Multiuser Downlink Beamforming in Multicell Wireless Systems: A Game Theoretical Approach

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***Abstract*—This paper is concerned with the game theoretical approach in designing the multiuser downlink beamformers in multicell systems. Sharing the same physical resource, the base-station of each cell wishes to minimize its transmit power subject to a set of target signal-to-interference-plus-noise ratios (SINRs) at the multiple users in the cell. In this context, at first, the paper considers a strategic noncooperative game (SNG) where each base-station greedily determines its optimal downlink beamformer strategy in a distributed manner, without any coordination between the cells. Via the game theory framework, it is shown that this game belongs to the framework of standard functions. The conditions guaranteeing the existence and uniqueness of a Nash Equilibrium (NE) in this competitive design are subsequently examined. The paper then makes a revisit to the fully coordinated design in multicell downlink beamforming, where the optimal beamformers are jointly designed between the base-stations. A comparison between the competitive and coordinated designs shows the benefits of applying the former over the latter in terms of each design's distributed implementation. Finally, in order to improve the efficiency of the NE in the competitive design, the paper considers a more cooperative game through a pricing mechanism. The pricing consideration enables a base-station to steer its beamformers in a more cooperative manner, which ultimately limits the interference induced to other cells. The study on the existence and uniqueness of the new game's NE is then given. The paper also presents a condition on the pricing factors that allow the new NE point to approach the performance established by the coordinated design, while retaining the distributed nature of the multicell game.**

***Index Terms*—Competitive design, coordinated design, downlink beamforming, game theory, multiple-input multiple-output (MIMO), multicell system, multiuser, Nash equilibrium, optimization.**

I. Introduction

A defining characteristic of a wireless channel is its broadcast nature. In addition, the limited spectrum resource constrains many wireless devices to share the same communication channel, thus inducing the mutual-interference with each other. In a mutual interference multiuser channel, power control is a critical issue as each user's performance depends not only on its own power allocation scheme, but also on the power allocation of other users. An optimal power control strategy helps to manage the interference between adjacent cells and also to facilitate efficient spectral reuse. However, such an optimal strategy, generally implemented in a centralized manner with full channel information, often inflicts a highly computational burden and tends to be inflexible in large-scale networks. Alternately, a distributed approach, where the power control is performed with local information and low computational requirement, becomes a more attractive option.

Recently, the study of power control in mutual interference using game theory has attracted considerable research attention. By considering the multiuser system as a strategic noncooperative game (SNG), each player (a wireless device) greedily adapts its strategy to maximize its own utility, given the strategies from other players [1]–[10]. In general, these works focus on studying the existence and uniqueness of the stable operating point of the system, i.e., the Nash equilibrium (NE). The uplink power control problem in a single-cell code-division multiple-access (CDMA) data system with multiple competing users was studied in [1] and [2], where the utility function was defined as the ratio of throughput to transmit power. A pricing mechanism was investigated in [2] to obtain a more efficient solution of the power control game. In orthogonal frequency division multiplexing (OFDM) system over a shared band, the work in [3] has inspired various works on the iterative water-filling algorithm, such as [4]–[7] with sum-rate as the utility function, or [8] with transmit power as the utility function. More recently, the water-filling game has also been considered in [11] for a system utilizing both protected and shared bands.

In multiple antenna systems, the work in [9] considered a multicell system, where each cell consisted of *one* multiple-antenna base-station (BS) and *one* single-antenna mobile station (MS). The objective of [9] was to study the precoding beamforming vector at the two BSs in competitive and cooperative manners. In the multiuser multiple-input multiple-output (MIMO) channels, the work in [10] studied the competitive precoding design, where each player wished to maximize its mutual information. It is noted that these works only considered the system where each transmitter (base-station) communicates with only *one* receiver (mobile station).

Inspired by the mentioned works, this paper considers a game theoretical approach to study the competitive precoding design in a multiuser multicell system, where each BS concurrently serves multiple MSs (or users). Sharing the same frequency band, the BS of each cell wishes to design the optimal downlink beamformers for its users in order to minimize its transmit

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power, given a set of target signal-to-interference-plus-noise ratios (SINRs) for the users in its cell. Under the similar setup, Ren and La [12] studied scheduling schemes to handle the inter-cell interference and provide a quality of service in the form of packet error rate. Multicell downlink beamforming with coordination was considered in [13], where the total weighted transmit power across multiple BSs is jointly minimized. Via the concept of uplink–downlink duality, it is shown in [13] that such a jointly optimal design can be implemented in a distributed manner under certain requirements, including perfect channel reciprocal from each BS to each MS (not necessarily in the same cell) and synchronization between the BSs. These requirements, which may be difficult to meet in practice, are the drawbacks of the distributed implementation in the coordinated design. Conversely, in the competitive design, where the multicell beamformers are devised on per-cell basis with no centralized control, these requirements can be alleviated.

Using the game-theory framework, we establish the best response strategy of a cell, given the beamforming strategies from other cells. Then, it is shown that such best response strategy is a *standard* function [14],[1] which guarantees the uniqueness of the NE and the convergence of the distributed algorithm. This is the distinction of this power minimization game, compared to typical n-person concave games in an OFDM system with water-filling as the optimal strategy [3]–[7]. In addition, necessary and sufficient conditions for the existence of the NE are also given. A comparison to the fully coordinated multicell downlink beamforming design is then presented in this work.

It is worth mentioning that the NE of the multicell game needs not to be Pareto-optimal, i.e., it may not stay in the surface established the coordinated design. Moreover, it may happen that a beamformer design in one cell is highly correlated to the channel to other cell, which then causes significant inter-cell interference. To avoid this undesired effect, we consider a new multicell downlink beamforming game with pricing consideration, where each BS voluntarily attempts to minimize the interference induced to other cells. This pricing technique allows a BS to steer its beamformers in a more cooperative way, which results in a more Pareto-efficient NE. The characterization of the new game's NE reveals that under certain conditions, the new NE point is able to approach the performance established by the coordinated design, while retaining the distributed nature of the SNG.

Notations: Superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ stand for transpose, complex conjugate, and complex conjugate transpose operations, respectively; upper-case bold face letters are used to denote matrices whereas lower-case bold face letters are used to denote column vectors; $\boldsymbol{I}$ stands for an identity matrix; $\mathrm{diag}(d_1, d_2, \ldots, d_M)$ denotes an $M \times M$ diagonal matrix with diagonal elements $d_1, d_2, \ldots, d_M$; $[\cdot]_{i,j}$ denotes the (i, j) element of the matrix argument; $x^\star$ indicates the optimal value of the variable x; $\succeq$ denotes matrix inequality defined on the cone of positive semi-definite matrices; the sets $\mathbb{C}$ and $\mathbb{R}$ stand for the set of complex and real numbers, respectively.

[1]The framework of *standard* functions, proposed by Yates [14], shall be discussed for our context in Section III.

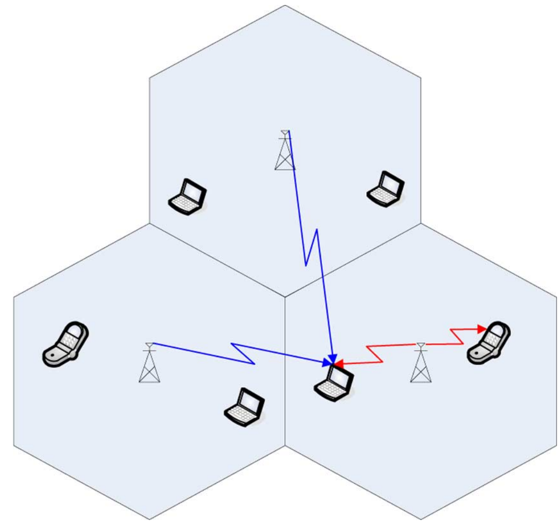

Fig. 1. An example of a multicell system with three base-stations and two users per cell.

II. System Model

We consider a multiuser downlink beamforming system with Q separate cells operating on the same frequency channel, as illustrated in Fig. 1. At each cell, one multiple-antenna BS is concurrently sending independent information to several remote single-antenna MSs. Let $\Omega = \{1, \ldots, Q\}$ denote the set of the cells (players).[2] For the simplicity of presentation, it is assumed that each BS is equipped with M antennas and is serving K MSs. It is noted each user (MS) is now subject to the co-channel interference from other cells (inter-cell interference), in addition to the interference caused by the signals intended for other users in the same cell (intra-cell interference). In a competitive design for this multicell system, it is assumed that each BS has full knowledge of the downlink channels within its cell, but not the inter-cell channels. Thus, each BS is only able to manage the intra-cell interference. On the other hand, the BS treats the inter-cell interference as background noise. In the later parts of this works, this channel assumption is relaxed, in which a BS may possess the full or partial channel information to all the users in the whole system. The additional channel knowledge then allows a BS to control the inter-cell interference as well.

Considering the transmission at a particular cell, say cell-q, its downlink channel can be modeled as

$$y_{q_i} = \boldsymbol{h}_{qq_i}^H \boldsymbol{x}_q + \sum_{m=1, m\neq q}^{Q} \boldsymbol{h}_{mq_i}^H \boldsymbol{x}_m + z_{q_i} \tag{1}$$

where $\boldsymbol{x}_m$ is an $M \times 1$ complex vector representing the transmitted signal at BS-m, $\boldsymbol{h}_{mq_i}^*$ is an $M \times 1$ complex channel vector from BS-m to user-i of cell-q, y_{q_i} represents the received signal at user-i and z_{q_i} is the AWGN with the power spectral density σ^2. It is assumed that the channel vector $\boldsymbol{h}_{qq_i}^*$ is known at both the BS and user-i of cell-q, whereas the cross-cell channel $\boldsymbol{h}_{mq_i}, m \neq q$ is unknown.

[2]Hereafter, a cell or a base-station is referred to as a player interchangeably, where a mobile station is referred to a user.

In a beamforming design, the transmitted signal $\boldsymbol{x}_q$ is of the form

$$\boldsymbol{x}_q = \sum_{i=1}^{K} x_{q_i} \boldsymbol{w}_{q_i} \quad (2)$$

where x_{q_i} is a complex scalar representing the signal intended for user-i and $\boldsymbol{w}_{q_i}$ is an $M \times 1$ beamforming vector for user-i. Without loss of generality, let $\mathbb{E}[|x_{q_i}|] = 1$. It is easy to verify that the SINR at user-i of cell-q is

$$\mathrm{SINR}_{q_i} = \frac{\left|\boldsymbol{w}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \left|\boldsymbol{w}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + \sum_{m\neq q}^{Q} \sum_{j=1}^{K} \left|\boldsymbol{w}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2 + \sigma^2}. \quad (3)$$

Note that the received signal at user-i of cell-q is corrupted by the intra-cell interference $\sum_{j\neq i}^{K} \left|\boldsymbol{w}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2$, the inter-cell interference $\sum_{m\neq q}^{Q} \sum_{j=1}^{K} \left|\boldsymbol{w}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2$, as well as the AWGN. Although the channel state information from other cells is not known at both the users and BS of cell-q, each user can measure its total interference and report back to the BS. The BS, having known the channel to the users in its cell, can determine the total inter-cell interference plus AWGN at each user.

