Sum-Rate Maximization in the Multicell MIMO Broadcast Channel With Interference Coordination

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***Abstract*—This paper studies the precoding designs to maximize the weighted sum-rate (WSR) in a multicell multiple-input multiple-output (MIMO) broadcast channel (BC). We consider a multicell network under universal frequency reuse with multiple mobile stations (MS) per cell. With *interference coordination* (IC) between the multiple cells, the base-station (BS) at each cell only transmits information signals to the MSs within its cell using the dirty paper coding (DPC) technique, while coordinating the inter-cell interference (ICI) induced to other cells. The main focus of this work is to jointly optimize the encoding covariance matrices across the BSs in order to maximize the network-wide WSR. Since this optimization problem is shown to be nonconvex, obtaining its globally optimal solution is highly complicated. To address this problem, we consider two low-complexity solution approaches with distributed implementation to obtain at least locally optimal solutions. In the first approach, by applying a successive convex approximation technique, the original nonconvex problem is decomposed into a sequence of simpler problems, which can be solved optimally and separately at each BS. In the second approach, the WSR problem is solved via an equivalent problem of weighted sum mean squared error minimization. Both solution approaches will unfold the control signaling among the coordinated BSs to allow their distributed implementation. Simulation results confirm the convergence of the proposed algorithms, as well as their superior performances over schemes with linear precoding or no interference coordination among the BSs.**

***Index Terms*—Broadcast channel, convex optimization, coordinated multipoint transmission/reception, dirty-paper coding, interference coordination, MMSE, multicell, multiple-input multiple-output.**

I. Introduction

In order to better utilize the spectrum resource and control the interference, fractional frequency reuse has been widely adopted in many wireless cellular systems. For a fractional frequency reuse system, adjacent cells are guaranteed to operate in different frequencies. On the contrary, cells operating on the same frequency are sufficiently apart such that inter-cell interference (ICI) is kept sufficiently low. Thus, in this conventional multicell system, the ICI is controlled by deploying the frequency reuse pattern and setting the maximum power spectral density levels at all the base stations (BS). As a result, the interference management is usually relegated to on a per-cell basis management, and the ICI is treated as background noise. In a wireless multicell system with universal frequency reuse, each BS may utilize all the radio frequency resources. In this case, the multicell system emulates an interference network where the management of inter-cell interference is of particular importance and should not be neglected. Mobile stations (MS), especially the ones in the cell-edge region, often exhibit a high level of ICI, which shall dramatically degrade their link performances. To better control the ICI, the base-stations may jointly coordinate their downlink transmissions to the MSs. Known as coordinated multi-point transmission/reception (CoMP), BS coordination has been considered as a key technology to improve the coverage, throughput, and efficiency of the 3GPP LTE-Advanced [1].

Depending on the extent of coordination between the BSs, various CoMP modes have been proposed in the 3GPP LTE-Advanced [1]. In the *full coordination* mode, user data and channel state information (CSI) are exchanged between the coordinated BSs such that the multiple BSs are simultaneously transmitting data signals to the MSs within the coordinated areas. Apparently, *full coordination* is the most complex CoMP mode as it demands a significant amount of signaling to be exchanged among the BSs via an ideal backhaul [2]. Thus, this highly complex mode may not be suitable to a large scale network. In a lesser extent of coordination, the *interference coordination* (IC) mode allows a BS to transmit the data only to the MSs within its cell limit [3], [4]. Nonetheless, the ICI are still jointly controlled between the coordinated BSs through the means of precoding. By coordinating the ICI, significant power reduction or rate enhancement can be realized [3], [5], [6].

Optimizing the precoding designs in an interference network is a challenging task due to the nonconcavity of the weighted sum-rate (WSR) function. Different numerical methods for designing the precoders that maximize this WSR have been investigated in the literature [7]–[9]. Specifically, the gradient projection method was applied in [7] to search for a locally optimal transmit strategy. Successive convex approximation was applied in [8], [9] to decompose the original nonconvex problem into a sequence of simpler convex problems, which can be solved separately at the transmitters. Note that these works only considered the network with one MS per cell. For a more general case of multiple MSs per cell, recent works in

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[3], [4] studied the optimal linear precoding to maximize the WSR with per-BS constraints. Specifically, an iterative algorithm was proposed in [3] to solve the Karush-Kuhn-Tucker conditions of the non-convex WSR maximization problem. Another solution approach to the nonconvex WSR maximization problem is to transform it into a minimization of the weighted mean squared error (WMMSE) problem [4], [10], [11]. The WMMSE problem then can be solved by iteratively optimizing the weight matrices, the MMSE precoders, and the MMSE decoders [10]. Thus, by establishing the equivalence between the WSR maximization problem and the WMMSE minimization problem, a locally optimal solution to the former can be found from the solution of the latter.

In this work, we consider a coordinated multicell system in a general setting with multiple MSs per cell, where each BS or MS is equipped with multiple transmit antennas. At each cell, the BS concurrently transmits information signals to its connected MSs, which emulates a multiple-input multiple-output (MIMO) broadcast (BC) system. The main focus of this work is to jointly optimize the precoding covariance matrices at the BSs in order to maximize the network-wide WSR under the IC mode. While most of the works considered linear precoding at each BS for the multicell MIMO-BC system [3]–[9], our focus in this paper is on nonlinear precoding design. Specifically, in the BC with multiple MSs per cell, each BS utilizes dirty paper coding (DPC) to encode the data for the MSs within its cell. It is well-known that DPC is the capacity-achieving multiuser precoding technique for a single-cell system [12]–[15]. In this work, we extend the study of DPC onto the multicell system with interference coordination. Although DPC only remains as a theoretical benchmark due to its high complexity implementation, our consideration of DPC potentially allows the multicell network to realize extra performance from the nonlinear precoding over the linear precoding. The achievable rate by DPC then can be used as the benchmark for practical nonlinear precoding techniques, such as Tomlinson-Harashima precoding (THP) [16] and vector perturbation (VP) [17]. Since the maximization of WSR in a multicell MIMO-BC with DPC is shown to be a nonconvex problem, finding its globally optimal solution is computationally complex. To address this concern, we consider two low-complexity solution approaches, namely iterative linear approximation (ILA) and WMMSE, to numerically search for at least locally optimal solutions of the problem.

