently studied. In the *coordination* mode, the BD precoders are jointly designed across the multiple BSs to maximize the network weighted sum-rate (WSR). Since this WSR maximization problem is nonconvex, we consider a distributed algorithm to obtain at least a locally optimal solution. Finally, we extend our analysis of the multicell BD precoding to the case of BD-Dirty Paper Coding (BD-DPC) precoding. We characterize BD-DPC precoding game for the multicell system in the competition mode and propose an algorithm to jointly optimize BD-DPC precoders for the multicell system in the coordination mode. Simulation results show significant network sum-rate improvements by jointly designing the BD or BD-DPC precoders across the multicell system in the coordination mode over the competition mode.**

***Index Terms*—Multicell transmissions, multiuser, block-diagonalization precoding, non-cooperative game, Nash equilibrium, optimization, CoMP, interference coordination.**

I. INTRODUCTION

IN a multiple-input multiple-output (MIMO) system, space division multiple access (SDMA) can be applied at the base-station (BS) to concurrently multiplex data streams for multiple mobile-stations (MS). With appropriate downlink precoding techniques at the BS, SDMA can significantly improve the system spectral efficiency. The research on downlink precoding for a multiple-input multiple-output (MIMO) system has been an active area for many years. Dirty paper coding (DPC) [1]–[4] has been proved to be the capacity-achieving multi-user precoding strategy. However, due to its high complexity implementation that involves random nonlinear encoding and decoding, DPC only remains as a theoretical benchmark. Consequently, linear precoding techniques, such as zero-forcing (ZF), block-diagonalization (BD) [5]–[8], become appealing alternatives due to their simplicity and good performance. With BD precoding, the transmitted signal from the BS intended for a particular MS is restricted to be in the null space created by the downlink channels associated with all the other MSs. Therefore, all inter-user interference within the cell at the MSs can be fully suppressed.

In a multicell system, current designs of wireless networks adopt universal frequency reuse where all cells have the potential to use all available radio resources. Conventionally, most of the related works in downlink precoding designs focus on the single-cell setting where the inter-cell interference (ICI), i.e., co-channel interference, is simply treated as additional background noise at the MSs. Thus, the effect of ICI is rather neglected in the precoding design process. However, with universal frequency-reuse, the level of ICI is much more pronounced and should not be ignored. Consequently, existing research in single-cell precoding designs may need a rework to take into account the adverse effect of ICI when applying to a multicell system. Recently, precoding designs for coordinated multipoint transmission/reception (CoMP) in a multicell system have attracted a lot of research attention [9]–[13]. In these studies, the multiple BSs are fully cooperated to form a single large system with distributed antenna elements. This form of cooperation, also termed as *network MIMO*, requires all the BSs to share the channel state information (CSI) and data information among themselves via backhaul links as well as to coordinate their concurrent data information transmissions to all the MSs. Under the network MIMO context, BD precoding was investigated in [11], [12] with per-base-station power constraints. Specifically, the works in [11], [12] considered a multicell system where all the BSs form a single large BD precoder to remove all intra-cell and inter-cell interferences. A modified BD multicell precoding scheme was proposed in [13] to improve the performance of BD precoding in the low-to-medium signal-to-noise (SNR) region. While extracting a good performance from the multicell network,

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D. H. N. Nguyen and T. Le-Ngoc are with the Department of Electrical and Computer Engineering, McGill University, 3480 University Street, Montreal, QC, Canada, H3A 0E9 (e-mail: huu.n.nguyen@mail.mcgill.ca, tho.le-ngoc@mcgill.ca).

H. Nguyen-Le is with both the Department of Science, Technology and Environment, the University of Danang, 41 Le Duan, Danang, Vietnam, and the Department of Electronics and Telecommunications Engineering, the University of Danang, University of Science and Technology, 54 Nguyen Luong Bang, Danang, Vietnam (e-mail: nlhung@dut.udn.vn).

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the benefits of network MIMO come at the expense of high complexity in joint precoding/decoding and ideal backhaul transmissions among the BSs for data and control signaling exchange [14].

Different from the studies in [11]–[13], this work investigates the multicell system where BD precoding is applied on a per-cell basis. Specifically, we consider the BD precoding schemes for the multicell system under the two operating modes: interference aware (IA) and interference coordination (IC) [15]. Under these two operating modes, each BS is required to transmit information data *only* to the MSs within its cell limits. The reasons for selecting BD precoding on a per-cell basis in our work are three-fold. First, using this approach, the multicell system is relieved of the requirement for data exchange in the backhaul links among the BSs. Second, due to the relatively close distance between co-located MSs, the intra-cell interference deems to be the more dominant source of interference than the ICI. With BD precoding being enforced at each cell, the intra-cell interference can be fully suppressed. Finally, BD precoding is much simpler than DPC in implementation and capable of achieving measurably close to DPC performance in many scenarios [16].

Under the IA mode, each interference-aware MS shall measure the level of ICI and feed back this information to its connected BS [15]. Given the strategies from other BSs reflected by the ICI, each BS selfishly adjusts its precoding strategy to maximize the sum-rate for its connected MSs. Thus, the multicell system is said to be in *competition* since the BSs are competing with each other for the radio resource.[1] Naturally, the IA mode represents a strategic noncooperative game (SNG) with the BSs being the rational players. The study of precoding design for the multicell system under the IA mode using game theory is plentiful in literature [17]–[26]. In general, these works focus on studying the existence and uniqueness of the game's Nash equilibrium (NE). For a single-input single-output (SISO) multi-carrier system, the work in [17] has led to various works on the iterative water-filling game, such as [18]–[21]. These cited works examined the SNG where each BS competes against the others by adapting its power allocation strategy over the multiple carriers to maximize its data rate. The work in [22] studied the precoding game of a two-cell multiple-input single-output (MISO) system. In the multicell MIMO system, [23], [24] studied the competitive precoding design, where each cell selfishly maximizes its mutual information. It is noted that [22]–[24] only considered the system where each BS communicates with only one MS. For the case of multiple MSs per cell, [25] studied a multicell SNG where each BS selfishly minimizes its transmit power. The work in [26] investigated a SNG where each BS utilizes ZF precoding to maximize the sum-rate to the MSs within its cell. Both studies in [25], [26] only considered the case of single-antenna MSs. Different from these cited works, the consideration of BD precoding in this paper allows us to examine a multicell SNG under a more general setting where there are multiple MSs per cell and each MS is equipped with multiple antennas. In order to characterize the multicell BD precoding game, we first present the best response strategy at the each BS in a closed-form water-filling (WF) solution. We then show that this WF best response strategy can be interpreted as a projection onto a closed and convex set. This interpretation shall allow us to study the uniqueness of the game's NE later on. It shall be shown that the game's NE always exists and is guaranteed to be unique under a certain condition on the ICI.

Under the IC mode, while each BS only transmits data information to the MSs within its cell limits, the precoders from all BSs are jointly designed to fully control the ICI [15]. Thus, the multicell system is said to be in *coordination* since the transmissions from the BSs are coordinated.[2] In this work, we examine the BD precoding strategy that jointly maximizes the weighted sum-rate (WSR) of the multicell system under the IC mode. We then show that this joint WSR maximization problem is a nonconvex problem, which is generally difficult and computationally complex to find its globally optimal solution. Thus, our focus is on proposing a numerical algorithm to approximate the nonconvex WSR maximization into a sequence of simpler convex problems. We then show that each approximated problem can be solved separately at the corresponding BS. In particular, each BS attempts to optimize its BD precoder to maximize the sum-rate for its connected MSs while doing its best in limiting the ICI induced to other cells through an interference-penalty mechanism.

In the later part of this paper, we extend our analysis of the multicell BD precoding to the system where Block-Diagonalization - Dirty Paper Coding (BD-DPC) is utilized in a per-cell basis. In BD-DPC, the information signals sent to the multiple users are encoded in sequence such that the receiver at any user does not see any intra-cell interference due to the use of BD and DPC at the BS [2]. Thus, BD-DPC can take advantage of DPC to enhance the performance of BD precoding. Under the IA mode, we attempt to characterize the NE of the BD-DPC multicell precoding game by examining the conditions for its existence and uniqueness. It shall be shown that the game may have multiple NEs, depending on the encoding order in the BD-DPC precoding design at each BS. In addition, the condition for the uniqueness of the BD-DPC multicell precoding game is generally stricter than that of the BD one. Under the IC mode, we propose a numerical algorithm to maximize the WSR of the multicell system with BD-DPC precoding. In a conventional single-cell system, BD-DPC precoding can yield a better sum-rate performance over the BD precoding. Numerical simulations then confirm this observation for the multicell system under both IA and IC modes. In comparing the IA mode with the IC mode, simulation results show a significant improvement in the network sum-rate by jointly coordinating the BD or BD-DPC precoders, especially at the high ICI region.

Notations: $\mathbf{X}^H$ and $\mathbf{X}^\sharp$ denote the conjugate transpose (Hermitian operator) and the Moore-Penrose pseudo-inverse of the matrix $\mathbf{X}$, respectively; $[\mathbf{X}]_{m,n}$ stands for the (m,n)th entry of the matrix $\mathbf{X}$; $[\mathbf{X}]^+$ denotes the component-wise

[1]Hereafter, the "competition" mode or the "interference aware" mode is referred interchangeably.

[2]Hereafter, the "coordination" mode or the "interference coordination" mode is referred interchangeably.

operation $\max\{[\mathbf{X}]_{m,n}, 0\}$; $\mathbf{X} \succeq \mathbf{0}$ means that $\mathbf{X}$ is a positive semi-definite matrix; $\mathrm{Tr}\{\mathbf{X}\}$, $|\mathbf{X}|$ and $\|\mathbf{X}\|_F$ denote the trace, determinant, and Frobenius norm of the matrix $\mathbf{X}$, respectively; $\rho(\mathbf{X})$, denoting the spectral radius of the matrix $\mathbf{X}$, is defined as $\rho(\mathbf{X}) \triangleq \max\{|\lambda_i|\}$, where λ_i's are eigenvalues of $\mathbf{X}$; $\mathrm{blk}\{\mathbf{X}_1, \ldots, \mathbf{X}_K\}$ denotes a square block-diagonal matrix with the main diagonal blocks as square matrices $\mathbf{X}_1, \ldots, \mathbf{X}_K$; $\mathcal{N}(\mathbf{X})$ denotes the null space of matrix $\mathbf{X}$ whereas $\mathbf{P}_{\mathcal{N}(\mathbf{X})}$ denotes the orthogonal projection onto the null space of matrix $\mathbf{X}$; $[\mathbf{X}]_{\mathcal{S}}$ denotes the matrix projection of matrix $\mathbf{X}$ onto the (closed and convex) set $\mathcal{S}$ with respect to the Frobenius norm.

II. System Model

We consider a multiuser multicell downlink system with Q separate cells operating on the same frequency channel. At a particular cell, say cell-q, a multiple-antenna BS is concurrently sending independent information streams to K_q remote MSs, each equipped with multiple receive antennas. Let M_q and N_{q_i} be the numbers of antennas of the BS and the ith MS at cell-q, respectively. Denote $\mathbf{x}_q \in \mathbb{C}^{M_q \times 1}$ as the transmitted signal vector from BS-q. Assuming linear precoding at the BS, $\mathbf{x}_q$ can be represented as $\mathbf{x}_q = \sum_{i=1}^{K_q} \mathbf{W}_{q_i}\mathbf{s}_{q_i}$, where $\mathbf{W}_{q_i} \in \mathbb{C}^{M_q \times L_{q_i}}$ is the precoding matrix and $\mathbf{s}_{q_i} \in \mathbb{C}^{L_{q_i} \times 1}$ is the data symbol vector intended for MS-i with L_{q_i} being the number of transmitted symbols . Without loss of generality, we assume $\mathbb{E}\left[\mathbf{s}_{q_i}\mathbf{s}_{q_i}^H\right] = \mathbf{I}, \forall i, \forall q$.