III. The Multicell Downlink Beamforming Game

A. Problem Formulation

In the first part of this work, we are interested in formulating the multicell downlink beamforming design within the framework of game theory. In particular, we consider a SNG, where the players are the cells and the payoff functions are the transmit powers of the BSs. More specifically, each player competes with each other by choosing the downlink beamformer design that greedily minimizes its own transmit power subject to a given set of target SINRs at the users within its cell. Each channel is assumed to vary sufficiently slowly such that it can be considered fixed during the game being played.

Define the precoding matrix $\boldsymbol{W}_q = [\boldsymbol{w}_{q_1}, \ldots, \boldsymbol{w}_{q_K}]$ as the strategy at BS-q and $\boldsymbol{W}_{-q}$ as the precoding strategy of all the BSs, except BS-q. The transmit power at BS-q, is then given by $\|\boldsymbol{W}\|_F^2$. Further define the set of admissible beamforming strategies $\boldsymbol{W}_q \in \mathcal{P}_q(\boldsymbol{W}_{-q})$ of cell-q as

$$\mathcal{P}_q(\boldsymbol{W}_{-q}) = \left\{\boldsymbol{W}_q \in \mathbb{C}^{M\times K} : \mathrm{SINR}_{q_i}(\boldsymbol{W}_q, \boldsymbol{W}_{-q}) \geq \gamma_{q_i}, \forall i\right\}$$

where γ_{q_i} is the target SINR at user-i of cell-q.

At cell-q, denote the total interference induced by Ω_{-q} plus background noise at the ith user as $r_{-q_i}(\boldsymbol{W}_{-q}) = \sum_{m\neq q}^{Q} \sum_{j=1}^{K} \left|\boldsymbol{w}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2 + \sigma^2 = \sum_{m\neq q}^{Q} \left\|\boldsymbol{W}_m^H \boldsymbol{h}_{mq_i}\right\|^2 + \sigma^2$. Furthermore, denote $\boldsymbol{r}_{-q} = [r_{-q_1}, \ldots, r_{-q_K}]^T$. Note that the set of feasible strategies $\mathcal{P}(\boldsymbol{W}_{-q})$ of cell-q depends on the beamforming strategies $\boldsymbol{W}_{-q}$ of all the other cells. Mathematically, the corresponding game has the following structure:

$$\mathcal{G} = \left(\Omega, \{\mathcal{P}_q(\boldsymbol{W}_{-q})\}_{q\in\Omega}, \{t_q(\boldsymbol{W}_q)\}_{q\in\Omega}\right)$$

where $t_q(\boldsymbol{W}_q) = \left\|\boldsymbol{W}_q\right\|_F^2$ is the transmit power at BS-q.[3] Given the beamforming design of the others, reflected by the background noise vector $\boldsymbol{r}_{-q}$, the optimal or best response strategy of the qth BS is the solution of the following optimization problem:

$$\begin{aligned} \underset{\boldsymbol{W}_q}{\text{minimize}} \quad & \left\|\boldsymbol{W}_q\right\|_F^2 \\ \text{subject to} \quad & \frac{\left|\boldsymbol{w}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \left|\boldsymbol{w}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i}, \ \forall i. \end{aligned} \quad (4)$$

We note that this downlink beamforming problem has been optimally solved by several approaches in literature. Uplink-downlink duality was exploited in [15]–[17], where the downlink problem under individual SINR constraints can be solved via the equivalent uplink problem, which is much easier to solve. In another approach, [18] relaxed the problem into a convex semi-definite program (SDP). In a more recent work [19], the authors formulated the downlink problem directly as a convex second-order cone programming (SOCP). A simple and fast fixed-point iterative algorithm was also proposed to find the optimal downlink beamformers. In a multicell configuration, the problem arisen here is when one player changes its beamforming matrix, the other players also need to change their own beamforming matrices in order to achieve its target SINRs. Our interest is to investigate whether game $\mathcal{G}$ eventually converges into a stable point, i.e., a NE; and if a NE exists, does its uniqueness hold? A feasible strategy profile $\boldsymbol{W}^\star = \{\boldsymbol{W}_q^\star\}_{q=1}^{Q}$ is a NE of game $\mathcal{G}$ if

$$t_q(\boldsymbol{W}_q^\star) \leq t_q(\boldsymbol{W}_q), \ \forall \boldsymbol{W}_q \in \mathcal{P}_q(\boldsymbol{W}_{-q}^\star), \quad \forall q \in \Omega. \quad (5)$$

At the NE point, given the beamforming matrices from other cells, a BS does not have the incentive to unilaterally change its own beamforming matrix, i.e., it will consume more power to obtain the same SINR targets. In the following sections, by first studying the best response strategy of each player, the NE of game $\mathcal{G}$ is subsequently characterized.

B. The Best Response Strategy

In this section, we first present some claims to simplify the analysis of the game and characterize the best response strategy.

Claim 1: *If $\boldsymbol{W}_q^\star$ is the optimal beamforming strategy for cell-q, then $\boldsymbol{W}_q^\star \boldsymbol{R}$, where $\boldsymbol{R} = \mathrm{diag}\left(\mathrm{e}^{j\theta_1}, \ldots, \mathrm{e}^{j\theta_K}\right)$, $\forall \theta_1, \ldots, \theta_K$ is also optimal.*

This claim stems from the fact that the effective SINR at each user is invariant to a constant phase change of the beamforming vector of any other user.

Claim 2: *With unlimited transmit power, the feasibility of the optimization problem (4) at cell-q is only dependent on the channel $\boldsymbol{h}_{qq_1}^*, \ldots, \boldsymbol{h}_{qq_K}^*$ and the set of target SINRs*

[3]According to recent use, this game may be referred to as a generalized Nash equilibrium problem where the admissible strategy set of a player depends on the other players' strategy [8].

$\gamma_{q_1}, \ldots, \gamma_{q_K}$. *It is independent of the inter-cell interference plus noise vector* $\boldsymbol{r}_{-q} > 0$.

This claim comes directly from Proposition 1 in [19], which shows that the rank of the matrix $\boldsymbol{H}_q = [\boldsymbol{h}_{qq_1}, \ldots, \boldsymbol{h}_{qq_K}]$ determines the feasibility of the optimization problem (4). In addition, when $\boldsymbol{H}_q$ is full-rank, any set of target SINRs $\gamma_{q_1}, \ldots, \gamma_{q_K}$ is feasible. In this work, we assume that $\boldsymbol{H}_q$ is full-rank, $\forall q$.

It is noted that Claim 2 is only applicable where no power constraint is imposed at the BS. In practice, there exists a power bound at the BS, which then effects the feasibility of the QoS problem (4). In this work, we relax this power constraint with the assumption that the power limit at each BS is high enough to accommodate the transmissions to its connected MSs at the targeted QoS. Nonetheless, it is possible that a bounded feasible solution to the whole multicell system might not be found. Thus, our focus in this work is on the study of the boundedness and existence of the NE.

Claim 3: *For two different inter-cell interference plus noise vectors at cell-q,* $\boldsymbol{r}_{-q}$ *and* $\bar{\boldsymbol{r}}_{-q}$*, the optimal beamforming vector* $\boldsymbol{w}_{q_i}^\star$*, corresponding to* $\boldsymbol{r}_{-q}$*, is a scaled version of the optimal beamforming vector* $\bar{\boldsymbol{w}}_{q_i}^\star$*, corresponding to* $\bar{\boldsymbol{r}}_{-q}$.

Proof: It is first observed that the interference plus noise components in $\boldsymbol{r}_{-q}$ are scalars, which should not impact on the direction of the beamformers at BS-q. To support this observation, we revisit the dual uplink problem of problem (4), which turns out to be [20]

$$\begin{aligned} &\underset{\lambda_{q_1},\ldots,\lambda_{q_K}}{\text{maximize}} \quad \sum_{i=1}^{K} \lambda_{q_i} r_{-q_i} \\ &\text{subject to} \quad \sum_{j\neq i}^{K} \lambda_{q_j} \boldsymbol{h}_{qq_j} \boldsymbol{h}_{qq_j}^H + \boldsymbol{I} \succeq \frac{\lambda_{q_i}}{\gamma_{q_i}} \boldsymbol{h}_{qq_i} \boldsymbol{h}_{qq_i}^H, \ \forall i. \end{aligned} \tag{6}$$

This optimization is then equivalent to the dual uplink problem [20]

$$\begin{aligned} &\underset{\substack{\lambda_{q_1},\ldots,\lambda_{q_K} \\ \boldsymbol{w}_{q_1},\ldots,\boldsymbol{w}_{q_K}}}{\text{minimize}} \quad \sum_{i=1}^{K} \lambda_{q_i} r_{-q_i} \\ &\text{subject to} \quad \frac{\lambda_{q_i} \left|\hat{\boldsymbol{w}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \lambda_{q_j} \left|\hat{\boldsymbol{w}}_{q_i}^H \boldsymbol{h}_{qq_j}\right|^2 + \hat{\boldsymbol{w}}_{q_i}^H \hat{\boldsymbol{w}}_{q_i}} \geq \gamma_{q_i}, \ \forall i, \end{aligned} \tag{7}$$

where the optimization is over a weighted sum-power of the uplink power λ_{q_i} and the receive beamformer vectors $\hat{\boldsymbol{w}}_{q_i}$. It is noted that the optimal uplink power $\lambda_{q_i}^\star$ is only dictated by the constraints, but not the objective function. As the constraints are met with equality at optimality, the solution of λ_{q_i} can be obtained by the fixed point iteration [14]

$$\lambda_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\boldsymbol{h}_{q_i}^H \left(\sum_{j=1}^{K} \lambda_{q_j}^{(n)} \boldsymbol{h}_{qq_j} \boldsymbol{h}_{qq_j}^H + \boldsymbol{I} \right)^{-1} \boldsymbol{h}_{q_i}}. \tag{8}$$

The optimal receive beamformer vector $\hat{\boldsymbol{w}}_{q_i}^\star$ is indeed the minimum mean-square error (MMSE) receiver, i.e., $\hat{\boldsymbol{w}}_{q_i}^\star = \left(\sum_{j=1}^{K} \lambda_{q_j}^\star \boldsymbol{h}_{qq_j} \boldsymbol{h}_{qq_j}^H + \boldsymbol{I}\right)^{-1} \boldsymbol{h}_{qq_i}$. Clearly, the optimal uplink power $\lambda_{q_i}^\star$ and the optimal receive beamformer $\hat{\boldsymbol{w}}_{q_i}^\star$ are independent of r_{-q_i}. The optimal downlink beamformer vectors $\boldsymbol{w}_{q_i}^\star$ then can be found to be a scaled version of the uplink receive beamformer $\hat{\boldsymbol{w}}_{q_i}^\star$ [19]. Thus, both the optimal beamforming vector $\boldsymbol{w}_{q_i}^\star$, corresponding to $\boldsymbol{r}_{-q}$ and the vector $\bar{\boldsymbol{w}}_{q_i}^\star$, corresponding to $\bar{\boldsymbol{r}}_{-q}$, are scaled versions of $\hat{\boldsymbol{w}}_{q_i}$. This concludes the proof for this claim.