In the ILA solution approach, the sum-rate function at all other cells except a particular cell under consideration, is approximated into a linear interference penalty. Thus, maximizing the network WSR is approximate to maximizing the BC sum-rate with DPC at the given cell while minimizing a penalty term on the ICI generated by its corresponding BS. Although this per-cell BC problem is yet to be convex, we show that the problem is equivalent to a counterpart one in the multiple access channel (MAC) via the so-called BC-MAC duality. Since the MAC problem is convex and thus optimally solved, the optimal solution to the BC problem is also obtained by the MAC-BC transformation [14]. Interestingly, it will be proved that the network WSR is always improved by optimizing the DPC precoders at any given BS. With the ILA algorithm, each BS is required to iteratively take turn and refine its precoders. We then prove the monotonic convergence of the ILA algorithm to at least a local maximum. In addition, we develop a message exchange mechanism that allows the proposed algorithm to be implemented in a fully distributed manner.

In the WMMSE solution approach, we devise a version of the WMMSE algorithm for the multicell MIMO-BC with DPC precoding. To avoid the confusion with the original WMMSE algorithm for the case of linear precoding in [4], the newly devised algorithm will be referred to as the DPC-WMMSE algorithm. Similar to the original WMMSE algorithm, we show that the DPC-WMMSE can obtain a locally optimal solution to the multicell MIMO-BC WSR maximization problem. In addition, the DPC-WMMSE can be implemented distributively via a message exchange mechanism among the coordinated BSs. Simulation results confirm the convergence analysis of the ILA and DPC-WMMSE algorithm, and show that the proposed algorithm significantly improves the network WSR, in comparison with linear precoding or with no IC between the BSs.[1]

Notations: $(\mathbf{X})^T$ and $(\mathbf{X})^H$ denote the transpose and conjugate transpose (Hermitian operator) of the matrix $\mathbf{X}$, respectively; $[\mathbf{X}]^+$ denotes the component-wise operation $\max\{[\mathbf{X}]_{m,n}, 0\}$; $\mathbf{X} \succeq \mathbf{0}$ means that $\mathbf{X}$ is a positive semi-definite matrix; $\mathrm{Tr}\{\mathbf{X}\}$, $|\mathbf{X}|$, and $\mathrm{rank}\{\mathbf{X}\}$ denote the trace, determinant, and rank of the matrix $\mathbf{X}$, respectively; and $x^\star$ denotes the optimal value of the variable x.

II. System Model and Problem Formulation

Consider the downlink transmission of a multicell system with Q separate cells operating on the same frequency. At each cell, a multiple-antenna BS concurrently sends independent data streams to multiple MSs, each equipped with multiple receive antennas. For the simplicity in presentation, it is assumed that the number of antennas at each BS and MS are M and N, respectively, and the number of MSs per cell is K. At a particular cell, say cell-q, the received signal at MS-i, denoted as $\mathbf{y}_{q_i}$, is given by

$$\mathbf{y}_{q_i} = \mathbf{H}_{qq_i} \sum_{j=1}^{K} \mathbf{x}_{q_j} + \sum_{r \neq q}^{Q} \mathbf{H}_{rq_i} \sum_{j=1}^{K} \mathbf{x}_{r_j} + \mathbf{z}_{q_i} \tag{1}$$

where $\mathbf{x}_{r_j} \in \mathbb{C}^{M\times 1}$ is the transmitted vector from the BS-r intended for its connected MS-j, $\mathbf{H}_{rq_i}$ models the channel from BS-r to MS-i of cell-q, and $\mathbf{z}_{q_i}$ is the zero-mean additive Gaussian noise vector with the covariance matrix $\mathbf{Z}_{q_i}$. Since the multicell system operates on the same frequency channel, the intended signal from BS-q to its MS-i is now subject to the intra-cell interference from the signaling for other co-located MSs in $\mathbf{H}_{qq_i} \sum_{j\neq i}^{K} \mathbf{x}_{q_j}$, as well as the ICI from other cells in $\sum_{r\neq q}^{Q} \mathbf{H}_{rq_i} \left(\sum_{j=1}^{K} \mathbf{x}_{r_j}\right)$.

Let $D = \min(M, N)$ be the number of data sub-streams for each MS. The transmit signal vector $\mathbf{x}_{q_i}$ for MS-i in BS-q can be expressed as

$$\mathbf{x}_{q_i} = \mathbf{V}_{q_i} \mathbf{s}_{q_i} \tag{2}$$

where $\mathbf{V}_{q_i} \in \mathbb{C}^{M\times D}$ is the precoding matrix and $\mathbf{s}_{q_i} \in \mathbb{C}^{D\times 1}$ represents the information signal vectors. Without loss

[1]Without IC, the multicell system is said to be in the *competitive* mode where each BS is a rational and selfish player.

of generality, it is assumed that $\mathbb{E}[\mathbf{s}_{q_i}\mathbf{s}_{q_i}^H] = \mathbf{I}$. Denote $\mathbf{Q}_{q_i} = \mathbb{E}\left[\mathbf{x}_{q_i}\mathbf{x}_{q_i}^H\right] = \mathbf{V}_{q_i}\mathbf{V}_{q_i}^H$ as the transmit covariance matrix intended for MS-i of cell-q. Let $\mathbf{Q}_q = \{\mathbf{Q}_{q_i}\}_{i=1}^K$ be the downlink precoder profile of the K users at cell-q. Likewise, let $\mathbf{Q}_{-q} = (\mathbf{Q}_1, \ldots, \mathbf{Q}_{q-1}, \mathbf{Q}_{q+1}, \ldots, \mathbf{Q}_Q)$ denote the precoding profile of all cells except cell-q. Denote $\mathbf{z}_{-q_i} = \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\sum_{j=1}^{K}\mathbf{x}_{r_j} + \mathbf{z}_{q_i}$ as the total ICI plus additive Gaussian noise at MS-i of cell-q, whose covariance $\mathbf{R}_{q_i}$ is defined as

$$\mathbf{R}_{q_i} = \mathbb{E}\left[\mathbf{z}_{-q_i}\mathbf{z}_{-q_i}^H\right] = \sum_{r\neq q}^{Q}\mathbf{H}_{rq_i}\left(\sum_{j=1}^{K}\mathbf{Q}_{r_j}\right)\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i}. \quad (3)$$