Let $\mathbf{H}_{rq_i} \in \mathbb{C}^{N_{q_i} \times M_r}$ model the channel coefficients from BS-r to MS-i of cell-q, and $\mathbf{z}_{q_i}$ model the zero-mean complex additive Gaussian noise vector with an arbitrary covariance matrix $\mathbf{Z}_{q_i}$. The transmission to MS-i at cell-q can be modeled as

$$\begin{aligned}\mathbf{y}_{q_i} &= \sum_{r=1}^{Q} \mathbf{H}_{rq_i}\mathbf{x}_r + \mathbf{z}_{q_i} \\ &= \mathbf{H}_{qq_i}\mathbf{W}_{q_i}\mathbf{s}_{q_i} + \mathbf{H}_{qq_i}\sum_{j \neq i}^{K_q}\mathbf{W}_{q_j}\mathbf{s}_{q_j} \\ &\quad + \sum_{r \neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K_r}\mathbf{W}_{r_j}\mathbf{s}_{r_j} + \mathbf{z}_{q_i}. \qquad (1)\end{aligned}$$

It is observed from (1) that the received signal at MS-i of cell-q comprises of 4 components: the useful information signal $\mathbf{H}_{qq_i}\mathbf{W}_{q_i}\mathbf{s}_{q_i}$, the intra-cell interference $\mathbf{H}_{qq_i}\sum_{j\neq i}^{K_q}\mathbf{W}_{q_j}\mathbf{s}_{q_j}$, the inter-cell interference $\sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K_r}\mathbf{W}_{r_j}\mathbf{s}_{r_j}$, and the Gaussian noise $\mathbf{z}_{q_i}$. In this work, it is assumed that each MS can measure its total interference and noise (IPN) power perfectly and constantly report back to its connected BS. The same assumption is made for the estimation and feed back of the downlink CSI from each MS. The BS then utilizes the CSI and IPN information to accordingly design its precoders for its connected MSs. If the BS can only acquire imperfect knowledge of the CSI or IPN at its connected MSs, the precoders have to be redesigned with robustness to this imperfection. It is to be noted that the consideration of robust precoding design is beyond the scope of this work.

In the competitive design of this system model, it is assumed that each BS only possesses full knowledge of the downlink channels to the MSs in its cell, but not the channels to the MSs in other cells. As a result, the BS cannot control its induced ICI to other cells, which is then treated as background noise at the MSs. On the contrary, in the coordinated design of this system model, the BS also possesses the CSI to the MSs in the other cells. This additional channel knowledge allows the BS to control the ICI as well. Note that the BS can always fully manage the intra-cell interference within its cell by performing certain precoding techniques on a per-cell basis. In this work, we focus on the precoding technique that completely suppresses the intra-cell interference, namely BD precoding for multiple-antenna MSs [6]. To implement the BD precoding on a per-cell basis, it is assumed that the total number of receive antennas at the MSs does not exceed the number of transmit antennas at their connected BS, i.e., $\sum_{i=1}^{K_q} N_{q_i} \leq M_q, \forall q$. If the number of receive antennas at a cell exceeds the number of transmit antennas, the BS can select a subset of MSs beforehand using low-complexity selection techniques such as [27], [28].

Let $\mathbf{Q}_{q_i} = \mathbf{W}_{q_i}\mathbf{W}_{q_i}^H$ be the transmit covariance matrix intended for MS-i of cell-q, and $\mathbf{Q}_q = \{\mathbf{Q}_{q_i}\}_{i=1}^{K_q}$ be the precoding profile for K_q MSs of cell-q. Likewise, let $\mathbf{Q}_{-q} = \{\mathbf{Q}_1, \ldots, \mathbf{Q}_{q-1}, \mathbf{Q}_{q+1}, \ldots, \mathbf{Q}_Q\}$ denote the precoding profile of all cells except cell-q. Denote $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ as the covariance matrix of the IPN (with no intra-cell interference) at the MS-i of cell-q, which is defined as

$$\mathbf{R}_{q_i}(\mathbf{Q}_{-q}) = \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\left(\sum_{j=1}^{K_r}\mathbf{Q}_{r_j}\right)\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i}. \qquad (2)$$

With BD precoding applied an a per-cell basis at BS-q, the achievable data rate R_{q_i} to MS-i is then given by [6]

$$R_{q_i}(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\left|\mathbf{I} + \mathbf{H}_{qq_i}^H\mathbf{R}_{q_i}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq_i}\mathbf{Q}_{q_i}\right|. \qquad (3)$$

III. The Multicell Block-Diagonalization Precoding - Competitive Design

A. Problem Formulation

This section examines the multicell BD precoding under the competition mode, i.e., the IA mode, where each BS selfishly designs its BD precoders without any coordination between the cells. We are interested in formulating this competitive multicell BD precoding design using the game-theory framework. In particular, we consider a SNG, where the players are the BSs and the payoff functions are the sum-rates of the cells. At each cell, the BS strategically adapts its BD precoder on a per-cell basis that greedily maximizes the sum-rate to its connected MSs, subject to a constraint on its transmit power. In this work, it is assumed that all the channels change sufficiently slow such that they are considered fixed during the game being played.

Let $\Omega = \{1, \ldots, Q\}$ be the set of Q players. Define $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{i=1}^{K_q} R_{q_i}(\mathbf{Q}_q, \mathbf{Q}_{-q})$ as the payoff function of player-q. Then, given a strategy profile $\mathbf{Q}_{-q}$ from other players, player-q selfishly maximizes its payoff function by solving the following optimization problem

$$\begin{aligned}\underset{\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_{K_q}}}{\text{maximize}} \quad & R_q(\mathbf{Q}_q,\mathbf{Q}_{-q}) \\ \text{subject to} \quad & \mathbf{H}_{qq_j}\mathbf{Q}_{q_i}\mathbf{H}_{qq_j}^H=\mathbf{0},\forall j\neq i \\ & \mathbf{Q}_{q_i}\succeq\mathbf{0},\ \forall i \\ & \sum_{i=1}^{K_q}\mathrm{Tr}\{\mathbf{Q}_{q_i}\}\leq P_q,\end{aligned} \tag{4}$$

where P_q is the power budget at BS-q. To achieve the maximum sum data-rate at cell-q, it is assumed that the IPN matrix $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ is perfectly measured at the corresponding MS-i and reported back to its connected BS. Note that the IPN matrix $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ at MS-i captures the precoding strategy $\mathbf{Q}_{-q}$, as indicated in Equation (2). Thus, whenever a BS other than BS-q changes its strategy, the IPN matrix $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ is also changed. In this case, MS-i needs to feed back the updated $\mathbf{R}_{q_i}(\mathbf{Q}_{-q})$ to its corresponding BS-q. Clearly, the optimization problem (4) shows the dependence of the optimal strategy at BS-q on the strategies at other BSs. It is noted that the optimization (4) is carried with only local information (intra-cell CSI and IPN matrices between the MSs and its connected BS). Thus, the BD precoding game is implemented in a fully distributed manner without any signaling exchanges among the BSs.

Due to the constraints $\mathbf{H}_{qq_i}\mathbf{Q}_{q_j}\mathbf{H}_{qq_i}^H=\mathbf{0},\forall j\neq i$, each column of the precoder matrix $\mathbf{W}_{q_i}$ must be in the null space created by $\hat{\mathbf{H}}_{q_i}=[\mathbf{H}_{qq_1}^T,\ldots,\mathbf{H}_{qq_{i-1}}^T,\mathbf{H}_{qq_{i+1}}^T,\ldots,\mathbf{H}_{qq_{K_q}}^T]^T$. Suppose that one performs the singular value decomposition of the $(\sum_{j\neq i}^{K_q}N_{q_j})\times M_q$ matrix $\hat{\mathbf{H}}_{q_i}$ as

$$\hat{\mathbf{H}}_{q_i}=\mathbf{U}_{q_i}\mathbf{\Sigma}_{q_i}\mathbf{V}_{q_i}^H=\mathbf{U}_{q_i}\left[\tilde{\mathbf{\Sigma}}_{q_i},\ \mathbf{0}\right]\begin{bmatrix}\tilde{\mathbf{V}}_{q_i}^H\\ \hat{\mathbf{V}}_{q_i}^H\end{bmatrix}, \tag{5}$$

where $\tilde{\mathbf{\Sigma}}_{q_i}$ is diagonal, $\mathbf{U}_{q_i}$ and $\mathbf{V}_{q_i}$ are unitary matrices, and $\hat{\mathbf{V}}_{q_i}$ is formed by the last $\hat{N}_{q_i}\triangleq M_q-\sum_{j\neq i}^{K_q}N_{q_j}$ columns of $\mathbf{V}_{q_i}$. Then, any precoding covariance matrix $\mathbf{Q}_{q_i}$ formulated as $\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H$, where $\mathbf{D}_{q_i}\succeq\mathbf{0}$ is an arbitrary $\hat{N}_{q_i}\times\hat{N}_{q_i}$ matrix, would make $\mathbf{H}_{qq_j}\mathbf{Q}_{q_i}\mathbf{H}_{qq_j}^H=\mathbf{0},\forall j\neq i$. Thus, the set of admissible strategies for player-q can be defined as follows:

$$\mathcal{S}_q=\left\{\mathbf{Q}_{q_i}\in\mathbb{S}^{M_q\times M_q}:\mathbf{Q}_{q_i}=\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H,\mathbf{D}_{q_i}\succeq\mathbf{0},\ \sum_{i=1}^{K_q}\mathrm{Tr}\left\{\mathbf{D}_{q_i}\right\}\leq P_q\right\}. \tag{6}$$

Mathematically, the game has the following structure

$$\mathcal{G}=\left(\Omega,\ \{\mathcal{S}_q\}_{q\in\Omega},\ \{R_q\}_{q\in\Omega}\right). \tag{7}$$

A pure Nash equilibrium (NE) of game $\mathcal{G}$ is defined [29] when

$$R_q\left(\mathbf{Q}_q^\star,\mathbf{Q}_{-q}^\star\right)\geq R_q\left(\mathbf{Q}_q,\mathbf{Q}_{-q}^\star\right),\ \forall\mathbf{Q}_q\in\mathcal{S}_q,\quad\forall q\in\Omega. \tag{8}$$

At a NE, given the precoding strategy from other cells, a BS does not have the incentive to unilaterally change its precoding strategy, i.e., it shall achieve a lower sum-rate with the same power constraint.

B. Characterization of the BD Precoding Game's Nash Equilibrium

In this section, we study the two most fundamental questions in analyzing a SNG: the existence and uniqueness of the game's NE. The NE characterization allows us to predict a stable outcome of the noncooperative BD precoding design in game $\mathcal{G}$.