From Claim 3, it is noted that whenever the inter-cell interference plus noise vector $\boldsymbol{r}_{-q}$ is changed, BS-q only needs to adjust the allocated power for each user, but not the beam patterns to its users.[4] Thus, BS-q can determine its beam pattern first, then allocates the appropriate power to each user subject to the inter-cell interference in $\boldsymbol{r}_{-q}$.

The optimal beam patterns at each cell can be determined by solving problem (4) in the absence of inter-cell interference, which is

$$\begin{aligned} &\underset{\boldsymbol{W}_q}{\text{minimize}} \quad \sum_{i=1}^{K} \left\|\boldsymbol{w}_{q_i}\right\|^2 \\ &\text{subject to} \quad \frac{\left|\boldsymbol{w}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \left|\boldsymbol{w}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + \sigma^2} \geq \gamma_{q_i}, \ \forall i. \end{aligned} \tag{9}$$

This problem can be easily solved by the techniques mentioned in the previous section. The beamforming pattern for each user in cell-q is determined as $\tilde{\boldsymbol{w}}_{q_i} = \frac{\boldsymbol{w}_{q_i}}{\|\boldsymbol{w}_{q_i}\|}$. By Claim 3, we can restated the optimization problem (4) as

$$\begin{aligned} &\underset{p_{q_1},\ldots,p_{q_K}}{\text{minimize}} \quad \sum_{i=1}^{K} p_{q_i} \\ &\text{subject to} \quad \frac{p_{q_i} \left|\tilde{\boldsymbol{w}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} p_{q_j} \left|\tilde{\boldsymbol{w}}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i}, \ \forall i \end{aligned} \tag{10}$$

which is reduced back to a power allocation problem, where p_{q_i} is the allocated power for user-i. Note that the strategy set of player-q is redefined as

$$\mathcal{P}_q(\boldsymbol{p}_{-q}) = \left\{ \boldsymbol{p}_q \in \mathbb{R}_+^K : \mathrm{SINR}_{q_i}(\boldsymbol{p}_q, \boldsymbol{p}_{-q}) \geq \gamma_{q_i}, \ \forall i \right\}. \tag{11}$$

Obtaining the optimal solution $p_{q_i}^\star$ of problem (10), the optimal beamforming vectors corresponding to $\boldsymbol{r}_{-q}$ are $\sqrt{p_{q_i}^\star}\tilde{\boldsymbol{w}}_{q_i}$. The following claim presents the analytical solution to (10).

Claim 4: *The optimal solution of (10),* $\boldsymbol{p}_q^\star = [p_{q_1}^\star, \ldots, p_{q_K}^\star]^T$*, is given by*

$$\boldsymbol{p}_q^\star = \boldsymbol{G}_q^{-1} \boldsymbol{r}_{-q} \tag{12}$$

where $\boldsymbol{G}_q \in \mathbb{R}^{K\times K}$*, defined as* $[\boldsymbol{G}_q]_{i,i} = \left(\frac{1}{\gamma_{q_i}}\right) \left|\tilde{\boldsymbol{w}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2$ *and* $[\boldsymbol{G}_q]_{i,j} = -\left|\tilde{\boldsymbol{w}}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2$ *if* $i \neq j$*, is invertible. Furthermore,* $\boldsymbol{p}_q^\star > 0, \ \forall \boldsymbol{r}_{-q} > 0$.

Proof: Since all the SINR constraints in (10) are met with equality at optimality, they are equivalent to

$$p_{q_i}^\star \frac{\left|\tilde{\boldsymbol{w}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K} p_{q_j}^\star \left|\tilde{\boldsymbol{w}}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 = r_{-q_i}, \ i = 1, \ldots, K.$$

[4]By beam pattern, we mean the norm-1 $\frac{\boldsymbol{w}_{q_i}}{\|\boldsymbol{w}_{q_i}\|}$.

The solution of this set of K equations with K variables are the optimal solution of (10). Rewriting these K equations in matrix form, one has $\boldsymbol{G}_q\boldsymbol{p}_q^\star = \boldsymbol{r}_{-q}$. From Claim 2, as the problem (10) is always feasible $\forall \boldsymbol{r}_{-q} > 0$, $\boldsymbol{G}_q$ has to be invertible and $\boldsymbol{p}_q^\star = \boldsymbol{G}_q^{-1}\boldsymbol{r}_{-q} > 0, \forall \boldsymbol{r}_{-q} > 0$.

We proceed to denote $\boldsymbol{G}_{mq} \in \mathbb{R}^{K\times K}, m \neq q$ as the inter-cell interference matrix from cell-m to cell-q, where $[\boldsymbol{G}_{mq}]_{i,j} = \left|\tilde{\boldsymbol{w}}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2$. Then $\boldsymbol{G}_{mq}\boldsymbol{p}_m$ is the interference vector caused by BS-m to the K users of cell-q. Thus, one has $\boldsymbol{r}_{-q} = \sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m + \mathbf{1}\sigma^2$.

From Claim 4, the best response strategy of the qth cell subject to the strategy of Ω_{-q} is

$$\boldsymbol{p}_q^\star = \mathrm{BR}_q(\boldsymbol{p}_{-q}) = \boldsymbol{G}_q^{-1}\left(\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m + \mathbf{1}\sigma^2\right), \forall q. \quad (13)$$

The NEs of game $\mathcal{G}$ can now be redefined as the intersection points of the BRs, i.e.,

$$\boldsymbol{p}_q^\star = \boldsymbol{G}_q^{-1}\left(\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m^\star + \mathbf{1}\sigma^2\right), \forall q. \quad (14)$$

The next lemma shows that the best response function in (13) is *standard* [14], which guarantees the uniqueness of the NE if such the NE exists [21].

Lemma 1: *The best response function is a standard function.*

Proof: First, define $\boldsymbol{p} = [\boldsymbol{p}_1^T, \ldots, \boldsymbol{p}_Q^T]^T$, then $\mathrm{BR}_q(\boldsymbol{p}) \triangleq \mathrm{BR}_q(\boldsymbol{p}_{-q})$. We need to show that the best response function $\mathrm{BR}_q(\boldsymbol{p})$ meets the three requirements of a *standard* function:

1) *Positivity*: for any $\boldsymbol{p} \geq 0$, as $\boldsymbol{G}_{mq}$ is a positive matrix, then $\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m + \mathbf{1}\sigma^2 > 0, \forall q$. Thus, $\mathrm{BR}_q(\boldsymbol{p}) = \boldsymbol{G}_q^{-1}\left(\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m + \mathbf{1}\sigma^2\right) > 0, \forall q$, as a result from Claim 4.
2) *Monotonicity*: for $\boldsymbol{p} \geq \boldsymbol{p}'$, then

$$\mathrm{BR}_q(\boldsymbol{p}) - \mathrm{BR}_q(\boldsymbol{p}') = \boldsymbol{G}_q^{-1}\left[\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\left(\boldsymbol{p}_m - \boldsymbol{p}'_m\right)\right] \geq 0, \ \forall q$$

as a result from Claim 4 and each $\boldsymbol{p}_m - \boldsymbol{p}'_m \geq 0$.
3) *Scalability*: $\forall \epsilon > 1, \forall \boldsymbol{p} \geq 0$, one has

$$\begin{aligned}\epsilon\mathrm{BR}_q(\boldsymbol{p}) - \mathrm{BR}_q(\epsilon\boldsymbol{p}) &= \epsilon\boldsymbol{G}_q^{-1}\mathbf{1}\sigma^2 - \boldsymbol{G}_q^{-1}\mathbf{1}\sigma^2 \\ &= (\epsilon - 1)\boldsymbol{G}_q^{-1}\mathbf{1}\sigma^2 > 0.\end{aligned}$$

■

Since $\mathrm{BR}_q(\boldsymbol{p}), q = 1, \ldots, Q$, are *standard* functions, the iteration $\boldsymbol{p}_q^{(t+1)} = \mathrm{BR}_q(\boldsymbol{p}^{(t)}),\ q = 1, \ldots, Q$, will surely converge from any starting point $\boldsymbol{p}^{(0)} > 0$ to a fixed point (when it exists) [14], which is the NE of game $\mathcal{G}$ [21]. In addition, the iterative update can be implemented in a fully asynchronous manner between the cells. It is noted that due to the monotonicity of the standard function, if the fixed point does not exist, the transmit power at each BS will increase unboundedly. To this end, sufficient and necessary conditions guaranteeing the boundedness of the NE are examined. Such conditions equivalently guarantee the existence and uniqueness of the game's NE.

C. A Sufficient Condition for the Existence and Uniqueness of the NE

In this section, we consider the best response dynamic of the game as a mapping process. A sufficient condition is then presented such that the mapping is a contraction, which guarantees the existence of the fixed-point of the mapping, i.e., the NE of game $\mathcal{G}$. We summarize the obtained result in the following proposition.

Proposition 1: *The NE of game $\mathcal{G}$ exists if the spectral radius*[5]

$$(\mathrm{C}): \quad \rho(\boldsymbol{S}) < 1 \quad (15)$$

where the square matrix $\boldsymbol{S} \in \mathbb{R}^{Q\times Q}$ is defined as

$$[\boldsymbol{S}]_{q,m} = \begin{cases} 0, & \text{if} \quad m = q \\ \left\|\boldsymbol{G}_q^{-1}\boldsymbol{G}_{mq}\right\|_F, & \text{if} \quad m \neq q. \end{cases} \quad (16)$$

Proof: Let $\boldsymbol{T}_q(\boldsymbol{p}) = \mathrm{BR}_q(\boldsymbol{p})$ and $\boldsymbol{T}(\boldsymbol{p}) = \left(\mathrm{BR}_q(\boldsymbol{p})\right)_{q\in\Omega}$. Since $\boldsymbol{T}_q(\boldsymbol{p})$ is a mapping from $\mathbb{R}_+^{KQ}$ onto $\mathcal{P}_q(\boldsymbol{p}_{-q})$, which is a subset of $\mathbb{R}_+^K$, $\boldsymbol{T}(\boldsymbol{p})$ is a mapping from $\mathbb{R}_+^{KQ}$ onto a Cartesian product of Q $\mathbb{R}_+^K$ sets, i.e., $\mathbb{R}_+^{KQ}$.