As the multicell system operates in IC mode, each BS only attempts to encode and transmit information signals to the MSs within its cell. In this work, it is assumed that the BS implements the capacity-achieving multiuser encoding technique, namely dirty paper coding (DPC) [12], [15], for the downlink transmissions to its connected MSs. At cell-q, assuming the encoding order from user-K to user-1, DPC is utilized such that the intended codeword for user-i does not see the intra-cell interference from user-$(i+1)$ to user-K. Thus, the achievable data rate at user-i of cell-q is given by

$$R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\frac{\left|\mathbf{R}_{q_i} + \mathbf{H}_{qq_i}\left(\sum_{j=1}^{i}\mathbf{Q}_{q_j}\right)\mathbf{H}_{qq_i}^H\right|}{\left|\mathbf{R}_{q_i} + \mathbf{H}_{qq_i}\left(\sum_{j=1}^{i-1}\mathbf{Q}_{q_j}\right)\mathbf{H}_{qq_i}^H\right|}. \quad (4)$$

Let $R_q^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q})$ be the sum-rate at cell-q for its K connected MSs. Collectively, the network WSR is given by $\sum_{q=1}^{Q}w_q R_q^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q})$, where w_q denotes the nonnegative weight of cell-q. Given P_q as the maximum transmit power at BS-q, the network WSR is maximized by the following optimization

$$\begin{aligned} \underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_Q}{\text{maximize}} \; & \sum_{q=1}^{Q}w_q\sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{subject to} \; & \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q, \; \forall q \\ & \mathbf{Q}_{q_i} \succeq 0, \; \forall i, \forall q. \end{aligned} \quad (5)$$

It is observed that problem (5) is nonconvex because of the presence of $\mathbf{Q}_{q_i}$'s in the ICI terms $\mathbf{R}_{r_j}$'s with $r \neq q$, as well as the intra-cell interference term in $R_{q_j}^{\mathrm{BC}}(\mathbf{Q}_q, \mathbf{Q}_{-q})$ with $j < i$. In [18], the sum-rate maximization problem in an interference channel was shown to be strongly NP-hard (cf. Theorem 1 of [18]), even in the simplest case (single-carrier, optimizing the allocated power at the source). Intuitively, problem (5), which involves precoding designs, power allocation for multiple users per cells, is also NP-hard. However, to have a complete proof on this NP-hard problem requires a much more detailed analysis, which is beyond the scope of our work. Nonetheless, due to the nonconvexity of the considered problem, obtaining its globally optimal solution appears to be computationally complicated. It may also require a centralized solver unit to obtain such a solution. In this case, designing a low-complexity algorithm with distributed implementation to compute local optimizers becomes a more attractive option.

III. The Iterative Linear Approximation Solution Approach

A. The Iterative Linear Approximation (ILA) Algorithm

This section is to investigate a distributed and fast converging algorithm to obtain at least a locally optimal solution to the nonconvex problem (5). By formulating problem (5) as a difference of convex (DC) program [19], [20], we first take the approximation to the nonconcave part of the objective function, corresponding to one set of precoders at a particular BS. We then show that the approximated problem can be solved optimally at that BS.

Let $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \sum_{r\neq q}^{Q}w_r\sum_{j=1}^{K}R_{r_j}(\mathbf{Q}_q, \mathbf{Q}_{-q})$ denote the WSR of all other cells except cell-q. As $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is not concave in $\mathbf{Q}_{q_i}$, we shall take an approximation of f_q into a linear term. At a given value of $\mathbf{Q}_{q_i} = \bar{\mathbf{Q}}_{q_i}, \forall q, \forall i$, taking the Taylor expansion of f_q around $\bar{\mathbf{Q}}_{q_i}$ and retaining the first linear term, one has

$$f_q(\mathbf{Q}_q, \bar{\mathbf{Q}}_{-q}) \approx f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q(\mathbf{Q}_{q_i} - \bar{\mathbf{Q}}_{q_i})\} \quad (6)$$

where $\mathbf{A}_q$ is the negative partial derivative of f_q with respect to $\mathbf{Q}_{q_i}$, evaluated at $\mathbf{Q}_{q_i} = \bar{\mathbf{Q}}_{q_i}$, as given in (7) at the bottom of the page. Note that this partial derivative has the same form with respect to each $\mathbf{Q}_{q_1}, \ldots, \mathbf{Q}_{q_K}$, and is positive semi-definite, i.e., $\mathbf{A}_q \succeq \mathbf{0}$.

With the other variables fixed as $\mathbf{Q}_{-q} = \bar{\mathbf{Q}}_{-q}$, the objective function in problem (5) can be approximated around $\bar{\mathbf{Q}}_q$ as $w_q\sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q, \bar{\mathbf{Q}}_{-q})$ −

$$\begin{aligned} \mathbf{A}_q &= -\left.\frac{\partial f_q}{\partial \mathbf{Q}_{q_i}}\right|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}} \\ &= -\sum_{r\neq q}^{Q}w_r\sum_{j=1}^{K}\left.\frac{\partial R_{r_j}^{\mathrm{BC}}}{\partial \mathbf{Q}_{q_i}}\right|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}} \\ &= \sum_{r\neq q}^{Q}w_r\sum_{j=1}^{K}\mathbf{H}_{qr_j}^H\left[\left(\mathbf{R}_{r_j} + \sum_{k=1}^{j-1}\mathbf{H}_{rr_j}\mathbf{Q}_{r_k}\mathbf{H}_{rr_j}^H\right)^{-1} - \left(\mathbf{R}_{r_j} + \sum_{k=1}^{j}\mathbf{H}_{rr_j}\mathbf{Q}_{r_k}\mathbf{H}_{rr_j}^H\right)^{-1}\right]\mathbf{H}_{qr_j}\Bigg|_{\mathbf{Q}_{q_i}=\bar{\mathbf{Q}}_{q_i}}. \end{aligned} \quad (7)$$