The existence of a pure NE in game $\mathcal{G}$ can be deduced straightforwardly from the work in [30] for N-person quasi-concave games. First, it is easy to verify that the strategy set $\mathcal{S}_q$ for player-q, defined in (6), is compact and convex, $\forall q$. It remains to show that the utility function $R_q(\mathbf{Q}_q,\mathbf{Q}_{-q})$ is a continuous function in the profile of strategies $(\mathbf{Q}_q,\mathbf{Q}_{-q})$ and concave in $\mathbf{Q}_q$. Since the utility function $R_q(\mathbf{Q}_q,\mathbf{Q}_{-q})$, decomposed as

$$\begin{aligned}R_q(\mathbf{Q}_q,\mathbf{Q}_{-q}) &= \sum_{i=1}^{K_q}\log\left|\mathbf{R}_{q_i}(\mathbf{Q}_{-q})+\mathbf{H}_{qq_i}\mathbf{Q}_{q_i}\mathbf{H}_{qq_i}^H\right| \\ &\quad -\sum_{i=1}^{K_q}\log\left|\mathbf{R}_{q_i}(\mathbf{Q}_{-q})\right|,\end{aligned} \tag{9}$$

is a summation (and subtraction) of continuous functions in $(\mathbf{Q}_q,\mathbf{Q}_{-q})$, it is also continuous in $(\mathbf{Q}_q,\mathbf{Q}_{-q})$. In addition, the first summation given in (9) is a concave function in $\mathbf{Q}_q$ due to the fact that the composition of the concave log-determinant function [31, p. 74] with an affine mapping does preserve its concavity [31, p. 79], whereas the the second summation is independent of $\mathbf{Q}_q$. Thus, $R_q(\mathbf{Q}_q,\mathbf{Q}_{-q})$ is a concave function in $\mathbf{Q}_q$. Consequently, Theorem 1 in [30] indicates that there always exists at least one pure NE in game $\mathcal{G}$.

In order to study the uniqueness of a NE in game $\mathcal{G}$, we first investigate the best response strategy at each player. As defined in $\mathcal{S}_q$, the best response strategy of player-q must be in the form $\mathbf{Q}_{q_i}=\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H,\forall i$. Let $\mathbf{D}_q\triangleq\mathrm{blk}\{\mathbf{D}_{q_i}\}$, $\mathbf{D}=\{\mathbf{D}_q\}_{q\in\Omega}$. Then, the best response strategy $\mathbf{D}_q$ at BS-q can be obtained from the following optimization problem

$$\begin{aligned}\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_{K_q}}}{\text{maximize}} \quad & \sum_{i=1}^{K_q}\log\left|\mathbf{I}+\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\right| \\ \text{subject to} \quad & \mathbf{D}_{q_i}\succeq\mathbf{0},\forall i, \\ & \sum_{i=1}^{K_q}\mathrm{Tr}\{\mathbf{D}_{q_i}\}\leq P_q,\end{aligned} \tag{10}$$

where $\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q})$ is defined as

$$\begin{aligned}\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q}) &= \mathbf{R}_{q_i}(\mathbf{Q}_{-q}) \\ &= \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\left(\sum_{j=1}^{K_r}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\right)\mathbf{H}_{rq_i}^H+\mathbf{Z}_{q_i}.\end{aligned} \tag{11}$$

By eigen-decomposing $\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}=\hat{\mathbf{U}}_{q_i}\mathbf{\Lambda}_{q_i}\hat{\mathbf{U}}_{q_i}^H$, the optimal solution to problem (10) can be easily obtained from the WF procedure

$$\mathbf{D}_{q_i}\triangleq\mathbf{WF}_{q_i}(\mathbf{D}_{-q})=\hat{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I}-\mathbf{\Lambda}_{q_i}^{-1}\right]^+\hat{\mathbf{U}}_{q_i}^H, \tag{12}$$

where the water-level μ_q is adjusted to meet the power constraint $\sum_{i=1}^{K_q}\mathrm{Tr}\left\{\left[\mu_q\mathbf{I}-\mathbf{\Lambda}_{q_i}^{-1}\right]^+\right\}=P_q$. Note that as $\hat{\mathbf{V}}_{q_i}$

only depends on in-cell channels at cell-q, BS-q only needs to strategically adapt its precoding matrices $\mathbf{D}_{q_i}, \forall i$ as in (12).

While the best response strategy of each player can be obtained in a closed-form solution in (12), the nonlinear structure of the WF operator is rather problematic in analyzing the uniqueness of the game's NE. Fortunately, the WF operator can be interpreted as a projection onto a closed and convex set [21], [24]. In particular, it was shown in [21] that the WF operator for the SISO multi-carrier system is a projection onto a simplex set. Then, this interpretation was generalized to the case of single-user MIMO WF operation [24]. In the following theorem, we show that interpretation of the WF operator as a projection can be further generalized to the case of multiuser MIMO system.

Theorem 1. *The optimization problem* (10) *is equivalent to the following optimization problem*

$$\begin{aligned}\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_{K_q}}}{\text{minimize}} \quad & \sum_{i=1}^{K_q} \Big\| \mathbf{D}_{q_i} + \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\sharp} \\ & \qquad + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \Big\|_F^2 \qquad (13) \\ \text{subject to} \quad & \sum_{i=1}^{K_q} \mathrm{Tr}\left\{\mathbf{D}_{q_i}\right\} = P_q, \ \mathbf{D}_{q_i} \succeq \mathbf{0},\end{aligned}$$

where $\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})}$ *is the orthogonal projection onto the null space of* $\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}$, *and* c_q *is an arbitrarily large constant satisfying* $c_q \geq P_q + \max_{\forall i, \forall k} [\mathbf{\Lambda}_{q_i}]_{kk}^{-1}$.

Proof: Please see Appendix A. ■

From Theorem 1, the WF solution in (12) is indeed the solution of the optimization (13). Thus, the block-diagonal WF solution $\mathbf{WF}_q(\mathbf{D}_{-q}) \triangleq \mathrm{blk}\{\mathbf{WF}_{q_i}(\mathbf{D}_{-q})\}$, can be interpreted as a projection

$$\mathbf{WF}_q(\mathbf{D}_{-q}) = \Big[-\mathrm{blk}\Big\{ \left(\mathbf{V}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^{\sharp} + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \Big\} \Big]_{\mathcal{D}_q}, \qquad (14)$$

where $\mathcal{D}_q \triangleq \left\{ \mathbf{D}_{q_i} \in \mathbb{S}^{\hat{N}_{q_i} \times \hat{N}_{q_i}} : \sum_{i=1}^{K_q} \mathrm{Tr}\{\mathbf{D}_{q_i}\} = P_q \right\}$ is a closed and convex set. Note that a projection onto a closed and convex set holds the following non-expansive property [32]

$$\left\| [\mathbf{X}_1]_{\mathcal{S}} - [\mathbf{X}_2]_{\mathcal{S}} \right\|_F \leq \left\| \mathbf{X}_1 - \mathbf{X}_2 \right\|_F, \qquad (15)$$

where $\mathbf{X}_1$ and $\mathbf{X}_2$ are arbitrary matrices, and $\mathcal{S}$ is closed and convex. This non-expansive property was crucial to prove the uniqueness of and the convergence to the NE for the case of SISO multi-carrier system [21] and multicell MIMO system [24]. Thanks to the interpretation in (14), we can also utilize the non-expansive property (15) to analyze the NE's uniqueness of the multicell BD precoding game $\mathcal{G}$, as being shown next.

Define the multicell mapping $\mathbf{WF}(\mathbf{D}) = \{\mathbf{WF}_q(\mathbf{D}_{-q})\}_{q\in\Omega}$. Let $e_{\mathrm{WF}_q} = \left\| \mathbf{WF}_q(\mathbf{D}_{-q}^{(1)}) - \mathbf{WF}_q(\mathbf{D}_{-q}^{(2)}) \right\|_F$ and $e_q = \left\| \mathbf{D}_q^{(1)} - \mathbf{D}_q^{(2)} \right\|_F$, for any given $\mathbf{D}^{(1)} \neq \mathbf{D}^{(2)}$ and $\mathbf{D}_q^{(1)}, \mathbf{D}_q^{(2)} \in \mathcal{D}_q, \forall q$. Then, the set of inequalities, given at the top of the following page, prompt

$$e_{\mathrm{WF}_q} \leq \sum_{r \neq q}^{Q} [\mathbf{S}]_{q,r}\, e_r,$$

where $\hat{\mathbf{V}}_r \triangleq [\hat{\mathbf{V}}_{r_1}, \ldots, \hat{\mathbf{V}}_{r_{K_r}}]$ and $\mathbf{S} \in \mathbb{C}^{Q\times Q}$ is defined as

$$[\mathbf{S}]_{q,r} = \begin{cases} \sum_{i=1}^{K_q} \rho\left(\hat{\mathbf{V}}_r^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp} \mathbf{H}_{qq_i}^{\sharp} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_r \right), & \text{if } r \neq q \\ 0, \quad \text{if } r = q. \end{cases} \qquad (17)$$

Note that the inequality (16a) holds due to (15), inequalities (16b) and (16d) hold because $\mathbf{X} = \mathrm{diag}\{\mathbf{X}_1, \ldots, \mathbf{X}_K\}$ implies that $\|\mathbf{X}\|_F \leq \sum_{i=1}^K \|\mathbf{X}_i\|_F$, inequality (16c) holds due to the reverse order law for the Moore-Penrose pseudo-inverse [24], and inequality (16e) holds because the Frobenius norm is consistent [33].

Define the vectors $\mathbf{e}_{\mathrm{WF}} = [e_{\mathrm{WF}_1}, \ldots, e_{\mathrm{WF}_Q}]^T$ and $\mathbf{e} = [e_1, \ldots, e_Q]^T$. The set of inequalities (16f) implies that

$$\mathbf{0} \leq \mathbf{e}_{\mathrm{WF}} \leq \mathbf{S}\mathbf{e}. \qquad (18)$$

We now provide some definitions of the matrix norms and vector norms applied to study the contraction property of game $\mathcal{G}$. Given the mapping $\mathbf{D}^{(t+1)} = \mathbf{WF}(\mathbf{D}^{(t)})$ and a vector $\mathbf{w} = [w_q, \ldots, w_Q]^T > 0$, the block-maximum norm [32] on the mapping $\mathbf{WF}(\mathbf{D})$ is defined as

$$\|\mathbf{WF}(\mathbf{D})\|_{F,\text{block}}^{\mathbf{w}} \triangleq \max_{q\in\Omega} \frac{\|\mathbf{WF}_q(\mathbf{D}_{-q})\|_F}{w_q}. \qquad (19)$$

The weighted maximum norm of a vector $\mathbf{x}$, induced by a positive vector $\mathbf{w}$, is defined as [32]

$$\|\mathbf{x}\|_{\infty,\text{vec}}^{\mathbf{w}} = \max_{q\in\Omega} \frac{|x_q|}{w_q}, \quad \mathbf{x} \in \mathbb{R}^Q. \qquad (20)$$

Finally, define the matrix norm of a matrix $\mathbf{A}$ induced by $\|\cdot\|_{\infty,\text{vec}}^{\mathbf{w}}$ as [33]

$$\|\mathbf{A}\|_{\infty,\text{mat}}^{\mathbf{w}} = \max_{q\in\Omega} \frac{1}{w_q} \sum_{r=1}^{Q} [\mathbf{A}]_{q,r} w_r, \quad \mathbf{A} \in \mathbb{R}^{Q\times Q}. \qquad (21)$$

The mapping $\mathbf{WF}(\mathbf{D})$ is a block-contraction mapping of rate α with respect to the norm $\|\cdot\|_{F,\text{block}}^{\mathbf{w}}$, if there exists a nonnegative constant $\alpha < 1$, such that

$$\left\| \mathbf{WF}(\mathbf{D}^{(1)}) - \mathbf{WF}(\mathbf{D}^{(2)}) \right\|_{F,\text{block}}^{\mathbf{w}} \leq \alpha \left\| \mathbf{D}^{(1)} - \mathbf{D}^{(2)} \right\|_{F,\text{block}}^{\mathbf{w}}, \quad \forall \mathbf{D}^{(1)}, \mathbf{D}^{(2)}. \qquad (22)$$