We now provide some definitions of the vector norms and matrix norms applied in this work. Given a mapping $\boldsymbol{p}^{(t+1)} = \boldsymbol{T}(\boldsymbol{p}^{(t)}) : \mathbb{R}_+^{KQ} \mapsto \mathbb{R}_+^{KQ}$ and some $\boldsymbol{w} = [w_1, \ldots, w_Q]^T > 0$, the block-maximum norm of a vector [23], denoted as $\|\cdot\|_{2,\mathrm{block}}^{\boldsymbol{w}}$, is defined as

$$\left\|\boldsymbol{T}(\boldsymbol{p})\right\|_{2,\mathrm{block}}^{\boldsymbol{w}} = \max_{q\in\Omega} \frac{\|\mathrm{BR}_q(\boldsymbol{p})\|_2}{w_q} \quad (17)$$

where $\|\cdot\|_2$ is the Euclidean norm. The weighted maximum norm of a vector $\boldsymbol{x}$ for a positive vector $\boldsymbol{w}$ is defined as [23]

$$\|\boldsymbol{x}\|_{\infty,\mathrm{vec}}^{\boldsymbol{w}} = \max_{q\in\Omega} \frac{|x_q|}{w_q}, \quad \boldsymbol{x} \in \mathbb{R}^Q. \quad (18)$$

Furthermore, define the matrix norm induced by $\|\cdot\|_{\infty,\mathrm{vec}}^{\boldsymbol{w}}$ of a matrix $\boldsymbol{A}$ as [22]

$$\|\boldsymbol{A}\|_{\infty,\mathrm{mat}}^{\boldsymbol{w}} = \max_{q\in\Omega} \frac{1}{w_q} \sum_{r=1}^{Q} [\boldsymbol{A}]_{q,r} w_r, \quad \boldsymbol{A} \in \mathbb{R}^{Q\times Q}. \quad (19)$$

The mapping $\boldsymbol{T}$ is a block-contraction mapping of rate ζ, with respect to the norm $\|\cdot\|_{\infty,\mathrm{block}}^{\boldsymbol{w}}$, if there exists a nonnegative constant $\zeta < 1$ such that

$$\|\boldsymbol{T}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{p}')\|_{2,\mathrm{block}}^{\boldsymbol{w}} \leq \zeta \|\boldsymbol{p} - \boldsymbol{p}'\|_{2,\mathrm{block}}^{\boldsymbol{w}}, \ \forall \boldsymbol{p}, \boldsymbol{p}' \geq 0. \quad (20)$$

The condition $\zeta < 1$ is sufficient to guarantee the existence and uniqueness of the fixed point $\boldsymbol{p} = \boldsymbol{T}(\boldsymbol{p})$ as well as the convergence of the mapping to the fixed point. It is worth mentioning that this property has been commonly applied to the

[5]The spectral radius ρ of the matrix $\boldsymbol{S}$ is defined as $\rho(S) \triangleq \max\{|\lambda_i|\}$, with λ_i's are eigenvalues of $\boldsymbol{S}$[22].

analysis of games with best response dynamics, such as the well-known water-filling game in a multichannel system [4], [6], [7], [10]. To this end, this contraction mapping property is exploited to establish a sufficient condition on the boundedness of the power update in game $\mathcal{G}$.

Let $e_{\boldsymbol{T}_q} = \left\|\boldsymbol{T}_q(\boldsymbol{p}) - \boldsymbol{T}_q(\boldsymbol{p}')\right\|_2$ and $e_q = \left\|\boldsymbol{p}_q - \boldsymbol{p}'_q\right\|_2$, then

$$\begin{aligned} e_{\boldsymbol{T}_q} &= \left\|\mathrm{BR}_q(\boldsymbol{p}) - \mathrm{BR}_q(\boldsymbol{p}')\right\|_2 \\ &= \left\| \boldsymbol{G}_q^{-1} \left[\sum_{m\neq q}^{Q} \boldsymbol{G}_{mq} \left(\boldsymbol{p}_m - \boldsymbol{p}'_m\right) \right] \right\|_2 \\ &\leq \sum_{m\neq q}^{Q} \left\|\boldsymbol{G}_q^{-1}\boldsymbol{G}_{mq}\right\| \|\boldsymbol{p}_m - \boldsymbol{p}'_m\|_2 \end{aligned} \tag{21}$$

where the inequality is satisfied if the matrix norm $\|\cdot\|$ applied to $\boldsymbol{G}_q^{-1}\boldsymbol{G}_{mq}$ is consistent [22]. Here, we can use the Frobenius norm $\|\cdot\|_F$ since it is consistent and easy to compute. Define the vectors $\boldsymbol{e_T} = [e_{\boldsymbol{T}_1}, \ldots, e_{\boldsymbol{T}_Q}]^T$ and $\boldsymbol{e} = [e_1, \ldots, e_Q]^T$. Furthermore, define the square matrix $\boldsymbol{S} \in \mathbb{R}^{Q\times Q}$, where

$$[\boldsymbol{S}]_{q,m} = \begin{cases} 0, & \text{if } m = q \\ \left\|\boldsymbol{G}_q^{-1}\boldsymbol{G}_{mq}\right\|_F, & \text{if } m \neq q. \end{cases} \tag{22}$$

Then, one has

$$\boldsymbol{e_T} \leq \boldsymbol{S}\boldsymbol{e}. \tag{23}$$

Thus,

$$\|\boldsymbol{e_T}\|^{\boldsymbol{w}}_{\infty,\mathrm{vec}} \leq \|\boldsymbol{S}\boldsymbol{e}\|^{\boldsymbol{w}}_{\infty,\mathrm{vec}} \leq \|\boldsymbol{S}\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}}\|\boldsymbol{e}\|^{\boldsymbol{w}}_{\infty,\mathrm{vec}}, \tag{24}$$

as the induced ∞-norm $\|\cdot\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}}$ is consistent [22]. Then, one has

$$\begin{aligned} \left\|\boldsymbol{T}(\boldsymbol{p}) - \boldsymbol{T}(\boldsymbol{p}')\right\|^{\boldsymbol{w}}_{2,\mathrm{block}} &= \max_{q\in\Omega} \frac{\|\mathrm{BR}_q(\boldsymbol{p}) - \mathrm{BR}_q(\boldsymbol{p}')\|_2}{w_q} \\ &= \|\boldsymbol{e_T}\|^{\boldsymbol{w}}_{\infty,\mathrm{vec}} \\ &\leq \|\boldsymbol{S}\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}}\|\boldsymbol{e}\|^{\boldsymbol{w}}_{\infty,\mathrm{vec}} \\ &= \|\boldsymbol{S}\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}}\, \|\boldsymbol{p} - \boldsymbol{p}'\|^{\boldsymbol{w}}_{2,\mathrm{block}}. \end{aligned} \tag{25}$$

Thus, if $\|\boldsymbol{S}\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}} < 1$, the existence and the convergence to the fixed point are guaranteed because the mapping in (25) is a contraction. It is noted that $\boldsymbol{S}$ is a nonnegative matrix, there exists a positive vector $\boldsymbol{w}$ such that [23]

$$\|\boldsymbol{S}\|^{\boldsymbol{w}}_{\infty,\mathrm{mat}} < 1 \quad \Leftrightarrow \quad \rho(\boldsymbol{S}) < 1. \tag{26}$$

■

Remarks: A physical interpretation of the sufficient condition (C) is as follows. Assuming the path loss fading model $\boldsymbol{h}_{mq_i} = \bar{\boldsymbol{h}}_{mq_i} d_{mq_i}^{-\beta}$, where $\bar{\boldsymbol{h}}_{mq_i}$ contains normalized i.i.d. $\mathcal{CN}(0,1)$ channel gains, d_{mq_i} is the distance between BS-m to user-i of cell-q and β is the path loss exponent. When the distance d_{mq_i} increases, the cross channel gains $\boldsymbol{h}_{mq_i}$, $m \neq q$ are smaller, which result in a smaller cross interference matrix $\boldsymbol{G}_{mq}$. Thus, the positive off-diagonal elements of $\boldsymbol{S}$ also become smaller. This results in a smaller spectral radius of $\boldsymbol{S}$. Thus, the more apart a MS from the BSs of other cells, the higher chance of $\rho(\boldsymbol{S})$ being less than 1, which then guarantees the existence and uniqueness of the NE.

It is noted that while the sufficient condition (C) is obtained from the contraction mapping property of the power update, the direct characterization of the NE can be utilized to draw the necessary condition of the NE existence. We consider this necessary condition in the following section.

D. The Necessary Condition for the Existence and Uniqueness of the NE

This section is to examine the necessary condition for the existence and uniqueness of the NE of game $\mathcal{G}$. Before proceeding, we provide some definitions of related mathematic terms to be used in the section.

Definition 1 [24], [25]: *A square matrix $\boldsymbol{A}$ is a $\boldsymbol{Z}$-matrix if all its off-diagonal elements are nonpositive. A square matrix $\boldsymbol{A}$ is a $\boldsymbol{P}$-matrix if all its principle minors are positive. A square matrix that is both a $\boldsymbol{Z}$-matrix and a $\boldsymbol{P}$-matrix is called an $\boldsymbol{M}$-matrix.*

Besides the above definition, there are several equivalent characterizations of a $\boldsymbol{M}$-matrix [24]. It is noted that if a matrix ia a $\boldsymbol{M}$-matrix, it is invertible and the inverse is a positive matrix [24].

Proposition 2: *The NE of game $\mathcal{G}$ exists if and only if the following matrix:*

$$\text{(C1)}: \quad \boldsymbol{G} = \begin{bmatrix} \boldsymbol{G}_1 & -\boldsymbol{G}_{21} & \cdots & -\boldsymbol{G}_{Q1} \\ -\boldsymbol{G}_{12} & \boldsymbol{G}_2 & \cdots & -\boldsymbol{G}_{Q2} \\ \vdots & \vdots & \ddots & \vdots \\ -\boldsymbol{G}_{1Q} & -\boldsymbol{G}_{2Q} & \ldots & \boldsymbol{G}_Q \end{bmatrix} \tag{27}$$

is a $\boldsymbol{M}$-matrix.

Proof: First, if the NE of game $\mathcal{G}$ exists, then an intersection point of the BR functions must exist. At the NE, (14) is equivalent to

$$\boldsymbol{G}_q\boldsymbol{p}_q^\star - \sum_{m\neq q}^{Q} \boldsymbol{G}_{mq}\boldsymbol{p}_m^\star = \mathbf{1}\sigma^2,\ \forall q.$$

Reorganizing the above set of equations into a matrix form, one has

$$\boldsymbol{G}\boldsymbol{p}^\star = \mathbf{1}\sigma^2,$$

where $\boldsymbol{G}$ is previously defined. Note that $\boldsymbol{G}$ is a $\boldsymbol{Z}$-matrix, as its off-diagonal elements are all nonpositive. Since there exists $\boldsymbol{p}^\star > 0$ to make $\boldsymbol{G}\boldsymbol{p}^\star = \mathbf{1}\sigma^2 > 0$, this implies $\boldsymbol{G}$ being a $\boldsymbol{M}$-matrix by its characterization [24, Condition I_{28}, Theorem 6.2.3].

Conversely, if $\boldsymbol{G}$ is a $\boldsymbol{M}$-matrix, its inverse exists and is a positive matrix [24]. Thus, there exists a vector $\boldsymbol{p}^\star = \boldsymbol{G}^{-1}\mathbf{1}\sigma^2 > 0$ and $\boldsymbol{p}^\star$ satisfies the condition of being an intersection point of the BR functions in (14). As a result, a NE must exist. ■

As previously mentioned, there are various characterizations of a $\boldsymbol{M}$-matrix [24] that one can utilize to verify whether matrix $\boldsymbol{G}$ is one of the type.