$\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}+\left[f_q(\bar{\mathbf{Q}}_q,\bar{\mathbf{Q}}_{-q})+\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\bar{\mathbf{Q}}_{q_i}\}\right]$. By omitting the known terms from the objective function, we examine the optimization problem (5) over the set of variables $\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}$ by the approximation

$$\begin{aligned}\underset{\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}}{\text{maximize}}\ & w_q\sum_{i=1}^{K}R_{q_i}^{\mathrm{BC}}(\mathbf{Q}_q,\bar{\mathbf{Q}}_{-q})-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}\\ \text{subject to}\ & \sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_{q_i}\}\le P_q\\ & \mathbf{Q}_{q_i}\succeq\mathbf{0},\forall i,\end{aligned}\tag{8}$$

which can be performed solely at cell-q. Similarly, one can take the above approach with other set of variables to approximate problem (5) into a set of Q per-cell problems.

Some remarks regarding the optimization problem (8) are provided as follows:

Remark 1: Problem (8) is similar to the sum-rate maximization problem in the BC with DPC, studied in [14], [15], [21], albeit the presence of the term $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}$. Since $\mathbf{A}_q$ is the negative rate of change in the data rate of the adjacent cells to cell-q with respect to $\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}$, minimizing $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}$ is equivalent to maximizing the contribution of $\mathbf{Q}_{q_1},\ldots,\mathbf{Q}_{q_K}$ to the data rate of the adjacent cells. In addition, $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}$ can be interpreted as a penalty term charged on ICI generated by BS-q. This penalty term encourages the BS to coordinately design its precoders by controlling its induced ICI to other cells. Should the penalty term be omitted, the BS would only maximize the downlink capacity for its connected users. As a result, the precoding design in this multicell system is a noncooperative game between the BSs, where each BS acts as a rational and selfish player. This multicell precoding game is similar to the game studied in [22] for the case of 1 user per cell. It is noted that the study of the multicell precoding game with DPC for the case of multiple users per cell is beyond the scope of this paper. Nonetheless, we shall present some numerical results for this noncooperative design in comparison to the considered coordinated design.

Remark 2: It is worth mentioning that [23], [24] studied the BC sum-rate maximization with strict constraints on the induced ICI $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}_q\mathbf{Q}_{q_i}\}$. The considered problem (8) is different from the studies in [23], [24], since we attempt to minimize the ICI penalty term with a sum-power constraint on the transmit covariances.

Remark 3: Although problem (8) is not a convex optimization problem, its resemblance to the BC's sum-rate maximization problem enables its transformation into a dual MAC maximization problem via the so-called BC-MAC duality. In a conventional multiuser MIMO system with the objective of maximizing the system sum-rate, BC-MAC duality was investigated for the case of a single sum-power constraint [14], [15], [25], a set of linear power constraints [15], [26], and multiple general transmit covariance constraints [23]. In the following, it will be shown shortly that the BC-MAC duality also holds for the multiuser MIMO system with the objective of maximizing the system sum-rate while minimizing the penalty term imposed on the transmit covariances. As a result, the nonconvex problem (8) can be solved optimally via the convex MAC problem by utilizing this BC-MAC duality.

To this end, we consider solving the optimization (8) under two cases: without and with the power constraint $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_{q_i}\}\le P$. Since the optimization is performed at a particular BS, without loss of generality, the subscript indicating the BS and the variables $(\mathbf{Q}_q,\mathbf{Q}_{-q})$ annexed to the user data rates are dropped for the simplicity in presentation.

1) Case 1: Without the Power Constraint: Without the sum-power constraint $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i\}\le P$, the optimization (8) can be stated as

$$\begin{aligned}\underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}}\ & w\sum_{i=1}^{K}R_i^{\mathrm{BC}}-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i\}\\ \text{subject to}\ & \mathbf{Q}_i\succeq\mathbf{0},\ \forall i.\end{aligned}\tag{9}$$

In the following, we are interested in obtaining the globally optimal solution $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ to problem (9). Should $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ meet the sum-power constraint in (8), i.e., $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}\le P$, they must be the global maximizer of problem (8) as well.

By changing the variables $\tilde{\mathbf{Q}}_i=\mathbf{A}^{1/2}\mathbf{Q}_i\mathbf{A}^{1/2}$ and denoting $\tilde{\mathbf{H}}_i=\mathbf{R}_i^{-1/2}\mathbf{H}_i\mathbf{A}^{-1/2}$, the data-rate to user-$i$ can be rewritten as

$$R_i^{\mathrm{BC}}=\log\frac{\left|\mathbf{I}+\tilde{\mathbf{H}}_i\left(\sum_{j=1}^{i}\tilde{\mathbf{Q}}_j\right)\tilde{\mathbf{H}}_i^H\right|}{\left|\mathbf{I}+\tilde{\mathbf{H}}_i\left(\sum_{j=1}^{i-1}\tilde{\mathbf{Q}}_j\right)\tilde{\mathbf{H}}_i^H\right|}.\tag{10}$$

Thus, problem (9) is equivalent to

$$\begin{aligned}\underset{\tilde{\mathbf{Q}}_1,\ldots,\tilde{\mathbf{Q}}_K}{\text{maximize}}\ & w\sum_{i=1}^{K}R_i^{\mathrm{BC}}-\sum_{i=1}^{K}\mathrm{Tr}\{\tilde{\mathbf{Q}}_i\}\\ \text{subject to}\ & \tilde{\mathbf{Q}}_i\succeq\mathbf{0},\ \forall i.\end{aligned}\tag{11}$$