From the inequality (18), one has

$$\|\mathbf{e}_{\mathrm{WF}}\|_{\infty,\text{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\mathbf{e}\|_{\infty,\text{vec}}^{\mathbf{w}} \leq \|\mathbf{S}\|_{\infty,\text{mat}}^{\mathbf{w}} \|\mathbf{e}\|_{\infty,\text{vec}}^{\mathbf{w}}, \qquad (23)$$

as the induced ∞-norm $\|\cdot\|_{\infty,\text{mat}}^{\mathbf{w}}$ is consistent [33]. Then,

$$\begin{aligned} &\left\| \mathbf{WF}(\mathbf{D}^{(1)}) - \mathbf{WF}(\mathbf{D}^{(2)}) \right\|_{F,\text{block}}^{\mathbf{w}} \\ &= \max_{q\in\Omega} \frac{\left\| \mathbf{WF}_q(\mathbf{D}^{(1)}) - \mathbf{WF}_q(\mathbf{D}^{(2)}) \right\|_2}{w_q} \\ &= \|\mathbf{e}_{\mathrm{WF}}\|_{\infty,\text{vec}}^{\mathbf{w}} \\ &\leq \|\mathbf{S}\|_{\infty,\text{mat}}^{\mathbf{w}} \|\mathbf{e}\|_{\infty,\text{vec}}^{\mathbf{w}} \\ &= \|\mathbf{S}\|_{\infty,\text{mat}}^{\mathbf{w}} \left\| \mathbf{D}^{(1)} - \mathbf{D}^{(2)} \right\|_{F,\text{block}}^{\mathbf{w}}. \end{aligned} \qquad (24)$$

$$
\begin{aligned}
e_{\mathrm{WF}_q} &= \Bigg\| \left[-\mathrm{blk}\left\{ \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \right\} \right]_{\mathcal{D}_q} \\
&\quad - \left[-\mathrm{blk}\left\{ \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp + c_q \mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} \right\} \right]_{\mathcal{D}_q} \Bigg\|_F \\
&\leq \left\| \mathrm{blk}\left\{ -\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp \right\} - \mathrm{blk}\left\{ -\left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp \right\} \right\|_F \qquad \text{(16a)} \\
&\leq \sum_{i=1}^{K_q} \left\| \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp - \left(\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \mathbf{H}_{qq_i} \hat{\mathbf{V}}_{q_i} \right)^\sharp \right\|_F \qquad \text{(16b)} \\
&\leq \sum_{i=1}^{K_q} \left\| \hat{\mathbf{V}}_{q_i}^\sharp \mathbf{H}_{qq_i}^\sharp \left(\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(1)}) - \hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q}^{(2)}) \right) \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \right\|_F \qquad \text{(16c)} \\
&= \sum_{i=1}^{K_q} \left\| \hat{\mathbf{V}}_{q_i}^\sharp \mathbf{H}_{qq_i}^\sharp \left[\sum_{r\neq q}^{Q} \mathbf{H}_{rq_i} \left[\sum_{j=1}^{K_r} \hat{\mathbf{V}}_{r_j} \left(\mathbf{D}_{r_j}^{(1)} - \mathbf{D}_{r_j}^{(2)} \right) \hat{\mathbf{V}}_{r_j}^H \right] \mathbf{H}_{rq_i}^H \right] \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \right\|_F \\
&= \sum_{i=1}^{K_q} \left\| \hat{\mathbf{V}}_{q_i}^\sharp \mathbf{H}_{qq_i}^\sharp \left[\sum_{r\neq q}^{Q} \mathbf{H}_{rq_i} \hat{\mathbf{V}}_r \left(\mathbf{D}_r^{(1)} - \mathbf{D}_r^{(2)} \right) \hat{\mathbf{V}}_r^H \mathbf{H}_{rq_i}^H \right] \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \right\|_F \\
&\leq \sum_{i=1}^{K_q} \sum_{r\neq q}^{Q} \left\| \hat{\mathbf{V}}_{q_i}^\sharp \mathbf{H}_{qq_i}^\sharp \mathbf{H}_{rq_i} \hat{\mathbf{V}}_r \left(\mathbf{D}_r^{(1)} - \mathbf{D}_r^{(2)} \right) \hat{\mathbf{V}}_r^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \right\|_F \qquad \text{(16d)} \\
&\leq \sum_{i=1}^{K_q} \sum_{r\neq q}^{Q} \rho\left(\hat{\mathbf{V}}_r^H \mathbf{H}_{rq_i}^H \mathbf{H}_{qq_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^{\sharp H} \hat{\mathbf{V}}_{q_i}^\sharp \mathbf{H}_{qq_i}^\sharp \mathbf{H}_{rq_i} \hat{\mathbf{V}}_r \right) \left\| \mathbf{D}_r^{(1)} - \mathbf{D}_r^{(2)} \right\|_F \qquad \text{(16e)} \\
&= \sum_{r\neq q}^{Q} [\mathbf{S}]_{q,r}\, e_r. \qquad \text{(16f)}
\end{aligned}
$$

Thus, if $\|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1$, the $\mathbf{WF}(\mathbf{D})$ mapping is a contraction, which implies the uniqueness of the NE in game $\mathcal{G}$ [32]. In addition, the condition $\|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1$ is also sufficient to guarantee the convergence of the NE from any starting precoding strategy $\mathbf{D}_q \in \mathcal{D}_q$. Note that $\mathbf{S}$ is a nonnegative matrix, there always exists a positive vector $\mathbf{w}$ satisfying [32]

$$(\mathrm{C}): \quad \|\mathbf{S}\|_{\infty,\mathrm{mat}}^{\mathbf{w}} < 1 \quad \Longleftrightarrow \quad \rho(\mathbf{S}) < 1. \qquad (25)$$

Remark 1: It is observed from the construction of matrix $\mathbf{S}$ in (17) that $\mathbf{S}$ comprises of all the channels in the system. Thus, validating condition (C) is a challenging task, unless there is a centralized unit, e.g., a base-station controller (BSC), which can collect all the channels from the BSs, to perform the job. For this reason, there is a need for an interpretation to condition (C) that can help us to expedite its validation. Assuming the path loss fading model in this multicell system, a physical interpretation of the sufficient condition (C) is as follows. When the intra-cell BS-MS distance gets smaller relatively to the distance between the BSs, the ICI becomes less dominant. Thus, the positive off-diagonal elements of $\mathbf{S}$ also become smaller. This results in a smaller spectral radius of $\mathbf{S}$. Therefore, as the MSs are getting closer to its connected BS, the probability of meeting condition (C) is higher, which then guarantees the uniqueness of the NE.

IV. The Multicell Block-Diagonalization Precoding - Coordinated Design

A. Problem Formulation

In Section III, we have examined the fully decentralized approach in the multicell BD precoding design and characterized the NE of the system. However, it is well-known that the NE needs not to be Pareto-efficient [34]. Via the coordination between the BSs, significant network sum-rate improvement can be obtained by jointly designing all the precoders at the same time. Nonetheless, this advantage may come with the expense of message passing between the BSs as explained later in this section. To this end, we investigate the coordinated multicell BD precoding design in order to jointly maximize the network WSR through the following optimization

$$
\begin{aligned}
\underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \alpha_q \sum_{i=1}^{K_q} \log\left| \mathbf{I} + \mathbf{H}_{qq_i}^H \mathbf{R}_{q_i}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq_i} \mathbf{Q}_{q_i} \right| \qquad (26) \\
\text{subject to} \quad & \mathbf{H}_{qq_j} \mathbf{Q}_{q_i} \mathbf{H}_{qq_j}^H = \mathbf{0}, \forall j \neq i, \forall q \\
& \mathbf{Q}_{q_i} \succeq \mathbf{0},\ \forall i, \forall q \\
& \sum_{i=1}^{K_q} \mathrm{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q, \forall q,
\end{aligned}
$$

where $\alpha_q \geq 0$ denotes the nonnegative weight associated with BS-q. Herein, α_q's allow a trade-off between the sum-rates allocated to the BSs. When the sum-rate at a particular BS, say BS-q, is given at a higher priority, its weight α_q is assigned

$$\begin{aligned}\mathbf{A}_{q_i} &= -\frac{\partial f_q}{\partial \mathbf{D}_{q_i}}\bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}} \\ &= -\sum_{r\neq q}^{Q}\alpha_r\sum_{j=1}^{K_r}\frac{\partial R_{r_j}}{\partial \mathbf{D}_{q_i}}\bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}} \\ &= \sum_{r\neq q}^{Q}\alpha_r\sum_{j=1}^{K_r}\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qr_j}^H\left[\hat{\mathbf{R}}_{r_j}^{-1}-\left(\hat{\mathbf{R}}_{r_j}+\mathbf{H}_{rr_j}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\mathbf{H}_{rr_j}^H\right)^{-1}\right]\mathbf{H}_{qr_j}\hat{\mathbf{V}}_{q_i}\bigg|_{\mathbf{D}_{q_i}=\bar{\mathbf{D}}_{q_i}}. \end{aligned} \tag{29}$$

at a larger value than the others. When all the weights are assigned as equal, the optimization in (26) is to maximize the social welfare of all BSs [29]. Since the BD constraints can be removed by formulating the precoding covariance matrix $\mathbf{Q}_{q_i}$ as $\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\hat{\mathbf{V}}_{q_i}^H$, where $\mathbf{D}_{q_i}$ is an arbitrary $\hat{N}_{q_i}\times\hat{N}_{q_i}$ and $\hat{\mathbf{V}}_{q_i}$ given in (5), the optimization problem (26) can be restated as

$$\begin{aligned}\underset{\mathbf{D}_1,\ldots,\mathbf{D}_Q}{\text{maximize}}\quad & \sum_{q=1}^{Q}\alpha_q\sum_{i=1}^{K_q}\log\left|\mathbf{I}+\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\right| \\ \text{subject to}\quad & \mathbf{D}_{q_i}\succeq\mathbf{0},\forall i,\forall q \\ & \sum_{i=1}^{K_q}\text{Tr}\{\mathbf{D}_{q_i}\}\le P_q,\forall q.\end{aligned} \tag{27}$$

It is observed that the objective function in problem (27) is not concave due to presence of $\mathbf{D}_{r_j}$'s in the ICI term $\hat{\mathbf{R}}_{q_i}(\mathbf{D}_{-q})$'s. Thus, the optimization problem (27) is not convex. Consequently, it is generally difficult and computationally complex to find its globally optimal solution. To this end, we focus on proposing a lower-complexity algorithm that can obtain at least a locally optimal solution.

B. Iterative Linear Approximation Solution Approach

This section presents a solution approach to the nonconvex problem (27) by considering it as a difference of convex (DC) program [35]. This DC solution approach, termed as the iterative linear approximation (ILA) algorithm, has been utilized in a recent work [36] to maximize the multicell network WSR with one MS per cell. For the multicell multiuser system under consideration, this section then shows how the DC programming can be applied to locally solve the WSR maximization problem (27). Specifically, by iteratively isolating and approximating the nonconvex part of the objective function into linear terms, one can decompose problem (27) into a sequence of simpler convex optimization problems. It will be shown that each approximated problem can be solved separately at the corresponding BS in a closed-form solution.