IV. A Comparison to the Coordinated Design

In Section III, we considered the fully decentralized approach in the multicell downlink design and established the NE of the system. However, it is well-known that the NE needs not to be Pareto-efficient [26]. Via the coordination between the BSs, significant power reduction can be obtained by jointly designing all the beamformers at the same time. Nonetheless, this advantage may come with the cost of message passing between the BSs as explained later in this section. To this end, we review such a fully coordinated multicell downlink beamforming system [13], where the weighted total transmit power of all the cells is jointly minimized. A comparison between the two designs is presented in the end of the section.

Let $\boldsymbol{u}_{q_i}$ be the beamforming vector for user-i of cell-q with the coordinated design and let $\boldsymbol{U}_q = [\boldsymbol{u}_{q_1}, \ldots, \boldsymbol{u}_{q_K}]$. The problem to jointly minimize the weighted total transmit power of the Q cells is stated as follows:

$$\begin{aligned}&\underset{\boldsymbol{U}_1,\ldots,\boldsymbol{U}_Q}{\text{minimize}} \quad \sum_{q=1}^{Q} \alpha_q \left\|\boldsymbol{U}_q\right\|_F^2 \\ &\text{subject to} \quad \frac{\left|\boldsymbol{u}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K}\left|\boldsymbol{u}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\left|\boldsymbol{u}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2 + \sigma^2} \geq \gamma_{q_i}\end{aligned} \tag{28}$$

where α_q is the weight factor at BS-q and $\sum_{q=1}^{Q}\alpha_q = 1$. For a given $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_Q] \geq 0$, the optimal solution to (28) represents an optimal trade-off point between each cell's power consumption. Certainly, this point is Pareto-optimal, i.e., one cannot further reduce the power consumption at one cell without increasing the power consumption at (one or more) other cells.

The optimization problem (28) is convex, since the objective function is convex and the SINR constraints can be transformed into convex second order conic (SOC) constraints [13]. Thus, its optimal solution can be obtained from any conic solution package or standard convex optimization algorithm. This approach, however, is fully centralized. On the other hand, by exploiting the dual problem, this problem can be solved in a distributed fashion with message passing between the BSs. Via the Lagrangian technique, the dual problem of (28) is equivalent to the virtual dual uplink problem [13]

$$\begin{aligned}&\underset{\{\nu_{q_i}\},\{\hat{\boldsymbol{u}}_{q_i}\}}{\text{minimize}} \quad \sum_{q=1}^{Q}\sum_{i=1}^{K} \nu_{q_i}\sigma^2 \\ &\text{subject to} \quad \frac{\nu_{q_i}\left|\hat{\boldsymbol{u}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{m=1}^{Q}\sum_{j=1}^{K}\nu_{m_j}\left|\hat{\boldsymbol{u}}_{q_i}^H \boldsymbol{h}_{qm_j}\right| + \alpha_q \hat{\boldsymbol{u}}_{q_i}^H \hat{\boldsymbol{u}}_{q_i}} \geq \frac{\gamma_{q_i}}{1+\gamma_{q_i}}\end{aligned} \tag{29}$$

where the optimization is taken over the sum transmit power of the uplink power ν_{q_i} and the receive beamformer vectors $\hat{\boldsymbol{u}}_{q_i}$, $\forall i$, $\forall q$. Note that the optimal solution of the uplink power ν_{q_i} can be obtained iteratively to a fixed point [13], where

$$\nu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \cdot \frac{1}{\boldsymbol{h}_{qq_i}^H \left(\boldsymbol{\Sigma}_q\left(\{\nu_{m_j}^{(n)}\}\right)\right)^{-1} \boldsymbol{h}_{qq_i}} \tag{30}$$

with

$$\boldsymbol{\Sigma}_q\left(\{\nu_{m_j}^{(n)}\}\right) = \sum_{m=1}^{Q}\sum_{j=1}^{K} \nu_{m_j}^{(n)} \boldsymbol{h}_{qm_j}\boldsymbol{h}_{qm_j}^H + \alpha_q \boldsymbol{I}.$$

This iteration function is shown to be *standard* [13], which guarantees its convergence to a unique solution, if the problem is feasible [14]. It is noted that the feasibility study of this coordinated design has not been done in [13]. However, if the NE of the competitive design exists, the coordinated design must be feasible. Similar to the single-cell problem, given the optimal uplink transmit power $\nu_{q_i}^\star$, the optimal receive beamformers $\hat{\boldsymbol{u}}_{q_i}$ is the MMSE receiver, i.e.,

$$\hat{\boldsymbol{u}}_{q_i} = \left(\boldsymbol{\Sigma}_q\left(\{\nu_{m_j}^\star\}\right)\right)^{-1} \boldsymbol{h}_{qq_i}. \tag{31}$$

In addition, the optimal beamformer of the coordinated design, $\boldsymbol{u}_{q_i}$, can be found as a scaled version of $\hat{\boldsymbol{u}}_{q_i}$ by a factor $\sqrt{\delta_{q_i}}$ [13], i.e., $\boldsymbol{u}_{q_i} = \sqrt{\delta_{q_i}}\hat{\boldsymbol{u}}_{q_i}$. As all the SINR constraints in (28) are met with equality at optimality, substitute $\boldsymbol{u}_{q_i} = \sqrt{\delta_{q_i}}\hat{\boldsymbol{u}}_{q_i}$, the KQ SINR constraints can be written as

$$\begin{aligned}\delta_{q_i}\frac{\left|\hat{\boldsymbol{u}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K}\delta_{q_j}\left|\hat{\boldsymbol{u}}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 \\ - \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\delta_{m_j}\left|\hat{\boldsymbol{u}}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2 = \sigma^2.\end{aligned} \tag{32}$$

In order to find δ_{q_i}, one needs to solve this set of KQ equations. Define a matrix $\boldsymbol{F}$ of size $KQ \times KQ$ and its components as

$$[\boldsymbol{F}]_{K(q-1)+i,K(m-1)+j} = \begin{cases} \frac{\left|\hat{\boldsymbol{u}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\gamma_{q_i}}, & \text{if } q=m, i=j \\ -\left|\hat{\boldsymbol{u}}_{m_j}^H \boldsymbol{h}_{mq_i}\right|^2, & \text{otherwise}\end{cases}$$

with $i,j = 1,\ldots,K$ and $q,m = 1,\ldots,Q$. Then, one has

$$\boldsymbol{\delta} = [\delta_{1_1}, \ldots, \delta_{1_K}, \ldots, \delta_{Q_1}, \ldots, \delta_{Q_K}]^T = \boldsymbol{F}^{-1}\boldsymbol{1}\sigma^2. \tag{33}$$

In summary, with the fully coordinated multicell system, the following three-step algorithm is needed to find the jointly optimal beamformers [13]:

i) Fixed-point iteration (30) to find $\nu_{q_i}^\star$.
ii) Find the receive beamformer $\hat{\boldsymbol{u}}_{q_i}$ for the dual uplink channel as in (31).
iii) Find the scaling factor δ_{q_i}.

In [13], the authors argued that the above algorithm can be implemented in a distributed manner under the condition on channel reciprocals. More specifically, when the uplink and downlink channels are reciprocals of each other, the virtual dual uplink is the real uplink. Thus, the iteration (30) in step i) can be performed locally at each BS with local information.

In particular, at BS-q, $\boldsymbol{h}_{qq_i}$ is typically known and $\boldsymbol{\Sigma}_q$ is effectively the covariance matrix of the received signal in the uplink direction. In step ii), the receive beamformer $\hat{\boldsymbol{u}}_{q_i}$ can easily obtained. Finally, step iii) can be implemented iteratively where each δ_{q_i} is determined locally to meet its corresponding SINR target (assuming all other δ_{q_i}'s are fixed) until convergence. In overall, the distributed implementation of the coordinated design requires channel reciprocals and the synchronization between the BSs to be in the uplink phase or the downlink phase together [13]. It is worth mentioning that in practice the uplink and downlink channels usually operate in separate frequency bands in the frequency-division duplexing (FDD) mode. Channel reciprocals are therefore hard to realize.

Obviously, if the condition on channel reciprocals is not true, each BS in the coordinated design needs to know all the channels from itself to all the MSs in the system. A message passing scheme between the cells is then required to jointly update the dual variables ν_{q_i}'s. In addition, certain synchronization between BSs is desired, i.e., step iii). These are the main differences to the competitive design considered previously in this work, where the beamforming design is performed locally at each cell without any message exchanges and synchronization. These differences prompt us to investigate a new game that retains the advantages of the power minimization game $\mathcal{G}$, i.e., fully distributed implementation, no message passing and no synchronization and possibly approaches the performance established by the coordinated design. We address this concern in the next session.

V. The Multicell Downlink Beamforming Game With Pricing Consideration

A. Problem Formulation

We begin this section with a numerical example of the power consumption at a two-cell system with both the coordinated and competitive designs. Considered is the system with two cells, two MSs per cell with the target SINR $\gamma_{q_i} = 10$ (10 dB). It is assumed that each BS is equipped with 3 antennas and distance between the two BSs is 1. Each MS is located between the two BSs at a distance $d = 0.3$ from its connected BS. All the intra-cell and inter-cell channel coefficients are generated from i.i.d. Gaussian random variables, using the path loss model with the path loss exponent of 2.5. The background noise power σ^2 is 0.01. Fig. 2 displays the power consumption at the two BS with the competitive design, i.e., the NE point of game $\mathcal{G}$, relatively to the coordinated design. It should be noted that the power consumption at the cell is lower-bounded by the minimum power requirement to meet its users' SINR target, in the absence of inter-cell interference. It can be drawn from the figure that the NE point of the competitive design is rather inefficient, compared to the Pareto-optimal curve established by the coordinated design.

An interesting question here is whether one can modify the utility function at each player such that the game becomes more cooperative and its equilibria possibly lie on the boundary of the Pareto-optimal tradeoff surface, e.g., point "○" in Fig. 2. In this section, we study a new game with pricing consideration, namely game $\mathcal{G}'$. By introducing a pricing component to

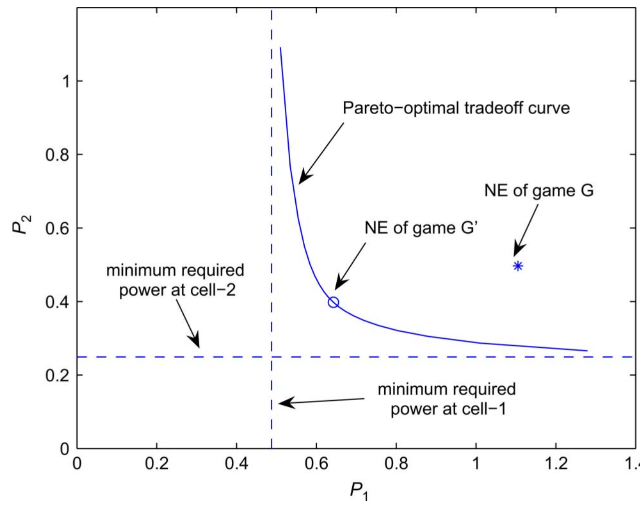

Fig. 2. Power consumption in two cells: competitive design versus coordinated design.

each player's utility function, the players now voluntarily cooperate with others by minimizing their inducing interference to the others as well as minimizing their own transmit power at the same time. In fact, it shall be shown that the point "○" can be obtained at the NE of the modified game $\mathcal{G}'$.