In order to solve the nonconvex problem (11), we utilize the known BC-MAC duality property as follows. Consider a dual MAC with K N-antenna MSs transmitting to an M-antenna BS, where the uplink channel from user-i to the BS is assumed to be $\tilde{\mathbf{H}}_i^H$ and background noise at the BS is AWGN with unit variance. It is assumed that the BS employs successive interference cancelation (SIC) to decode the signals from the K MSs. With the decoding order from user-1 to user-K, SIC ensures that the received signal from user-i is not interfered by the signals from user-1 to user-$(i-1)$. Denote $\mathbf{D}_i$ as the uplink precoding covariance matrix at user-i, the achievable data-rate for user-i in the MAC is thus given by

$$R_i^{\mathrm{MAC}}=\log\frac{\left|\mathbf{I}+\sum_{j=i}^{K}\tilde{\mathbf{H}}_j^H\mathbf{D}_j\tilde{\mathbf{H}}_j\right|}{\left|\mathbf{I}+\sum_{j>i}^{K}\tilde{\mathbf{H}}_j^H\mathbf{D}_j\tilde{\mathbf{H}}_j\right|}.\tag{12}$$

The key relationship between the BC and its dual MAC is presented in the following theorem.[2]

[2]The duality between the BC with a general linear constraint on $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i\}$ and the MAC was established in [23] using a technique called SINR duality. In this work, we apply a simple change of variables to show this duality.

Theorem 1: [14] For a given set of downlink covariance matrices $\tilde{\mathbf{Q}}_1, \ldots, \tilde{\mathbf{Q}}_K$ in the BC, it is always possible to find a set of uplink covariance matrices $\mathbf{D}_1, \ldots, \mathbf{D}_K$ such that $R_i^{\mathrm{MAC}} = R_i^{\mathrm{BC}}$ and $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{D}_i\} = \sum_{i=1}^K \mathrm{Tr}\{\tilde{\mathbf{Q}}_i\}$ through the BC-MAC transformation. Vice versa, for a given set of uplink covariance matrices $\mathbf{D}_1, \ldots, \mathbf{D}_K$, it is always possible to find a set of downlink covariances $\tilde{\mathbf{Q}}_1, \ldots, \tilde{\mathbf{Q}}_K$ such that $R_i^{\mathrm{BC}} = R_i^{\mathrm{MAC}}$ and $\sum_{i=1}^K \mathrm{Tr}\{\tilde{\mathbf{Q}}_i\} = \sum_{i=1}^K \mathrm{Tr}\{\mathbf{D}_i\}$ through the MAC-BC transformation.

From Theorem 1, instead of solving the nonconvex problem(8), one may consider the following optimization problem

$$\begin{aligned} &\underset{\mathbf{D}_1,\ldots,\mathbf{D}_K}{\text{maximize}}\; w\sum_{i=1}^K R_i^{\mathrm{MAC}} - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{D}_i\} \\ &\text{subject to } \mathbf{D}_i \succeq \mathbf{0},\ \forall i, \end{aligned} \tag{13}$$

which can be interpreted as a MAC sum-rate maximization with a penalty term on the transmit power at the MSs. Certainly, if the set $\mathbf{D}_1^\star, \ldots, \mathbf{D}_K^\star$ is optimal in (13), it is possible to find the set $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$ that is optimal in (11) with the same maximum value. By contradiction, if $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$ were not optimal, the BC-MAC transformation would ensure that $\mathbf{D}_1^\star, \ldots, \mathbf{D}_K^\star$ would not be optimal. Thus, the BC-MAC duality also holds for the considered problem with the objective of maximizing the sum-rate while minimizing the penalty term imposed on the transmit covariances. Consequently, by finding the optimal solution of problem (13), one shall obtain the optimal solution of problem(11) as well.

Note that the objective function in (13) can be simplified as

$$\begin{aligned} &w\sum_{i=1}^K R_i^{\mathrm{MAC}} - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{D}_i\} \\ &= w\log\left|\mathbf{I} + \sum_{i=1}^K \tilde{\mathbf{H}}_i^H \mathbf{D}_i \tilde{\mathbf{H}}_i\right| - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{D}_i\}, \end{aligned} \tag{14}$$

which is concave in $\mathbf{D}_1, \ldots, \mathbf{D}_K$. Consequently, problem (13) is convex. In addition, the inherently decoupled constraints for each variable matrix $\mathbf{D}_i$ allows the sequential maximization of the objective function over each variable matrix [27]. More specifically, MS-i optimizes its covariance matrix $\mathbf{D}_i$ by performing

$$\underset{\mathbf{D}_i \succeq \mathbf{0}}{\text{maximize}}\; w\log\left|\mathbf{I} + \left(\mathbf{I} + \sum_{j\neq i}^K \tilde{\mathbf{H}}_j^H \mathbf{D}_j \tilde{\mathbf{H}}_j\right)^{-1} \tilde{\mathbf{H}}_i^H \mathbf{D}_i \tilde{\mathbf{H}}_i\right| - \mathrm{Tr}\{\mathbf{D}_i\}, \tag{15}$$

while treating the signal from other MSs as noise. Using the eigen-decomposition $\tilde{\mathbf{H}}_i\big(\mathbf{I} + \sum_{j\neq i}^K \tilde{\mathbf{H}}_j^H \mathbf{D}_j \tilde{\mathbf{H}}_j\big)^{-1} \tilde{\mathbf{H}}_i^H = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{U}_i^H$, the optimal solution can be obtained in closed-form as $\mathbf{D}_i = \mathbf{U}_i\big[w\mathbf{I} - \mathbf{\Sigma}_i^{-1}\big]^+ \mathbf{U}_i^H$. Each MS-$i$ can iteratively update its covariance matrix while keeping other covariance matrices fixed [27]. Note that this procedure always improves the objective function (14).

Since the objective function (14) is a subtraction of a log function of $\mathbf{D}_1, \ldots, \mathbf{D}_K$ to a linear function of $\mathbf{D}_1, \ldots, \mathbf{D}_K$, it must have an upper bound. As a result, the sequential optimization of (15) over $\mathbf{D}_1, \ldots, \mathbf{D}_K$ is guaranteed to monotonically converge to the optimal solution $\mathbf{D}_1^\star, \ldots, \mathbf{D}_K^\star$ of problem (13). Consequently, one can obtain the optimal solution $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$ to problem (11) from $\mathbf{D}_1^\star, \ldots, \mathbf{D}_K^\star$ by the MAC-BC transformation [14]. The optimal solution of (9) is then given by $\mathbf{Q}_i^\star = \mathbf{A}^{-1/2}\tilde{\mathbf{Q}}_i^\star \mathbf{A}^{-1/2}$. As $\mathbf{D}_1^\star, \ldots, \mathbf{D}_K^\star$ is the globally optimal solution to the MAC problem (13), $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ must be the globally optimal solution to the BC problem (9). It is then straightforward to verify whether $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ meets the sum-power constraint $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\} \le P$. If it is not, we proceed to solve the optimization problem (8) in the remaining case, where the sum-power constraint is strictly imposed.