Denote $f_q(\mathbf{D}_q,\mathbf{D}_{-q}) = \sum_{r\neq q}^{Q}\alpha_r\sum_{j=1}^{K_r}R_{r_j}(\mathbf{D}_q,\mathbf{D}_{-q})$ as the WSR of all other cells except cell-q. Note that $f_q(\mathbf{D}_q,\mathbf{D}_{-q})$ is nonconcave in $\mathbf{D}_{q_i}, i=1,\ldots,K_q$. At a given value of $(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q})$, we take the approximation to f_q by using the Taylor expansion of f_q around $\bar{\mathbf{D}}_{q_i}, i=1,\ldots,K_q$, and retaining the first linear term

$$f_q(\mathbf{D}_q,\bar{\mathbf{D}}_{-q})\approx f_q(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q})-\sum_{i=1}^{K_q}\text{Tr}\left\{\mathbf{A}_{q_i}\left(\mathbf{D}_{q_i}-\bar{\mathbf{D}}_{q_i}\right)\right\}, \tag{28}$$

where $\mathbf{A}_{q_i}$ is the negative partial derivative of f_q with respect to the $\mathbf{D}_{q_i}$, evaluated at $\bar{\mathbf{D}}_{q_i}$, given at the top of this page.

Using (28), the network WSR around $(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q})$ can be approximated as $\alpha_q\sum_{i=1}^{K_q}R_{q_i} - f_q(\bar{\mathbf{D}}_q,\bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K_q}\text{Tr}\left\{\mathbf{A}_{q_i}\left(\mathbf{D}_{q_i}-\bar{\mathbf{D}}_{q_i}\right)\right\}$. Omitting the known terms in the objective function, the nonconvex problem (27) can be approximated as

$$\begin{aligned}\underset{\mathbf{D}_{q_1},\ldots,\mathbf{D}_{q_{K_q}}}{\text{maximize}}\quad & \alpha_q\sum_{i=1}^{K_q}\log\left|\mathbf{I}+\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\right| \\ & -\sum_{i=1}^{K_q}\text{Tr}\left\{\mathbf{A}_{q_i}\mathbf{D}_{q_i}\right\} \\ \text{subject to}\quad & \mathbf{D}_{q_i}\succeq\mathbf{0},\ \forall i \\ & \sum_{i=1}^{K_q}\text{Tr}\{\mathbf{D}_{q_i}\}\le P_q,\end{aligned} \tag{30}$$

which can be solved solely at BS-q. Thus, if the Q BSs take turn to approximate the original problem (27), it can be solved via Q per-cell separate problems (30).

It can be observed that the approximated problem (30) is similar to the sum-rate maximization problem with BD precoding, albeit the presence of the term $\sum_{i=1}^{K_q}\text{Tr}\left\{\mathbf{A}_{q_i}\mathbf{D}_{q_i}\right\}$. Herein, this term is the penalty charged on the ICI induced by BS-q to the MSs in other cells, whereas $\mathbf{A}_{q_i}$ acts as the interference price. If the ICI penalty term is not presented, the BS would only attempt to maximize the sum-rate for its connected MSs. As a result, the multicell system is in the *competition* mode, as studied in Section III. In contrast, in the *coordination* mode, each BS is doing its best in limiting the ICI induced to the other cells through this ICI penalty mechanism. Interestingly, problem (30) also indicates that BS-q will focus more on maximizing its own sum-rate than minimizing its induced ICI, if the weight α_q is set at a higher value than the other weights.

Since the approximated problem (30) is now convex, it can be readily solved by standard convex optimization techniques. In the following, we present the closed-form solution to the problem via the Lagrangian duality. The Lagrangian of problem (30) can be stated as

$$\begin{aligned}\mathcal{L}_q(\mathbf{D}_{q_i},\lambda_q) &= \alpha_q\sum_{i=1}^{K_q}\log\left|\mathbf{I}+\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{D}_{q_i}\right| \\ &\quad -\sum_{i=1}^{K_q}\text{Tr}\left\{\left(\mathbf{A}_{q_i}+\lambda_q\mathbf{I}\right)\mathbf{D}_{q_i}\right\}+\lambda_q P_q,\end{aligned} \tag{31}$$

where $\lambda_q \geq 0$ is the Lagrangian multiplier associated with the power constraint. The dual function is then given by

$$g_q(\lambda_q) = \max_{\mathbf{Q}_{q_i} \succeq \mathbf{0}} \mathcal{L}_q(\mathbf{D}_{q_i}, \lambda_q). \tag{32}$$

For a given λ_q, the optimal solution to the Lagrangian (31) is presented in the following proposition.

Proposition 1. *Let $\mathbf{G}_{q_i}$ be the generalized eigen-matrix of $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}$ and $(\mathbf{A}_{q_i} + \lambda_q \mathbf{I})$. The optimal solution, which maximizes the Lagrangian* (31)*, must have the structure $\mathbf{G}_{q_i}\mathbf{P}_{q_i}\mathbf{G}_{q_i}^H, i = 1, \ldots, K$, where $\mathbf{P}_{q_i}$ is a diagonal matrix with nonnegative elements.*

Proof: The proof for this proposition is similar to that of Proposition 1 in [36] for the case of single-user rate maximization with a penalty term. We omit the detailed proof of this proposition for brevity. ■

Given $\mathbf{G}_{q_i}$ as the generalized eigen-matrix of $\hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}$ and $(\mathbf{A}_{q_i} + \lambda_q \mathbf{I})$, one has

$$\begin{aligned} \mathbf{\Sigma}_{q_i}^{(1)} &= \mathbf{G}_{q_i}^H \hat{\mathbf{V}}_{q_i}^H \mathbf{H}_{qq_i}^H \hat{\mathbf{R}}_{q_i}^{-1}(\bar{\mathbf{D}}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\mathbf{G}_{q_i} \\ \mathbf{\Sigma}_{q_i}^{(2)} &= \mathbf{G}_{q_i}^H(\mathbf{A}_{q_i} + \lambda_q \mathbf{I})\mathbf{G}_{q_i}, \end{aligned} \tag{33}$$

where $\mathbf{\Sigma}_{q_i}^{(1)}$ and $\mathbf{\Sigma}_{q_i}^{(2)}$ are diagonal and positive semi-definite. Thus, the maximization of the Lagrangian (31) becomes

$$\underset{\mathbf{P}_{q_i} \succeq \mathbf{0}}{\text{maximize}}\ \alpha_q \sum_{i=1}^{K_q} \log \left| \mathbf{I} + \mathbf{P}_{q_i}\mathbf{\Sigma}_{q_i}^{(1)} \right| - \sum_{i=1}^{K_q} \mathrm{Tr}\left\{ \mathbf{P}_{q_i}\mathbf{\Sigma}_{q_i}^{(2)} \right\}, \tag{34}$$

whose optimal solution can be obtained by the well-known WF structure

$$[\mathbf{P}_{q_i}^\star]_{n,n} = \left[\frac{\alpha_q}{\left[\mathbf{\Sigma}_{q_i}^{(2)}\right]_{n,n}} - \frac{1}{\left[\mathbf{\Sigma}_{q_i}^{(1)}\right]_{n,n}} \right]^+, \forall i. \tag{35}$$

It remains to adjust the dual variable λ_q to impose the power constraint $\sum_{i=1}^{K_q} \mathrm{Tr}\{\mathbf{P}_{q_i}^\star\} \leq P_q$ for the above water-filling solution. One can easily verify for the case $\lambda_q = 0$ whether $\sum_{i=1}^{K_q} \mathrm{Tr}\{\mathbf{P}_{q_i}^\star\} < P_q$. If it holds, it means BS-q does not transmit at its full power limit. Otherwise, $\lambda_q > 0$ can be searched by the bisection method until $\sum_{i=1}^{K_q} \mathrm{Tr}\{\mathbf{P}_{q_i}^\star\} = P_q$.

C. Convergence of the ILA Algorithm and its Distributed Implementation

This section is to address the convergence of the ILA algorithm and its distributed implementation to maximize the network WSR with BD precoding. In order to solve the problem of Q cells in (27), the ILA algorithm requires each BS-q, $q = 1, \ldots, Q$, to update the parameters $\mathbf{A}_{q_i}$'s and sequentially take turns to solve its corresponding optimization (30). The convergence of the ILA algorithm is given in the following theorem.

Theorem 2. *The optimization* (30) *carried at any given BS-q always improves the network WSR. Thus, the Gauss-Seidel (sequential) iterative update across the Q BSs is guaranteed to converge to at least a local maximum.*

Proof: Please refer to Appendix B. ■

Remark 2: As stated in Theorem 2, the sequential update across the Q BSs will eventually converge to a local optimal solution. In practice, it is typical that nearby BSs are connected to a same *centralized* BSC. In this case, the BSC can pass tokens to the BSs so that each BS can take turn update its BD precoders. Note that the original optimization problem (27) may have many local maxima due to its nonconvexity. Since the proposed ILA algorithm converges monotonically to a local maximum, its convergence is dependent on the update order at the first few iterations. However, once the algorithm gets closer to a local maximum, the update order no longer impact on the obtained solution. From our observations of the simulation results with different updating orders, the ILA algorithm converges to slightly different suboptimal solutions. Nonetheless, the obtained local optimal values are almost the same.

As presented in Section III, the BD precoding design for a multicell system under the IA mode can be implemented in a fully decentralized manner. Interestingly, distributed implementation can also be realized for the BD precoding design under the IC mode. Via the coordination and message exchange between the BSs, the ILA can be implemented distributively as follows. Since the optimization problem (30) can be executed at the corresponding BS with only local information (CSI and IPN at the connected MSs), it remains to show that the pricing factors $\mathbf{A}_{q_i}$'s can also be computed in a distributed manner through a message exchange mechanism among the BSs. It is observed from equation (29) that in order to compute $\mathbf{A}_{q_i}$, BS-q has to know the channels $\mathbf{H}_{qr_j}$'s to all the MSs in the other cells. This is an important requirement for BS-q to coordinate its induced ICI. In addition, BS-q needs to acquire the factor $\mathbf{B}_{r_j} = \hat{\mathbf{R}}_{r_j}^{-1} - \left(\hat{\mathbf{R}}_{r_j} + \mathbf{H}_{rr_j}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\mathbf{H}_{rr_j}^H\right)^{-1}$ from other cells. Using the local measurement on the IPN covariance matrix $\hat{\mathbf{R}}_{r_j}$, MS-j of cell-r can feedback $\hat{\mathbf{R}}_{r_j}$ to its connected BS-r. The BS, having known its transmitted signal to its MS-j in $\mathbf{H}_{rr_j}\hat{\mathbf{V}}_{r_j}\mathbf{D}_{r_j}\hat{\mathbf{V}}_{r_j}^H\mathbf{H}_{rr_j}^H$, can easily compute the matrix factor $\mathbf{B}_{r_j}$. These factors $\mathbf{B}_{r_j}$'s are then exchanged among the BSs to evaluate the prices $\mathbf{A}_{q_i}$'s.

Remark 3: The message exchange mechanism for matrices $\mathbf{B}_{r_j}$'s among the coordinated BSs is the distinct feature of the IC mode, compared to the IA mode. On the other hand, the intra-cell MS to BS feedback messaging for the IPN covariance matrices remains the same for both IA and IC modes. Thus, the computation at the MSs can be kept at minimum even in the IC mode. All the computations of the prices and the updates of the precoders are simply performed at the more computationally efficient BSs.