Now, suppose that BS-q has additional information about the channel to the users in other cells, it performs the following optimization:

$$\begin{aligned} \underset{\boldsymbol{V}_q}{\text{minimize}} \quad & \sum_{i=1}^{K} \|\boldsymbol{v}_{q_i}\|^2 + \sum_{m\neq q}^{Q}\sum_{j=1}^{K} \pi_{qm_j} \left\|\boldsymbol{V}_q^H \boldsymbol{h}_{qm_j}\right\|^2 \\ \text{subject to} \quad & \frac{\left|\boldsymbol{v}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K} \left|\boldsymbol{v}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2 + r_{-q_i}} \geq \gamma_{q_i},\ \forall i \end{aligned} \tag{34}$$

where $\pi_{qm_j} \geq 0$ is the pricing factor and $\left\|\boldsymbol{V}_q^H \boldsymbol{h}_{qm_j}\right\|^2$ is the interference at user-j of cell-m, caused by BS-q.[6]

It is noted that unlike the coordinated design in Section IV, this downlink beamforming game can be implemented at the system where partial information is available. More specifically, if the channel to user-i at cell-m is known at BS-q, a pricing factor $\pi_{qm_j} > 0$ is set to motivate cell-q adopting a more sociable strategy by steering its beamformers to the directions that cause less interference (damage) to other cells. Otherwise, the pricing factor π_{qm_j} is set to 0. Certainly, the nature of the game being played between the players is no longer purely competitive. Through pricing, it is possible to improve system performance by inducing cooperation between the players and yet maintaining the decentralized nature of the game. In general, the pricing factors π_{qm_j} should be tuned such that a largest possible enhancement in the overall system is obtained [2]. In a dynamic pricing scheme, the pricing factors can be jointly decided and constantly exchanged between the BSs. However, in

[6]To avoid any confusion with the notation $\boldsymbol{w}_{q_i}$ used in Section III and $\boldsymbol{u}_{q_i}$ used in Section IV, we denote $\boldsymbol{v}_{q_i}$ as the beamformer for user-i of cell-q in the competitive design with pricing consideration within this section. Similarly, $\boldsymbol{V}_q$ is used instead of $\boldsymbol{W}_q$ and $\boldsymbol{U}_q$. Likewise, let $\boldsymbol{q}_q \in \mathbb{R}^K$ denote the allocated power vector for the K users in cell-q, instead of $\boldsymbol{p}_q$ in Section III.

order to reduce the system overhead, it is assumed that the prices are chosen a prior and fixed during the game being played. This assumption may be motivated in the system with a system designer, who informs prices to the players in advance.

The game with pricing consideration is practically the same game as $\mathcal{G}$ with different payoff function. Mathematically, the new game is defined as

$$\mathcal{G}' = \left(\Omega, \{\mathcal{P}_q(\boldsymbol{V}_{-q})\}_{q\in\Omega}, \{s_q(\boldsymbol{V}_q)\}_{q\in\Omega}\right)$$

where $s_q(\boldsymbol{V}_q) = t_q(\boldsymbol{V}_q) + \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\pi_{qm_j}\left\|\boldsymbol{V}_q^H\boldsymbol{h}_{qm_j}\right\|^2$ is the utility function at player-q. Our interest in this part is to study whether game $\mathcal{G}'$ eventually converges to an NE and whether the NE is unique. A feasible strategy profile $\{\boldsymbol{V}_q^\star\}_{q=1}^{Q}$ is a NE of game $\mathcal{G}'$ if

$$s_q(\boldsymbol{V}_q^\star) \leq s_q(\boldsymbol{V}_q),\ \forall \boldsymbol{V}_q \in \mathcal{P}_q(\boldsymbol{V}_{-q}^\star), \quad \forall q\in\Omega. \tag{35}$$

B. Existence and Uniqueness of the Nash Equilibrium

This section studies the existence and uniqueness of the NE of the new game $\mathcal{G}'$. First of all, given the strategy of other player $\boldsymbol{V}_{-q}$, we study the optimal strategy for player-q, i.e., solving the optimization problem (34). Note that problem(34) is convex, as the constraints are SOC and the objective function is quadratic. This useful observation enables us to find its optimal solution via convex optimization. In addition, uplink-downlink duality can be exploited to devise the optimal solution for this problem. The following theorem establishes the analytical steps to find the solution.

Theorem 1: The optimal transmit beamforming problem (34) can be solved via a dual virtual uplink channel

$$\begin{aligned} &\underset{\substack{\mu_{q_1},\ldots,\mu_{q_K}\\ \hat{\boldsymbol{v}}_{q_1},\ldots,\hat{\boldsymbol{v}}_{q_K}}}{\text{minimize}} \quad \sum_{i=1}^{K}\mu_{q_i}r_{-q_i}\\ &\text{subject to} \quad \frac{\mu_{q_i}\left|\hat{\boldsymbol{v}}_{q_i}^H\boldsymbol{h}_{qq_i}\right|^2}{\sum_{j\neq i}^{K}\mu_{q_j}\left|\hat{\boldsymbol{v}}_{q_i}^H\boldsymbol{h}_{qq_j}\right|^2 + \hat{\boldsymbol{v}}_{q_i}^H\boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right)\hat{\boldsymbol{v}}_{q_i}} \geq \gamma_{q_i},\ \forall i \end{aligned} \tag{36}$$

where $\boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right) = \sum_{m\neq q}^{Q}\sum_{j=1}^{K}\pi_{qm_j}\boldsymbol{h}_{qm_j}\boldsymbol{h}_{qm_j}^H + \boldsymbol{I}$ is treated as the noise covariance matrix at the BS and the optimization is taken over the weighted sum-power of the uplink power μ_{q_i} and the receive beamformer vectors $\hat{\boldsymbol{v}}_{q_i}$. The optimal $\boldsymbol{v}_{q_i}$ is a scaled version of the optimal $\hat{\boldsymbol{v}}_{q_i}$.

Proof: The proof of this theorem is based on the Lagrangian technique and similar to the one in [20]. The Lagrangian of (34) can be reorganized as

$$\mathcal{L}_q(\boldsymbol{V}_q,\boldsymbol{\mu}_q) = \sum_{i=1}^{K}\mu_{q_i}r_{-q_i} + \sum_{i=1}^{K}\boldsymbol{v}_{q_i}^H\left(\boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right) - \frac{\mu_{q_i}}{\gamma_{q_i}}\boldsymbol{h}_{qq_i}\boldsymbol{h}_{qq_i}^H + \sum_{j\neq i}^{K}\mu_{q_j}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H\right)\boldsymbol{v}_{q_i}$$

where $\boldsymbol{\mu}_q = [\mu_{q_1},\ldots,\mu_{q_K}]^T$'s are the Lagrangian multipliers associated with SINR constraints. The dual objective function is defined as

$$g_q(\boldsymbol{\mu}_q) = \min_{\boldsymbol{V}_q}\mathcal{L}_q(\boldsymbol{V}_q,\boldsymbol{\mu}_q).$$

Obviously, if $\boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right) - \frac{\mu_{q_i}}{\gamma_{q_i}}\boldsymbol{h}_{qq_i}\boldsymbol{h}_{qq_i}^H + \sum_{j\neq i}^{K}\mu_{q_j}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H$ is not positive semi-definite, there exists a set of $\boldsymbol{v}_{q_i}$ to make g_q unbounded below. Thus, the dual problem of (34) is

$$\begin{aligned} &\underset{\mu_{q_1},\ldots,\mu_{q_K}}{\text{maximize}} \quad \sum_{i=1}^{K}\mu_{q_i}r_{-q_i}\\ &\text{subject to} \quad \sum_{j=1}^{K}\mu_{q_j}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right)\\ &\qquad\qquad \succeq \left(1+\frac{1}{\gamma_{q_i}}\right)\mu_{q_i}\boldsymbol{h}_{qq_i}\boldsymbol{h}_{qq_i}^H,\ \forall i. \end{aligned} \tag{37}$$

Similar to the technique used to solve the traditional downlink beamforming problem [19], as mentioned at (6), the dual problem (37) is equivalent to the virtual uplink problem given in (36). Once again, the fixed point iteration can be utilized to find the optimal solution of μ_{q_i} as follows:

$$\mu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \times \frac{1}{\boldsymbol{h}_{qq_i}^H\left(\sum_{j=1}^{K}\mu_{q_j}^{(n)}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right)\right)^{-1}\boldsymbol{h}_{qq_i}}. \tag{38}$$

Using the *standard* function property, this iteration is guaranteed to converged to unique fixed-point if the primal problem (34) is feasible.[7]

The optimal receive beamformer for the virtual uplink channel is indeed the MMSE receiver

$$\hat{\boldsymbol{v}}_{q_i} = \left(\sum_{j=1}^{K}\mu_{q_j}^{(n)}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H + \boldsymbol{\Upsilon}_q\left(\{\pi_{qm_j}\}\right)\right)^{-1}\boldsymbol{h}_{qq_i}. \tag{39}$$

In addition, using the similar technique as in [20], it can be shown that $\boldsymbol{v}_{q_i}$ is a scaled version of $\hat{\boldsymbol{v}}_{q_i}$, i.e., $\boldsymbol{v}_{q_i} = \sqrt{\varepsilon_{q_i}}\hat{\boldsymbol{v}}_{q_i}$ where $\sqrt{\varepsilon_{q_i}}$ is the scaling factor. One can find the scaling factor by noticing that all the SINR constraints in (34) are met with equality at optimality. Substitute $\boldsymbol{v}_{q_i} = \sqrt{\varepsilon_{q_i}}\hat{\boldsymbol{v}}_{q_i}$ into the SINR constraints, one can rewrite them as

$$\varepsilon_{q_i}\frac{\left|\hat{\boldsymbol{v}}_{q_i}^H\boldsymbol{h}_{qq_i}\right|^2}{\gamma_{q_i}} - \sum_{j\neq i}^{K}\varepsilon_{q_j}\left|\hat{\boldsymbol{v}}_{q_j}^H\boldsymbol{h}_{qq_i}\right|^2 = r_{-q_i},\ i=1,\ldots,K. \tag{40}$$

Define $\boldsymbol{\varepsilon}_q = [\varepsilon_{q_1},\ldots,\varepsilon_{q_K}]^T$, then $\boldsymbol{\varepsilon}_q = \boldsymbol{E}^{-1}\boldsymbol{r}_{-q}$, where $\boldsymbol{E}\in\mathbb{R}^{K\times K}$ is defined as $[\boldsymbol{E}]_{i,i} = \left(\frac{1}{\gamma_{q_i}}\right)\left|\hat{\boldsymbol{v}}_{q_i}^H\boldsymbol{h}_{qq_i}\right|^2$ and $[\boldsymbol{E}]_{i,j} = -\left|\hat{\boldsymbol{v}}_{q_j}^H\boldsymbol{h}_{qq_i}\right|^2$ if $i\neq j$.