2) Case 2: With the Power Constraint: Consider the Lagrangian of original BC problem (8) as follows:

$$\mathcal{L}(\mathbf{Q}_1, \ldots, \mathbf{Q}_K, \lambda) = w\sum_{i=1}^K R_i^{\mathrm{BC}} - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A} + \lambda\mathbf{I})\mathbf{Q}_i\} + \lambda P \tag{16}$$

where $\lambda \ge 0$ is the Lagrangian multiplier associated with the power constraint $\sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i\} \le P$. The Lagrangian dual function is then given by

$$g(\lambda) = \sup_{\mathbf{Q}_1 \succeq \mathbf{0}, \ldots, \mathbf{Q}_K \succeq \mathbf{0}} \mathcal{L}(\mathbf{Q}_1, \ldots, \mathbf{Q}_K, \lambda) \tag{17}$$

and the dual problem is defined as

$$\begin{aligned} &\underset{\lambda}{\text{minimize}}\;\; g(\lambda) \\ &\text{subject to}\;\; \lambda \ge 0. \end{aligned} \tag{18}$$

We first focus on the maximization of the Lagrangian dual function for a given λ, which can be stated as

$$\begin{aligned} &\underset{\mathbf{Q}_1,\ldots,\mathbf{Q}_K}{\text{maximize}}\; w\sum_{i=1}^K R_i^{\mathrm{BC}} - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A} + \lambda\mathbf{I})\mathbf{Q}_i\} \\ &\text{subject to } \mathbf{Q}_i \succeq \mathbf{0},\ \forall i. \end{aligned} \tag{19}$$

Clearly, problem (19) is similar to problem(9). Thus, one can obtain the globally optimal solution to problem(19) by adopting the approach in solving problem(9). The difference is in the change of variables where $\tilde{\mathbf{Q}}_i = (\mathbf{A}+\lambda\mathbf{I})^{1/2}\mathbf{Q}_i(\mathbf{A}+\lambda\mathbf{I})^{1/2}$ and $\tilde{\mathbf{H}}_i = \mathbf{R}_i^{-1/2}\mathbf{H}_i(\mathbf{A} + \lambda\mathbf{I})^{-1/2}$. Then, by solving the dual MAC problem (13) and performing the MAC-BC transformation one can obtain the globally optimal solution $\tilde{\mathbf{Q}}_1^\star, \ldots, \tilde{\mathbf{Q}}_K^\star$. Subsequently, the optimal solution to the Lagrangian dual problem (19) is given by $\mathbf{Q}_i^\star = (\mathbf{A} + \lambda\mathbf{I})^{-1/2}\tilde{\mathbf{Q}}_i^\star(\mathbf{A} + \lambda\mathbf{I})^{-1/2}$.

It remains to minimize $g(\lambda)$ subject to the constraint $\lambda \ge 0$ in (18). By the Lagrangian duality theory, $g(\lambda)$ is convex in λ[28]. However, $g(\lambda)$ may not be differentiable. Nonetheless, it is possible to find the subgradient of $g(\lambda)$. Suppose that at λ, $\mathbf{Q}_1^\star, \ldots, \mathbf{Q}_K^\star$ is the optimal solution of (19). For any given $\lambda' > 0$, one has

$$\begin{aligned} g(\lambda') &= \max_{\{\mathbf{Q}_i\}} w\sum_{i=1}^K R_i^{\mathrm{BC}}\big(\{\mathbf{Q}_i\}\big) - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A}+\lambda'\mathbf{I})\mathbf{Q}_i\} + \lambda' P \\ &\ge w\sum_{i=1}^K R_i^{\mathrm{BC}}\big(\{\mathbf{Q}_i^\star\}\big) - \sum_{i=1}^K \mathrm{Tr}\{(\mathbf{A} + \lambda'\mathbf{I})\mathbf{Q}_i^\star\} + \lambda' P \\ &= g(\lambda) + \left(P - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_i^\star\}\right)(\lambda' - \lambda). \end{aligned} \tag{20}$$

Thus, $P-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}$ can be chosen as the subgradient of $g(\lambda)$. The subgradient search direction suggests to increase λ if $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}\geq P$ or decrease λ otherwise. Since λ is searched in a one-dimensional space, the bisection method can be efficiently applied to find the optimal $\lambda^\star$. We summarize the proposed algorithm to solve the nonconvex problem (8) in Algorithm 1. The optimality of the proposed algorithm is proved in the following theorem.

Algorithm 1 Iterative Algorithm for the MIMO-BC Sumrate Maximization with a Penalty Term

1 **For** a given $\lambda\geq 0$ **do**

2 Change the variables as $\tilde{\mathbf{Q}}_i=(\mathbf{A}+\lambda\mathbf{I})^{1/2}\mathbf{Q}_i(\mathbf{A}+\lambda\mathbf{I})^{1/2}$ and $\tilde{\mathbf{H}}_i=\mathbf{R}_i^{-1/2}\mathbf{H}_i(\mathbf{A}+\lambda\mathbf{I})^{-1/2}$;

3 Solve the equivalent uplink MAC problem $\underset{\mathbf{D}_1,\ldots,\mathbf{D}_K}{\text{maximize}}\ w\log\left|\mathbf{I}+\sum_{i=1}^{K}\tilde{\mathbf{H}}_i^H\mathbf{D}_i\tilde{\mathbf{H}}_i\right|-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{D}_i\}$ by;