V. Multicell BD-DPC Precoding: Competition and Coordination

A. BD-DPC Precoding on a Per-cell Basis

This section considers the multicell system where each BS utilizes BD-DPC to the downlink transmissions of its connected MSs. It is well-known that DPC is the capacity-achieving encoding scheme for the multi-user broadcast channel [2]–[4]. In [2], a suboptimal and simpler zero-forcing DPC (ZF-DPC) scheme was proposed for single-antenna receivers

$$[\mathbf{S}']_{q,r} = \begin{cases} \sum_{i=1}^{K_q} \rho \left(\hat{\mathbf{V}}_{\pi_r}^H \mathbf{H}_{r\pi_q(i)}^H \mathbf{H}_{q\pi_q(i)}^{\sharp H} \hat{\mathbf{V}}_{\pi_q(i)}^{\sharp H} \hat{\mathbf{V}}_{\pi_q(i)}^{\sharp} \mathbf{H}_{q\pi_q(i)}^{\sharp} \mathbf{H}_{r\pi_q(i)} \hat{\mathbf{V}}_{\pi_r} \right), & \text{if } r \neq q \\ 0, & \text{if } r = q, \end{cases} \tag{39}$$

that takes advantage of both DPC and ZF precoding. In ZF-DPC, the information signals sent to the multiple users are encoded in sequence such that the receiver at any user does not see any inter-user interference due to the use of ZF and DPC at the BS. In this work, we apply a similar technique to the encoding process at each BS. Due to the consideration of multi-antenna receivers, the technique shall be referred to as the BD-DPC precoding.

At any BS, say BS-q, denote the encoding sequence to its K_q connected MSs as $\boldsymbol{\pi}_q = [\pi_q(1), \ldots, \pi_q(K_q)]^T$. The concept of BD-DPC can be briefly explained as follows:

- BS-q freely designs the precoder $\mathbf{W}_{\pi_q(1)}$ for MS-$\pi_q(1)$.
- BS-q, having the noncausal knowledge of the codeword intended for MS-$\pi_q(1)$, uses DPC such that MS-$\pi_q(2)$ does not see the codeword for MS-$\pi_q(1)$ as interference. At the same time, the precoder $\mathbf{W}_{\pi_q(2)}$ for MS-$\pi_q(2)$ is designed on the null space caused by $\mathbf{H}_{q\pi_q(1)}$ to eliminate its induced interference to MS-$\pi_q(1)$.
- Similarly, to encode the signal for user-i, BS-q can utilize the noncausal knowledge of the codewords for MSs $\pi_q(1), \ldots, \pi_q(i-1)$, and design $\mathbf{W}_{\pi_q(i)}$ on the null space caused by $\hat{\mathbf{H}}'_{\pi_q(i)} = [\mathbf{H}_{q\pi_q(1)}, \ldots, \mathbf{H}_{q\pi_q(i-1)}]$.

B. The Multicell BD-DPC Precoding - Competitive Design

Similar to game $\mathcal{G}$ defined in Section III, we consider a new game $\mathcal{G}'$, where each BS strategically adapts its BD-DPC precoders to maximize the sum-rate to its connected MSs. Mathematically, game $\mathcal{G}'$ can be defined as

$$\mathcal{G}' = \left(\Omega, \left\{\mathcal{S}'_q(\boldsymbol{\pi}_q)\right\}_{q\in\Omega}, \left\{R_q\right\}_{q\in\Omega}\right), \tag{36}$$

The set of admissible strategies $\mathcal{S}'_q(\boldsymbol{\pi}_q)$ is now defined as

$$\mathcal{S}'_q(\boldsymbol{\pi}_q) = \Big\{\mathbf{Q}_{\pi_q(i)} \in \mathbb{S}^{M_q\times M_q} : \mathbf{Q}_{\pi_q(i)} = \hat{\mathbf{V}}_{\pi_q(i)} \mathbf{D}_{\pi_q(i)} \hat{\mathbf{V}}_{\pi_q(i)}^H, \mathbf{D}_{\pi_q(i)} \succeq \mathbf{0}, \sum_{i=1}^{K_q} \mathrm{Tr}\left\{\mathbf{D}_{\pi_q(i)}\right\} \leq P_q\Big\}, \tag{37}$$

where $\hat{\mathbf{V}}_{\pi_q(i)}$ is the null space created by $\hat{\mathbf{H}}'_{\pi_q(i)}$. Due to the similarity between games $\mathcal{G}$ and $\mathcal{G}'$, the characterization for game $\mathcal{G}$ presented in Section III-B can be directly applied to game $\mathcal{G}'$. In particular, it can be concluded that there always exists at least one NE in game $\mathcal{G}'$ and the NE is unique if

$$(\mathrm{C}') : \rho(\mathbf{S}') < 1, \tag{38}$$

where $\mathbf{S}' \in \mathbb{C}^{Q\times Q}$ is given at the top of this page, with $\hat{\mathbf{V}}_{\pi_r} \triangleq [\hat{\mathbf{V}}_{\pi_r(1)}, \ldots, \hat{\mathbf{V}}_{\pi_r(K_r)}]$.

Remark 4: Due to the dependence of the admissible strategy set $\mathcal{S}'_q(\boldsymbol{\pi}_q)$ on the encoding order $\boldsymbol{\pi}_q$ at BS-q, the characterization of game $\mathcal{G}'$ strictly depends on the encoding order at each BS. In addition, with different encoding orders at a BS, say BS-q, the optimal strategies, which maximize the sum-rate at BS-q, are also different. The condition (C') for the uniqueness of game $\mathcal{G}'$ also depends on the encoding order at each BS-q. In fact, for any permutation in $\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_Q$, we have at least a different NE of game $\mathcal{G}'$. Given $K_q!$ encoding order permutations at BS-q, it can be concluded that game $\mathcal{G}'$ has at least $\prod_{q=1}^{Q}(K_q!)$ NE points.

Remark 5: For a particular encoding order $\boldsymbol{\pi}_1, \ldots, \boldsymbol{\pi}_Q$ in game $\mathcal{G}'$, game $\mathcal{G}'$ provides a higher degree of freedom in designing the precoder at each BS. In fact, the size of matrix $\hat{\mathbf{V}}_{\pi_q(i)}$ in game $\mathcal{G}'$ is at least equal or larger than its counterpart $\hat{\mathbf{V}}_{q_i}$ in game $\mathcal{G}$. Intuitively, the off-diagonal elements of matrix $\mathbf{S}'$ are also larger than that of matrix $\mathbf{S}$. As a result, it is expected that the condition for the uniqueness of the NE in game $\mathcal{G}'$ is stricter than that in game $\mathcal{G}$.

C. The Multicell BD-DPC Precoding - Coordinated Design

In this section, we investigate the implementation of BD-DPC precoding in a multicell system under the IC mode. In this case, we consider the joint BD-DPC precoding design to maximize the network WSR as follows:

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_Q}{\text{maximize}} \quad & \sum_{q=1}^{Q} \alpha_q R_q \\ \text{subject to} \quad & \mathbf{Q}_q \in \mathcal{S}'_q, \forall q. \end{aligned} \tag{40}$$

Similar to the optimization problem (26) considered in Section IV, the above problem is also nonconvex. Thus, we apply the same ILA algorithm proposed in Section IV to solve problem (40). In particular, due to monotonic convergence of the ILA algorithm, we can obtain at least a locally optimal solution to the problem. The only difference here is that the solution $\mathbf{Q}_{q_i}$ of problem (40) must be in the form $\hat{\mathbf{V}}_{\pi_q(i)} \mathbf{D}_{q_i} \hat{\mathbf{V}}_{\pi_q(i)}^H$, where $\hat{\mathbf{V}}_{\pi_q(i)}$ is the null space created by $\hat{\mathbf{H}}'_{\pi_q(i)}$, and $\mathbf{D}_{q_i}$ is obtained from the ILA algorithm.

VI. Simulation Results and Discussions

This section presents the simulation results validating our studies on the uniqueness of a NE and the convergence to the NE in games $\mathcal{G}$ and $\mathcal{G}'$. We then compares the IA and IC modes with various precoding schemes in terms of the achievable sum-rates of the multicell system. In addition to the BD and BD-DPC precoding examined in this work, we also consider DPC precoding. Under the IA mode, the DPC precoding [2], [3] is performed on a per-cell basis in a non-cooperative manner (each BS selfishly maximizes its own sum-rate) until the multicell system converges to a stable state.[3] Under the IC mode, to jointly maximize the network sum-rate with DPC precoding, we utilize a numerical algorithm recently proposed in [37]. For the BD-DPC or DPC precoding, a fixed encoding

[3]The characterization of the multicell DPC game is beyond the scope of this paper.

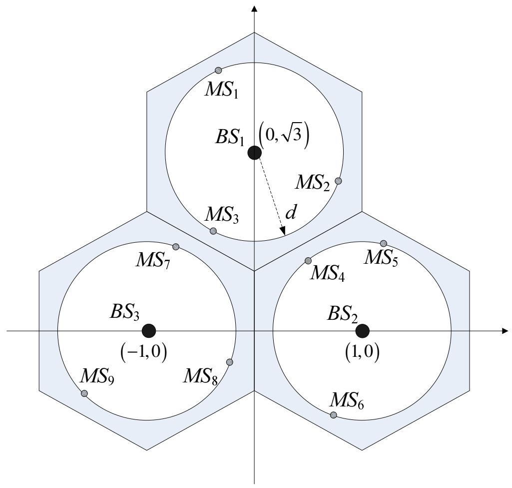

Fig. 1. A multicell system configuration with 3 cells, 3 users per cell.

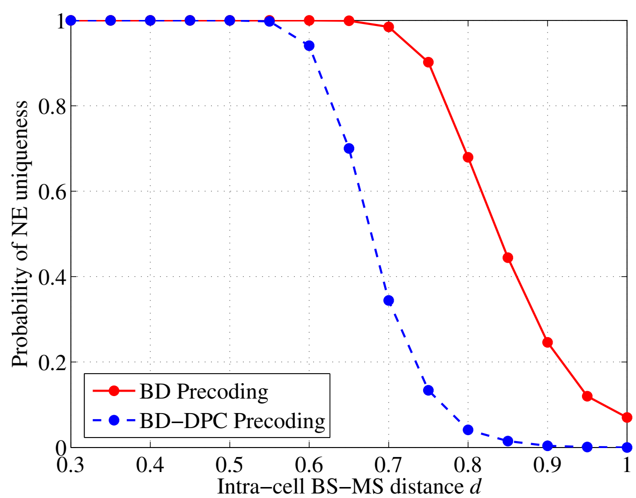

Fig. 2. Probability of NE's uniqueness versus the intra-cell BS-MS distance d.

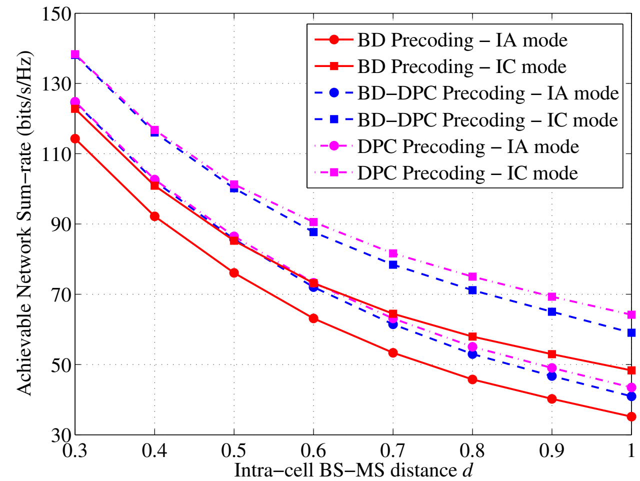

Fig. 3. Network sum-rate versus the intra-cell BS-MS distance d.

order from MS-1 to MS-K_q at BS-q is applied and similarly at other BSs.