[7]The proof for this function to be *standard* is similar to the ones in [19].

Having solved problem(34), it is clear that its solution resembles the solution of the typical downlink beamforming problem(4). Thus, many properties of problem(4) 's solution, i.e., Claims 1–4, also hold. We summarize this observation in the following lemma.

Lemma 2: *Given fixed pricing factors π_{qm_j}'s, Claims 1–4 associated with the solution of problem (4) are also applicable to the solution of problem (34).*

Proof: Claim 1 is straightforward. As the feasibility depends only on the constraints, if problem (4) is feasible, problem (34) is also feasible. Thus, Claim 2 also holds. Claim 3 comes directly from the fact that the solution $\boldsymbol{v}_{q_i}$ of problem (4) is a scaled version of $\hat{\boldsymbol{v}}_{q_i}$ given in (39). Claim 4 also holds, following the same procedure given in Section III-B. That is, given $\boldsymbol{r}_{-q}$ as the total interference induced by Ω_{-q} plus background noise and $\tilde{\boldsymbol{v}}_{q_i}$ as the beam pattern corresponding to π_{qm_j}, the optimal allocated power vector $\boldsymbol{q}_q \in \mathbb{R}^K$ for the K users at BS-q is $\boldsymbol{q}_q = \boldsymbol{K}^{-1}\boldsymbol{r}_{-q}$, where $\boldsymbol{K} \in \mathbb{R}^{K\times K}$ is defined as $[\boldsymbol{K}]_{i,i} = \left(\frac{1}{\gamma_{q_i}}\right)\left|\tilde{\boldsymbol{v}}_{q_i}^H \boldsymbol{h}_{qq_i}\right|^2$ and $[\boldsymbol{K}]_{i,j} = -\left|\tilde{\boldsymbol{v}}_{q_j}^H \boldsymbol{h}_{qq_i}\right|^2$ if $i \neq j$. ■

Denote $\boldsymbol{K}_{mq} \in \mathbb{R}^{K\times K}, m \neq q$ as the inter-cell interference matrix, where $[\boldsymbol{K}_{mq}]_{i,j} = |\tilde{\boldsymbol{v}}_{m_j}^H \boldsymbol{h}_{mq_i}|^2$. Then, one has $\boldsymbol{r}_{-q} = \sum_{m\neq q}^{Q} \boldsymbol{K}_{mq}\boldsymbol{q}_m + \boldsymbol{1}\sigma^2$. From Lemma 2, subject to the strategy of Ω_{-q}, the best response strategy of the player-q with pricing consideration is

$$\boldsymbol{q}_q^\star = \mathrm{BR}'_q(\boldsymbol{q}_{-q}) = \boldsymbol{K}_q^{-1}\left(\sum_{m\neq q}^{Q} \boldsymbol{K}_{mq}\boldsymbol{q}_m + \boldsymbol{1}\sigma^2\right). \quad (41)$$

Lemma 3: *With pricing consideration, the best response function of player-q is standard.*

Proof: The proof is the same as that in Lemma 1. ■

Since the best response function $\mathrm{BR}'_q(\boldsymbol{q}_{-q})$ is *standard*, from any starting point $\boldsymbol{q}^{(0)}$, the iteration $\boldsymbol{q}_q^{(t+1)} = \mathrm{BR}'_q(\boldsymbol{q}_{-q}^{(t)})$ will surely converge to a fixed point (if it exists), which is the NE of game $\mathcal{G}'$. The necessary condition for the existence of the NE in game $\mathcal{G}'$ is similar to that of game $\mathcal{G}$, established in Proposition 2, i.e., the matrix $\boldsymbol{K} \in \mathbb{R}^{KQ\times KQ}$, in the same form as $\boldsymbol{G}$ in (27) with $\boldsymbol{K}_q$ and $\boldsymbol{K}_{mq}$ replacing $\boldsymbol{G}_q$ and $\boldsymbol{G}_{mq}$, is a $\boldsymbol{M}$-matrix.

Likewise, thanks to Proposition 1, if $\rho(\boldsymbol{S}') < 1$, where $\boldsymbol{S}' \in \mathbb{R}^{K\times K}$ is defined as $[\boldsymbol{S}']_{qq} = 0$ and $[\boldsymbol{S}']_{q,m} = \left\|\boldsymbol{K}_q^{-1}\boldsymbol{K}_{mq}\right\|_F$ if $m \neq q$, game $\mathcal{G}'$ is also guaranteed to admit a unique NE.

To this point, one may wonder how efficient the NE of game $\mathcal{G}'$, compared to the NE of game $\mathcal{G}$ and the Pareto-optimal tradeoff curve. Although this work does not present a concrete proof to the claim that the NE of game $\mathcal{G}'$ is more efficient than that of game $\mathcal{G}$, all simulations show that with right pricing factors, this claim is true. In fact, with a certain pricing scheme deployed at all the BSs, the NE of game $\mathcal{G}'$ is able to approach the Pareto-optimal tradeoff curve. The characterization of this pricing scheme is given in the following.

Theorem 2: *Given the weight vector $\boldsymbol{\alpha}$ for the coordinated design and suppose that $\nu_{q_i}^\star$'s are the dual variables that satisfy the iteration (30), if the weight factors for game $\mathcal{G}'$ are set as*

$$\pi_{qm_j} = \frac{\nu_{m_j}^\star}{\alpha_q}, \quad \forall i, \forall q \quad (42)$$

the NE of game $\mathcal{G}'$ is exactly the solution of the coordinated design. That is, the NE lies on the Pareto-optimal tradeoff curve.

Proof: When the weight factors for game $\mathcal{G}'$ are set as in (42), the fixed-point iteration(38) becomes

$$\alpha_q\mu_{q_i}^{(n+1)} = \frac{\gamma_{q_i}}{1+\gamma_{q_i}} \times \frac{1}{\boldsymbol{h}_{qq_i}^H\left(\sum_{j=1}^{K}\alpha_q\mu_{q_j}^{(n)}\boldsymbol{h}_{qq_j}\boldsymbol{h}_{qq_j}^H + \alpha_q\boldsymbol{\Upsilon}_q\left(\left\{\frac{\nu_{m_j}^\star}{\alpha_q}\right\}\right)\right)^{-1}\boldsymbol{h}_{qq_i}}. \quad (43)$$

Comparing to fixed-point iteration (30), it is obvious that the unique fixed-point of the above iteration satisfies $\alpha_q\mu_{q_i}^\star = \nu_{q_i}^\star$. As a result, $\hat{\boldsymbol{v}}_{q_i} = \alpha_q\hat{\boldsymbol{u}}_{q_i}$. That is, the beam pattern set for the users of game $\mathcal{G}'$ is the same as the beam pattern set in the coordinated design. Thus, it is left to determine that the beamformers of the two designs are indeed the same. Note that the coordinated design determines the scaling factor δ_{q_i} to $\hat{\boldsymbol{u}}_{q_i}$ by either using matrix inversion, cf. (33), or each MS sets a δ_{q_i} to meet its corresponding SINR constraint assuming all other δ_{q_i}'s are fixed [13]. The convergence of the second method can be proved by the *standard* function technique [13], which effectively explains the convergence to a unique fixed-point. On the other hand, at each time instance, BS-q in game $\mathcal{G}'$ determines the allocated powers, equivalently the scaling factors to $\hat{\boldsymbol{v}}_{q_i}$, to satisfy the SINR constraints at its users. Due to the uniqueness of the fixed-point, the game played in $\mathcal{G}'$ has to converge to the same solution as the coordinated design. ■

Theorem 2 is significant in the sense that the fully coordinated design can still be interpreted as a competitive game with the *right* pricing scheme. Our next task is to study how to implement such the pricing scheme, under two game scenarios: i) game with complete information and ii) game with incomplete information. In a game with complete information, it is assumed that the system designer knows all the game parameters, including the channels and the QoS requirements. Each BS is also assumed to fully know its channels to all the MSs. The designer then can exactly decide the optimal dual variables $\{\nu_{q_i}^\star\}$, which are used to determine the optimal prices in (42). As stated in Theorem 2, the game with pricing consideration will be Pareto-optimal. Interestingly, it has been recently shown in [27] that pricing can allow the system designer to locate the NE point to any feasible point in a broad class of power allocation games under complete information. Our result given in Theorem 2 has established a similar result for the beamforming and power allocation game in a multicell system.

In a game with incomplete information, it is assumed that neither the BSs nor the system designer fully know all the channels. Thus, implementing the pricing scheme (42) is no longer possible. In this case, the designer may help the BSs to search for

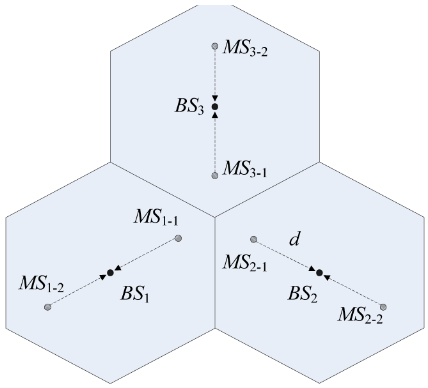

Fig. 3. A multicell system configuration with three cells, two users per cell. Of the two users at each cell, one stays close to the borders with other cells, one is far away.

the good pricing factors. Here, we employ the same mechanism exploited in [2] due to its simplicity. More specifically, the designer lets the game $\mathcal{G}$ (no pricing) play and obtain its NE. Then, each BS sets its pricing factors π_{qm_j} to a same value $c > 0$ (initially large), which is informed by the designer and lets game $\mathcal{G}'$ play. After dividing the pricing factor c by a positive factor of Δc, game $\mathcal{G}'$ is played again and its NE is re-measured. The procedure is repeated if the sum of the utility functions at the new equilibrium is smaller than that of the previous instance. Otherwise, the procedure is stopped and the all the pricing factors are set to the same factor, called c_{BEST}. As shall be shown in the simulation, this technique performs very well in improving the NE efficiency.

VI. Numerical Results

This section presents some numerical results validating our findings. In particular, we compare the feasibility of the coordinated design and the probability of existence of a NE of games $\mathcal{G}$ and $\mathcal{G}'$. Also compared are the average total transmit powers (of all the cells) of the three designs. We consider a multicell network as illustrated in Fig. 3, composed of 3 cells with 2 users per cell. It is assumed that the BSs are equidistant and their distance is normalized to 1. The distance between a MS and its serving BS is also set the same, at d. Of the two MSs at each cell, one is located closely to the borders with other cells, whereas the second one is far away. The same target SINRs are set at each MS, either $\gamma_{q_i} = 0$ dB or $\gamma_{q_i} = 10$ dB. The AWGN power spectral density σ^2 is set at 0.01. The intra-cell and inter-cell channel coefficients are generated from i.i.d. Gaussian random variables using the path loss model with the path loss exponent $\beta = 2.5$. As we vary the distance d, 100 000 channel realizations at each value d are used to plot the probability of the existence of a stable operating point in Fig. 4 and the average total transmit power in Fig. 5.