4 **repeat**

5 **for** $i=1,2,\ldots,K$ **do**

6 Perform the eigen-decomposition $\tilde{\mathbf{H}}_i\big(\mathbf{I}+\sum_{j\neq i}^{K}\tilde{\mathbf{H}}_j^H\mathbf{D}_j\tilde{\mathbf{H}}_j\big)^{-1}\tilde{\mathbf{H}}_i^H=\mathbf{U}_i\mathbf{\Sigma}_i\mathbf{U}_i^H$;

7 Update $\mathbf{D}_i=\mathbf{U}_i\big[w\mathbf{I}-\mathbf{\Sigma}_i^{-1}\big]^+\mathbf{U}_i$;

8 **end**

9 **until** *convergence to* $\mathbf{D}_1^\star,\ldots,\mathbf{D}_K^\star$;

10 Compute $\tilde{\mathbf{Q}}_1^\star,\ldots,\tilde{\mathbf{Q}}_K^\star$ from $\mathbf{D}_1^\star,\ldots,\mathbf{D}_K^\star$ by the MAC-BC transformation;

11 Compute $\mathbf{Q}_i^\star=(\mathbf{A}+\lambda\mathbf{I})^{-1/2}\tilde{\mathbf{Q}}_i^\star(\mathbf{A}+\lambda\mathbf{I})^{-1/2}, i=1,\ldots,K$;

12 **end**

13 case $\lambda=0$ *(without the power constraint)*

14 Follow step 1 to 12 to obtain $\mathbf{Q}_i^\star$;

15 **if** $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}\leq P$ **then** stop the algorithm;

16 case $\lambda>0$ *(with the power constraint)*

17 Set $\lambda_{\min}=0$ and $\lambda_{\max}$ large;

18 **repeat**

19 $\lambda=(\lambda_{\min}+\lambda_{\max})/2$;

20 Follow step 1 to 12 to obtain $\mathbf{Q}_i^\star$;

21 **if** $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}>P$ **then** set $\lambda_{\min}=\lambda$; **otherwise**, set $\lambda_{\max}=\lambda$

22 **until** $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}=P$ or $(\lambda_{\max}-\lambda_{\min})$ is small enough;

Theorem 2: The proposed Algorithm 1 achieves the globally optimal solution to problem(8).

Proof: Per the proposed Algorithm 1, if the obtained solution set $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ for the case $\lambda=0$ meets the power constraint $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}\leq P$, then $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ is the globally optimal solution to problem (8). This is due to the equivalence between the BC problem(11) and the MAC problem(13).

We now focus on the case $\lambda^\star>0$. Suppose that the obtained solution $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ from the proposed algorithm is not globally optimal, and there is another solution set $\hat{\mathbf{Q}}_1,\ldots,\hat{\mathbf{Q}}_K$ satisfying the conditions:

(i) $\sum_{i=1}^{K}\mathrm{Tr}\{\hat{\mathbf{Q}}_i\}\leq P$

(ii) $w\sum_{i=1}^{K}R_i^{\mathrm{BC}}(\hat{\mathbf{Q}})-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\hat{\mathbf{Q}}_i\}>w\sum_{i=1}^{K}R_i^{\mathrm{BC}}(\mathbf{Q}^\star)-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i^\star\}$.

Since $\mathbf{Q}_1^\star,\ldots,\mathbf{Q}_K^\star$ globally maximizes the Lagrangian as given in problem (19), one has

$$w\sum_{i=1}^{K}R_i^{\mathrm{BC}}(\mathbf{Q}^\star)-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\mathbf{Q}_i^\star+\lambda^\star\mathbf{Q}_i^\star\}\geq w\sum_{i=1}^{K}R_i^{\mathrm{BC}}(\hat{\mathbf{Q}})-\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{A}\hat{\mathbf{Q}}_i+\lambda^\star\hat{\mathbf{Q}}_i\}. \quad (21)$$

Thus, condition (ii) then guarantees

$$\lambda^\star\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}<\lambda^\star\sum_{i=1}^{K}\mathrm{Tr}\{\hat{\mathbf{Q}}_i\}. \quad (22)$$

Since Algorithm 1 guarantees $\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}=P$ for the case $\lambda^\star>0$, one has

$$P=\sum_{i=1}^{K}\mathrm{Tr}\{\mathbf{Q}_i^\star\}<\sum_{i=1}^{K}\mathrm{Tr}\{\hat{\mathbf{Q}}_i\}, \quad (23)$$

which contradicts condition (i). Thus, the proof for this theorem follows by contradiction. ■

As proved in Theorem 1, the optimization(8) performed at cell-q can be effectively and optimally solved. For the network WSR maximization problem (5), the proposed ILA algorithm requires each cell-q, $q=1,\ldots,Q$ to iteratively update the matrix $\mathbf{A}_q$ and solve its approximated optimization problem (8).

Theorem 3: The optimization (8) performed at any given BS-q always improves the network WSR $\sum_{q=1}^{Q}w_qR_q^{\mathrm{BC}}$.

Proof: Please refer to Appendix A. ■

As indicated in Theorem 3, the network WSR is strictly nondecreasing after an update of the covariance matrices at any given BS. More specifically, at a current iteration where $\mathbf{Q}_q=\bar{\mathbf{Q}}_q,\forall q$, at any given BS, say BS-q, the update to $\mathbf{Q}_{q_i}=\mathbf{Q}_{q_i}^\star,\forall i$ always improves the network WSR. This iterative procedure can be performed sequentially across all BSs until the whole system reaches to a stable state. Since the network sum-rate is upper-bounded, the Gauss-Seidel (sequential) iterative update is guaranteed to converge to at least a local maximum. We summarize the ILA algorithm for the multicell MIMO-BC in Algorithm 2. In Algorithm 2, we refer the iterative procedure 2–8 as an outer-loop iteration and refer the update at a particular BS using the iterative Algorithm 1 in step 6 as an inner-loop iteration. It is worth mentioning that certain optimization steps in Algorithm 2 can be assigned and executed in a distributed manner across the coordinated BSs. We address the distributed implementation of the ILA algorithm in Section III-B.