We considered a 3-cell system with 3 MSs per cell sharing the same channel frequency, as illustrated in Fig. 1. The numbers of antennas at each BS and each MS are set at $M_q = 8$ and $N_{q_i} = 2$. The same power constraint $P_q = 1$ is set at each BS, unless stated otherwise. The AWGN at each MS is set as $\mathbf{Z}_{q_i} = \sigma^2\mathbf{I}$ with $\sigma^2 = 0.01$. The distance between any two BSs is normalized to 2. In each cell, the MSs are assumed to be randomly located on a circle from its connected BS with the radius of d. The channels from a BS to a MS are generated from i.i.d. Gaussian random variables using the path-loss model with the path-loss exponent of 3 and the reference distance of 1 corresponding to MSs at the cell edge. The variance of the small-scale fading (shadowing) is set at 0 dB such that the expected average SNR at the cell edge is $P_q/\sigma^2 = 20$ dB. In each figure, each plotted point is obtained by averaging over 10000 independent channel realizations.

Fig. 2 displays the probability of the NE's uniqueness versus intra-cell BS-MS distance d by evaluating condition (C) for game $\mathcal{G}$ and (C′) for game $\mathcal{G}'$. Since (C) and (C′) are only sufficient conditions for the corresponding games, Fig. 2 should present a lower bound on the chance of getting the NE's uniqueness. Corresponding to a small distance d is the low-ICI region (and high signal-to-interference-plus-noise (SINR) as a result). In contrast, at high d, each MS is more susceptible to a higher level of ICI (low-SINR region). As observed from the figure, the uniqueness of the NE (in both games $\mathcal{G}$ and $\mathcal{G}'$) is guaranteed if the ICI is sufficiently small, as suggested in our analytical result in Section III-B. In addition, as the MSs approach the cell-edge, corresponding to an increase of the ICI levels, the chance of getting the NE's uniqueness of both games $\mathcal{G}$ and $\mathcal{G}'$ decreases. Fig. 2 also confirms our analysis in Section V-B that game $\mathcal{G}'$ admits a much stronger uniqueness condition than that in game $\mathcal{G}$.

Figs. 3 and 4 compare the IA and IC modes in terms of the achievable network sum-rates. In the IA mode, if conditions (C) and (C′) are not fulfilled, the game may have one or multiple NEs. In general, if a game has multiple NEs, it is better to choose the NE that maximizes the total utility, i.e., the social welfare [29]. However, finding all NEs in the game under consideration is a challenging task, which is beyond the scope of our current work. For the simulation in Figs. 3 and 4, we simply let the game be played by the BSs. When the game converges to one of the NEs, the total sum-rate of the system is measured and plotted. It is observed from Fig. 3 that increasing the ICI powers (increasing d) results in significant sum-rate reductions in the multicell system. On the other hand, the network sum-rates can be significantly improved by coordinating the precoders in the IC mode, especially at the high-ICI region. However, this performance advantage comes with the requirement of control signaling and CSI exchange among the coordinated BSs, as noted in Remark 3. In comparing the BD, BD-DPC, and DPC precoding, DPC outperforms the former two schemes in both IA and IC modes due to its optimality on a per-cell basis. However, the performance difference between DPC and BD-DPC precoding is fairly small. It is worth noting that DPC precoding in this multicell setting is more complex in terms of analysis and numerical optimization than BD and

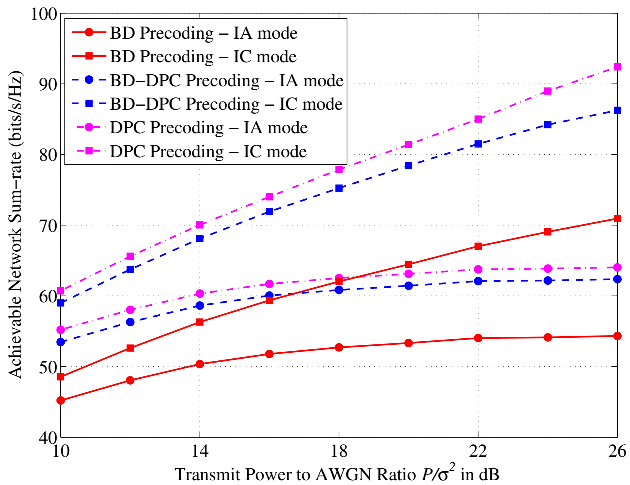

Fig. 4. Network sum-rate versus the transmit power to AWGN ratio at each BS for $d = 0.7$.

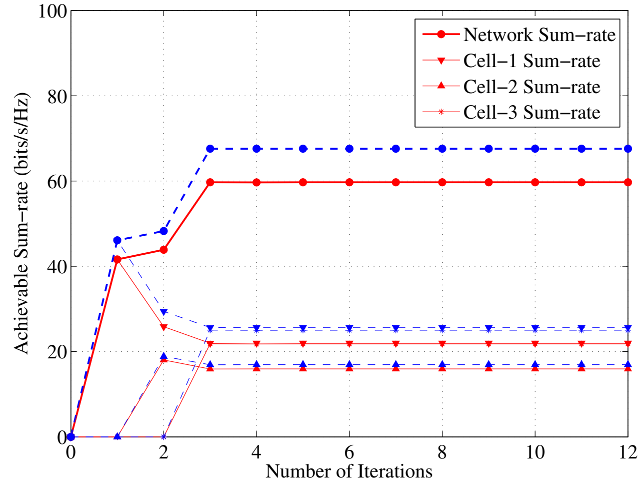

Fig. 5. Sum-rate versus number of iterations for $d = 0.7$ in the IA mode (*solid* lines are for the BD precoding game and *dashed-dotted* lines are for the BD-DPC precoding game).

BD-DPC precoding. In fact, under the IA mode, the BD and BD-DPC precoding games are much easier to analyze than the DPC precoding game. Furthermore, under the IC mode, the numerical algorithm proposed in Section IV to optimize the BD or BD-DPC precoding is computationally simpler than the algorithm proposed in [37] for DPC precoding.

Fig. 4 illustrates the network sum-rates under IA and IC modes versus the transmit power to AWGN ratio P/σ^2 for $d = 0.7$ (assuming the same power budget P at all Q BSs). It is observed that increasing the transmit power at each BS does improve the network sum-rates in both modes. However, at very high level of transmit power, the network sum-rates obtained from the multicell precoding games become saturated. This is due to the reason that the ICI is also increased relatively with the intra-cell information signal powers. In this case, it is desirable to coordinate and limit the amount of ICI by the IC mode. Apparently, the IC mode does perform much better than the IA mode with all 3 precoding designs at the high ICI region.

To illustrate the convergence of the multicell precoding games $\mathcal{G}$ and $\mathcal{G}'$, we randomly select a channel realization and plot the achievable sum-rates versus the number of iterations in Fig. 5. In both games, the BSs perform sequential precoder updates. The network sum-rates and the sum-rates at each cell are then plotted after each instance of updating. It is observed that both games converge very quickly in a few iterations. As expected, the BD-DPC game results in a higher network sum-rate over the BD game due to the superior performance of BD-DPC precoding over BD precoding on a per-cell basis.

Finally, Fig. 6 illustrates the convergence of the proposed ILA algorithm to maximize the network sum-rate under the IC mode. For the same channel realization utilized to generate Fig. 5, we plot the network sum-rates and sum-rates at each cell after each time instance. As observed in the figure, the ILA algorithm monotonically converges with the sequential updates at the coordinated BSs for both the cases of BD and BD-DPC precoding. At each update, even though the sum-rate at one of the cells may decrease, the network sum-rate is always improved. This convergence behavior of the ILA algorithm agrees with our analysis in Theorem 2. As expected, the BD-DPC precoding converges to a better sum-rate than the BD precoding due to its superior performance on a per-cell basis. Compared to Fig. 5, the ILA algorithm provides better sum-rate performances than the multicell games with BD and BD-DPC precoding. However, the ILA algorithm takes more iterations to converge. From our observations with other randomized channel realizations, the games and the ILA algorithms do converge with a similar number of iterations and a similar behavior as indicated in Figs. 5 and 6. In other words, the channel realization used to obtain Figs. 5-6 was chosen randomly to illustrate the typical convergence behavior of the algorithms (the best response dynamics in the game and ILA algorithm).

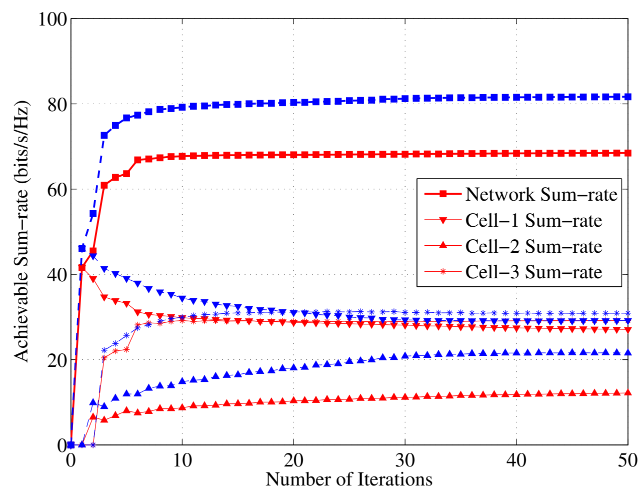

Fig. 6. Sum-rate versus number of iterations for $d = 0.7$ in the IC mode (*solid* lines are for the BD precoding and *dashed-dot* lines are for the BD-DPC precoding).

VII. Conclusion

This paper studied the multicell system with universal frequency reuse where BD or BD-DPC precoding is performed on a per-cell basis. When the multicell system is under *competition* mode, we investigated the conditions on the existence and uniqueness of the multicell games' NE. Simulation results confirmed that the NE of the multicell games is unique if the ICI is sufficiently small. They also indicated that the BD-DPC multicell precoding game outperforms the BD game while achieving a sum-rate very close to that of the DPC precoding game. When the multicell system is under *coordination* mode, we proposed the distributed ILA algorithm to obtain at least a local optimal solution to the nonconvex WSR maximization problems. Simulation results then show that the network sum-rate can be improved over the *competition* mode by coordinating the BD or BD-DPC precoders across the multicell system. It is noted that the treatment in this paper is limited to single-carrier systems. In multi-carrier systems, it is possible that one can allocate the resource over the frequency domain first (e.g., equal power allocation to each carrier), then the study in this work (competitive and coordinated designs) can be applied on a per-carrier basis. Extending our work to the scenario of joint resource allocation (power allocation and precoding design) over frequency and space is possible and will be interesting future work.