In the competitive design with pricing consideration, it is assumed that each BS also knows the two nearly MSs at the other two cells. For example, base-station BS-1 knows the its channels to MS-2_1 and MS-3_1. Certainly, these two MSs are subject to a much higher inter-cell interference level from cell-1 than the others. Each BS then takes advantage of this extra information to the pricing scheme in game $\mathcal{G}'$ to improve efficiency of its NE. The pricing scheme for games with incomplete information as discussed in Section V is applied in this simulation.

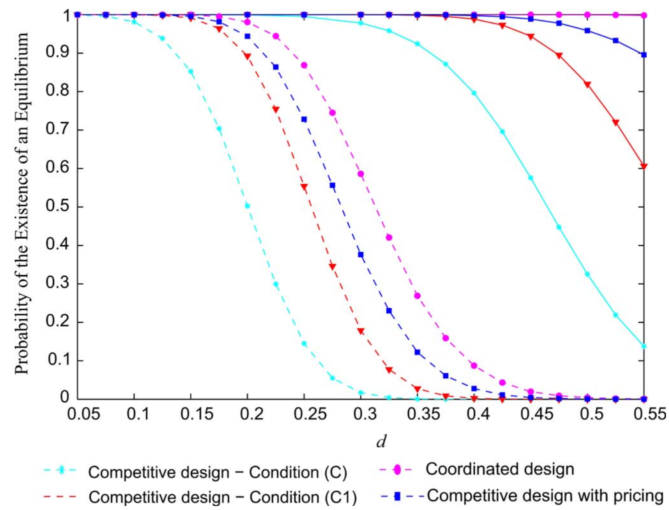

Fig. 4. Probability of existence of a stable operating point versus d by evaluating conditions (C) and (C1) and numerically examining the convergence of game $\mathcal{G}$, the coordinated design and game $\mathcal{G}'$ to meet the target SINRs: $\gamma_{q_i} = 10$ dB (dashed lines) and $\gamma_{q_i} = 0$ dB (solid lines).

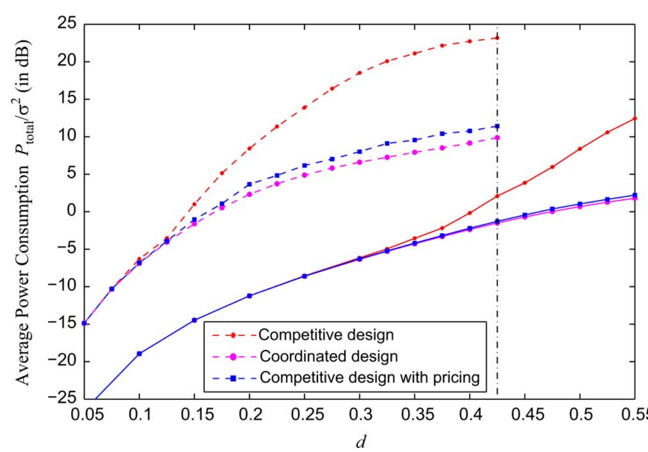

Fig. 5. Average total transmit power versus d of the competitive design, the coordinated design and the competitive design with pricing consideration to meet the target SINRs: $\gamma_{q_i} = 10$ dB (dashed lines) and $\gamma_{q_i} = 0$ dB (solid lines).

Fig. 4 displays the probability of existence of a stable operating point as the function of the MS-BS distance d by evaluating whether condition (C) are satisfied and numerically examining the convergence of game $\mathcal{G}$ [which matches with condition (C1)], the coordinated design and game $\mathcal{G}'$. From the figure, as the MSs get closer to its BS, a higher probability of the existence of a stable operating point for all three designs is observed. This is due to the fact that the stronger intra-cell channels allow a BS to transmit at a lower power level to meet its target SINRs, which then causes lower inter-cell interference. With the competitive design, a lower level of inter-cell interference certainly guarantees a higher probability of existence of a NE. On the other hand, with the pricing consideration, the whole system may further reduce the inter-cell interference. As a result, the existence probability of game $\mathcal{G}'$ is higher than that of game $\mathcal{G}$. Finally, with the coordinated design, where the inter-cell interference is fully managed, the feasibility of finding an solution that meets all the target SINRs is certainly higher than finding one in both games $\mathcal{G}$ and $\mathcal{G}'$. Fig. 4 also shows that conditions (C) and (C1) are good indications to quickly verify the existence of the NE of the competitive design. In case of not meeting conditions (C) or (C1), one may attempt to switch the network into the design with pricing consideration or the fully coordinated design to improve the convergence probability of the whole system. Fig. 4 also shows an unsurprised result that a lower target SINR, which requires lower transmit power (lower inter-cell interference), would induce a higher chance of finding the solutions in all three designs.

Of all cases where game $\mathcal{G}$ converges, the total transmit power $P_{\rm total}$ at the 3 BSs with both three designs are averaged and compared in Fig. 5 in the form of $\frac{P_{\rm total}}{\sigma^2}$. Note that the weight factors α_q's of the coordinated design are set equal with each others to minimize the design's sum transmit power. At small inter-cell MS-BS distances, it can be seen that the power usages of all the designs are very low and their difference are rather marginal. Again, this is due to the fact that the intra-cell channels are strong and the inter-cell interference are too small. However, as d increases, the effect of inter-cell interference becomes significant. Since the competitive design does not attempt to control the inter-cell interference, its NE point becomes inefficient compared to the Pareto-optimal frontier established by the coordinated design. On the other hand, should a BS know the channels to the MSs at other cells, it can alter its strategy by playing the game with pricing consideration $\mathcal{G}'$. In fact, using the aforementioned procedure to determine the pricing factor, the NE of game $\mathcal{G}'$ is almost Pareto-optimal. It is worth noting that this impressive result is obtained when each BS does not possess full channel knowledge from itself to all the users.

To illustrate the convergence behaviors of the multicell downlink beamforming games $\mathcal{G}$ and $\mathcal{G}'$ and compare the transmit powers of the two designs, we select two examples and display them in Figs. 6 and 7. In both games, it is assumed that all the BSs perform simultaneous power update at each time instance. The transmit power of each cell is then displayed after each iteration. The system configuration is the one in Fig. 3, with $\gamma_{q_i} = 10$ dB and $d = 0.3$.

In the first example, $\rho(\boldsymbol{S})$ and $\rho(\boldsymbol{S}')$ are calculated at 0.7332 and 0.6256, respectively. The power updates of both games displayed in Fig. 6 clearly show the convergence of the two designs. Fig. 6 also shows the benefit of using the design with pricing consideration, where the transmit power at each cell is reduced, compared to that of the purely competitive design. For this particular example, the price is set at 0.1388.

In the second example, $\rho(\boldsymbol{S})$ and $\rho(\boldsymbol{S}')$ are calculated at 2.0016 and 0.9815, respectively. In can be seen from Fig. 7 that the power updates of game $\mathcal{G}$ do not converge. Interestingly, with the price set at 0.551, the design with pricing consideration eventually converges. This behavior clearly indicates the benefit of adopting a more cooperative strategy at each cell by exploiting the extra channel information to other cells.

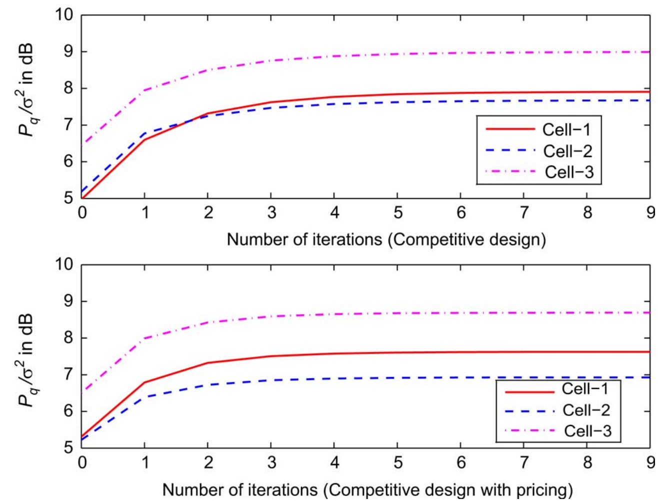

Fig. 6. A converging example of the downlink beamforming games $\mathcal{G}$ and $\mathcal{G}'$ in a multicell system: the sum power of each cell versus the number of iterations with $\rho(\boldsymbol{S}) = 0.7332$, $\rho(\boldsymbol{S}') = 0.6256$ and the corresponding matrices $\boldsymbol{G}$ and $\boldsymbol{K}$ are $\boldsymbol{M}$-matrices.

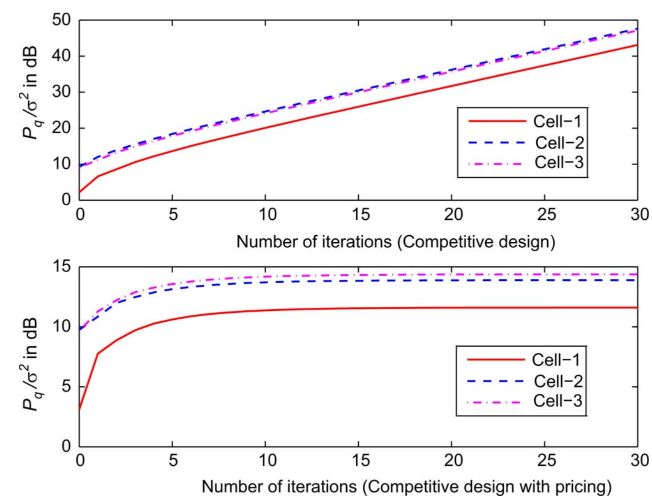

Fig. 7. An example of the downlink beamforming game that diverges in game $\mathcal{G}$ and but converges in game $\mathcal{G}'$: the sum power of each cell versus the number of iterations with $\rho(S) = 2.0016$ and $\rho(S') = 0.9815$. In this case, the corresponding matrix $\boldsymbol{K}$ is a $\boldsymbol{M}$-matrix, but $\boldsymbol{G}$ is not.

VII. Conclusion

This paper has studied the problem of downlink beamformer designs in a multicell system via game theory. Given the QoS requirements at the users in its cell, each base-station determines its optimal downlink beamformer strategy in a distributed manner, without any coordination between the cells. At first, we considered a fully competitive game, where each base-station greedily minimizes its transmit power. We has examined necessary and sufficient conditions guaranteeing the existence and uniqueness of the NE of the game. In addition, a comparison between the competitive and coordinated designs were also presented. Finally, to improve the efficiency of the competitive game, a new game with pricing consideration was studied. It was shown that the new game is able to obtain the same optimal performance as the coordinated design, while retaining the distributed nature of a multicell game.

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