Appendix A
Proof of Theorem 1

The proof for this theorem is similar to that of Lemma 1 in [24] for the case of single-user MIMO WF. Given that $\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i} = \hat{\mathbf{U}}_{q_i}\mathbf{\Lambda}_{q_i}\hat{\mathbf{U}}_{q_i}^H$, one has

$$\left(\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\right)^\sharp = \hat{\mathbf{U}}_{q_i}\mathbf{\Lambda}_{q_i}^{-1}\hat{\mathbf{U}}_{q_i}^H. \tag{41}$$

Note that $\hat{\mathbf{U}}_{q_i}$ is a $\hat{N}_{q_i}\times N_{q_i}$ unitary matrix, i.e., $\hat{\mathbf{U}}_{q_i}^H\hat{\mathbf{U}}_{q_i} = \mathbf{I}$, and $\mathbf{\Lambda}_{q_i}$ is a $N_{q_i}\times N_{q_i}$ diagonal matrix. By the assumption that $\sum_{i=1}^{K_q} N_{q_i} \le M_q$, i.e., $N_{q_i} \le \hat{N}_{q_i}$, one may form a unitary matrix $\check{\mathbf{U}}_{q_i} = [\hat{\mathbf{U}}_{q_i}, \tilde{\mathbf{U}}_{q_i}]$, where $\tilde{\mathbf{U}}_{q_i}$ is a $\hat{N}_{q_i}\times(\hat{N}_{q_i}-N_{q_i})$ matrix satisfying $\hat{\mathbf{U}}_{q_i}^H\tilde{\mathbf{U}}_{q_i} = \mathbf{0}$ and $\tilde{\mathbf{U}}_{q_i}^H\tilde{\mathbf{U}}_{q_i} = \mathbf{I}$. In addition, $\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}) = \mathcal{N}(\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})$ implies that $\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})} = \tilde{\mathbf{U}}_{q_i}\tilde{\mathbf{U}}_{q_i}^H$. Thus, for a given c_q, one has

$$\begin{aligned}&\left(\hat{\mathbf{V}}_{q_i}^H\mathbf{H}_{qq_i}^H\hat{\mathbf{R}}_{q_i}^{-1}(\mathbf{D}_{-q})\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i}\right)^\sharp + c_q\mathbf{P}_{\mathcal{N}(\mathbf{H}_{qq_i}\hat{\mathbf{V}}_{q_i})}\\ &= \check{\mathbf{U}}_{q_i}\check{\mathbf{\Lambda}}_{q_i}^{-1}\check{\mathbf{U}}_{q_i}^H,\end{aligned}$$

where $\check{\mathbf{\Lambda}}_{q_i} = \mathrm{blk}\{\mathbf{\Lambda}_{q_i}, (1/c_q)\mathbf{I}\}$.

The optimization problem (13) then can be rewritten as

$$\begin{aligned}\underset{\check{\mathbf{D}}_{q_1},\ldots,\check{\mathbf{D}}_{q_{K_q}}}{\text{minimize}} \quad & \sum_{i=1}^{K_q}\left\|\check{\mathbf{D}}_{q_i} + \check{\mathbf{\Lambda}}_{q_i}^{-1}\right\|_F^2 \\ \text{subject to} \quad & \sum_{i=1}^{K_q}\mathrm{Tr}\left\{\check{\mathbf{D}}_{q_i}\right\} = P_q,\ \check{\mathbf{D}}_{q_i}\succeq\mathbf{0},\end{aligned}\tag{42}$$

where $\check{\mathbf{D}}_{q_i} \triangleq \check{\mathbf{U}}_{q_i}^H\mathbf{D}_{q_i}\check{\mathbf{U}}_{q_i}$. Due to the fact the objective function is lower-bounded by diagonal matrices $\{\check{\mathbf{D}}_{q_i}\}$, the optimal solution set $\{\check{\mathbf{D}}_{q_i}\}$ to problem (42) has to be diagonal. Thus, the optimization (42) can be reduced to

$$\begin{aligned}\underset{\check{\mathbf{D}}_{q_1},\ldots,\check{\mathbf{D}}_{q_{K_q}}}{\text{minimize}} \quad & \sum_{i=1}^{K_q}\sum_k\left([\check{\mathbf{D}}_{q_i}]_{k,k} + [\check{\mathbf{\Lambda}}_{q_i}^{-1}]_{k,k}\right)^2 \\ \text{subject to} \quad & \sum_{i=1}^{K_q}\sum_k[\check{\mathbf{D}}_{q_i}]_{k,k} = P_q,\ [\check{\mathbf{D}}_{q_i}]_{k,k}\ge 0,\end{aligned}\tag{43}$$

whose (unique) optimal solution has the WF structure such that $\check{\mathbf{D}}_{q_i} = \left[\mu_q\mathbf{I} - \check{\mathbf{\Lambda}}_{q_i}^{-1}\right]^+$, where μ_q is the water-level to meet the power constraint $\sum_{i=1}^{K_q}\mathrm{Tr}\left\{\check{\mathbf{D}}_{q_i}\right\} = P_q$. Thus, the optimal solution to the original problem (13) is given by

$$\begin{aligned}\mathbf{D}_{q_i} &= \check{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I} - \mathrm{blk}\{\mathbf{\Lambda}_{q_i}^{-1}, c_q\mathbf{I}\}\right]^+\check{\mathbf{U}}_{q_i}^H\\ &= \hat{\mathbf{U}}_{q_i}\left[\mu_q\mathbf{I} - \mathbf{\Lambda}_{q_i}^{-1}\right]^+\hat{\mathbf{U}}_{q_i}^H,\end{aligned}$$

if c_q is chosen to be large enough such that $[\mu_q - c_q]^+ = 0$. As suggested in [24], choosing $c_q \ge P_q + \max_{\forall i,\forall k}[\mathbf{\Lambda}_{q_i}]_{kk}^{-1}$ is sufficient to meet this requirement. This concludes the proof for Theorem 1.

Appendix B
Proof for Theorem 2

Suppose that $\mathbf{D}_q = \bar{\mathbf{D}}_q = \left\{\bar{\mathbf{D}}_{q_i}\right\}_{i=1}^K, \forall q$ is obtained from the previous iteration, and $\mathbf{D}_q^\star = \left\{\mathbf{D}_{q_i}^\star\right\}_{i=1}^K, \forall q$ is the optimal solution obtained from the optimization problem (30) at BS-q. Similar to the technique applied in [36], [38], it can be derived that $f_q(\mathbf{D}_q, \bar{\mathbf{D}}_{-q})$ is a convex function with respect to $\mathbf{D}_q$. Thus, by the first-order condition for the convex function $f_q(\mathbf{D}_q, \bar{\mathbf{D}}_{-q})$ [31], one has

$$f_q(\mathbf{D}_q^\star, \bar{\mathbf{D}}_{-q}) \ge f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K_q}\mathrm{Tr}\left\{\mathbf{A}_{q_i}(\mathbf{D}_{q_i}^\star - \bar{\mathbf{D}}_{q_i})\right\}. \tag{44}$$

After the optimization (30) carried at BS-q, the network WSR is updated such that

$$\begin{aligned}&\sum_{q=1}^{Q}\alpha_q\sum_{i=1}^{K_q}R_{q_i}(\mathbf{D}_q^\star, \bar{\mathbf{D}}_{-q})\\ &= \alpha_q\sum_{i=1}^{K_q}R_{q_i}(\mathbf{D}_q^\star, \bar{\mathbf{D}}_{-q}) + f_q(\mathbf{D}_q^\star, \bar{\mathbf{D}}_{-q})\\ &\ge \alpha_q\sum_{i=1}^{K_q}R_{q_i}(\mathbf{D}_q^\star, \bar{\mathbf{D}}_{-q})\\ &\quad + f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K_q}\mathrm{Tr}\left\{\mathbf{A}_{q_i}(\mathbf{D}_{q_i}^\star - \bar{\mathbf{D}}_{q_i})\right\}\\ &\ge \alpha_q\sum_{i=1}^{K_q}R_{q_i}(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q})\\ &\quad + f_q(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}) - \sum_{i=1}^{K_q}\mathrm{Tr}\left\{\mathbf{A}_{q_i}(\bar{\mathbf{D}}_{q_i} - \bar{\mathbf{D}}_{q_i})\right\}\\ &= \sum_{q=1}^{Q}\alpha_q\sum_{i=1}^{K_q}R_{q_i}(\bar{\mathbf{D}}_q, \bar{\mathbf{D}}_{-q}),\end{aligned}\tag{45}$$

where the first inequality is due to the first-order condition in (44) and the second inequality is due to the fact that $\mathbf{D}_q^\star$ is the optimal solution of problem (30). Clearly, the optimization carried at a given BS-q always improves the network WSR. In the ILA algorithm, each BS, say BS-q, updates the parameters $\mathbf{A}_{q_i}$'s and sequentially takes turn to solve its corresponding optimization (30). Since the network WSR is upper-bounded, the Gauss-Seidel (sequential) updates across the Q BSs must converge monotonically to at least a local maximum. This concludes the proof for Theorem 2.

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Duy H. N. Nguyen (S'07) received the B.Eng. degree (with First Class Honours) from Swinburne University of Technology, Hawthorn, Victoria, Australia, in 2005, the M.Sc. degree from University of Saskatchewan, Saskatoon, SK, Canada, in 2009, and the Ph.D. degree from McGill University, Montréal, QC, Canada, in 2013, all in electrical engineering. Since September 2013, he has been a Research Associate at the Department of Electrical and Computer Engineering, McGill University and a Postdoctoral Research Fellow at the Institut National de la Recherche Scientifique - Énergie, Matériaux et Télécommunications, Université du Québec, Montréal, QC, Canada. His research interests include resource allocation in wireless networks, signal processing for communications, convex optimization, and game theory.

Dr. Nguyen was a recipient of the Australian Development Scholarship (ADS) for his undergraduate study in Australia, a recipient of the McGill Engineering Doctoral Award, and a recipient of the Fonds de recherche du Québec - Nature et technologies (FRQNT) Doctoral Fellowship. Since September 2013, he has been a recipient of the FRQNT Postdoctoral Fellowship.

Hung Nguyen-Le received the B.Eng. degree and the M.Eng. degree in Electrical Engineering from Hochiminh City University of Technology, Vietnam, in 2001 and 2003, respectively. He obtained the Ph.D. degree in Electrical Engineering from the National University of Singapore in 2008. From 2008 to 2010, he worked as a Postdoctoral Research Fellow at the Department of Electrical and Computer Engineering, McGill University, Montreal, Canada. Since 2010, he has been with the Department of Electronics and Telecommunications Engineering, the University of Danang, University of Science and Technology, Vietnam. Since 2013, he has been with the Department of Science, Technology and Environment, the University of Danang, Vietnam. His research interests include array signal processing, multiuser/multicell transmissions, channel estimation and synchronization in broadband wireless communications.

Tho Le-Ngoc (F'97) obtained his B.Eng. (with Distinction) in Electrical Engineering in 1976, his M.Eng. in 1978 from McGill University, Montreal, and his Ph.D. in Digital Communications in 1983 from the University of Ottawa, Canada. During 1977-1982, he was with Spar Aerospace Limited and involved in the development and design of satellite communications systems. During 1982-1985, he was an Engineering Manager of the Radio Group in the Department of Development Engineering of SRTelecom Inc., where he developed the new point-to-multipoint DA-TDMA/TDM Subscriber Radio System SR500. During 1985-2000, he was a Professor at the Department of Electrical and Computer Engineering of Concordia University. Since 2000, he has been with the Department of Electrical and Computer Engineering of McGill University. His research interest is in the area of broadband digital communications. He is a Fellow of the Institute of Electrical and Electronics Engineers (IEEE), the Engineering Institute of Canada (EIC), the Canadian Academy of Engineering (CAE) and the Royal Society of Canada (RSC). He is the recipient of the 2004 Canadian Award in Telecommunications Research, and recipient of the IEEE Canada Fessenden Award 2005. He holds a Canada Research Chair (Tier I) on Broadband Access Communications, and a Bell Canada/NSERC Industrial Research Chair on Performance & Resource Management in Broadband xDSL Access Networks.