Algorithm 2 ILA Algorithm for the Multicell MIMO-BC with DPC

Initialize $\{\mathbf{Q}_{q_i}\}_{\forall q, \forall i}$, such that $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{Q}_{q_i}\} = P_q$;

repeat

  $\bar{\mathbf{Q}}_{q_i} \leftarrow \mathbf{Q}_{q_i}$;

  for $q = 1, 2, \ldots, Q$ **do**

    At the BS, update the matrix $\mathbf{A}_q$ as given in **(7)**;

    Update $\mathbf{Q}_{q_i}, i = 1, \ldots, K$ by executing Algorithm 1;

  end

until *convergence;*

B. Distributed Implementation of the Proposed ILA Algorithm

In order to implement the proposed ILA algorithm distributively, the following assumptions are taken in consideration:

- Assumption 1: Each BS, say BS-q, knows the channel matrices $\mathbf{H}_{qr_i}$'s to all the MSs in the network. This assumption allows the BS to control its induced ICI to other cells.
- Assumption 2: The coordinated BSs have reliable backhaul links to exchange control information among themselves.
- Assumption 3: The channels are in block-fading or vary sufficiently slow such that they can be considered fixed during the optimization process.

It is noted that the optimization (8) can be performed distributively at the corresponding BS with local information. Thus, it remains to show that the factors $\mathbf{A}_q$'s can also be computed in a distributed manner through a message exchange mechanism among the BSs. It is observed from (7) that in order to compute $\mathbf{A}_q$, BS-q has to possess the channels $\mathbf{H}_{qr_j}$'s to all the MSs in the other cells, as stated in Assumption 1. Although (7) indicates the dependence of $\mathbf{A}_q$ on the channels $\mathbf{H}_{rr_j}$ at other cells, the knowledge of $\mathbf{H}_{rr_j}$ is not necessarily required at BS-q. Define

$$\mathbf{B}_{r_j} = w_r \left[\mathbf{C}_{r_j}^{-1} - \left(\mathbf{C}_{r_j} + \mathbf{H}_{rr_j} \mathbf{Q}_{r_j} \mathbf{H}_{rr_j}^H \right)^{-1} \right] \Bigg|_{\mathbf{Q}_{q_i} = \bar{\mathbf{Q}}_{q_i}} \quad (24)$$

where $\mathbf{C}_{r_j} = \mathbf{R}_{r_j} + \sum_{k=1}^{j-1} \mathbf{H}_{rr_j} \mathbf{Q}_{r_k} \mathbf{H}_{rr_j}^H$. Then, it can be deduced from (7) that

$$\mathbf{A}_q = \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathbf{H}_{qr_j}^H \mathbf{B}_{r_j} \mathbf{H}_{qr_j}. \quad (25)$$

As stated in Remark 1, $\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_q \mathbf{Q}_{q_i}\}$ can be interpreted as the penalty term charged on the ICI generated by BS-q. The following decomposition

$$\sum_{i=1}^{K} \mathrm{Tr}\{\mathbf{A}_q \mathbf{Q}_{q_i}\} = \sum_{i=1}^{K} \sum_{r \neq q}^{Q} \sum_{j=1}^{K} \mathrm{Tr}\left\{ \mathbf{B}_{r_j} \mathbf{H}_{qr_j} \mathbf{Q}_{q_i} \mathbf{H}_{qr_j}^H \right\} \quad (26)$$

shows that $\mathbf{B}_{r_j}$ is essentially the price charged on the ICI generated to the particular MS-j at cell-r. Interestingly, the pricing matrix $\mathbf{B}_{r_j}$ can be computed using only local measurements at the corresponding MS-j at cell-r. Note that $\mathbf{C}_{r_j}$ is the total interference plus noise and $\mathbf{C}_{r_j} + \mathbf{H}_{rr_j} \mathbf{Q}_{r_j} \mathbf{H}_{rr_j}^H$ is the total signal, interference plus noise, pertaining to MS-j of cell-r, which can be locally measured by the MS at the instance $\mathbf{Q}_{q_i} = \bar{\mathbf{Q}}_{q_i}$. After computing the pricing matrix $\mathbf{B}_{r_j}$, the MS can feed back this parameter to its connected BS. These pricing matrices $\mathbf{B}_{r_j}$'s are then the messages to be exchanged among the BSs. Any given BS, say BS-q, after acquiring the pricing matrices $\mathbf{B}_{r_j}$, can compute $\mathbf{A}_q$ using (25).

Remark 4: It is proved in Theorem 3 that the optimization performed at a given BS always improves the network WSR, which leads to the convergence of the ILA algorithm with the Gauss-Seidel update. However, at each iteration, all the MSs are required to compute their pricing matrices $\mathbf{B}_{r_j}$'s and exchange them within the whole network before the precoding update. As a result, the Gauss-Seidel update may demand a lot of computation at the MSs and message exchanges in the network. To reduce the number of iterations, the proposed ILA algorithm can be also implemented by the Jacobi (simultaneous) iterative update. Specifically, at each iteration, all the BSs simultaneously update their covariance matrices. Through numerous simulations, we observe a much faster convergence rate by the Jacobi update than that by the Gauss-Seidel update. Although the convergence of the ILA algorithm with the Jacobi update is not analytically proved, we have not observed any instance in simulations that the Jacobi update fails to converge. However, there is a reason to believe that the Jacobi update may not always converge. That is the case when the Jacobi update jumps backward and forward between multiple peaks of the objective function. In this case, the Jacobi will not converge to any particular local maximum. The Gauss-Seidel update can avoid this behavior because it forces the optimization to converge monotonically once it gets closer to a local maximum.

IV. The Weighted Minimum Mean Squared Error Algorithm Solution Approach

A. The DPC Weighted Minimum Mean Squared Error (DPC-WMMSE) Algorithm

In Section III, we have examined a linear convex approximation technique to solve the nonconvex optimization problem(5) by successively improving the downlink covariance matrices at the BSs. In this section, we examine the second approach to solve this nonconvex problem by transforming it into a matrix-weighted sum-MSE minimization problem. Following the approach proposed in [4], [10], we develop a WMMSE-based algorithm for the multicell MIMO-BC with DPC. The newly developed algorithm will be referred to as the DPC-WMMSE algorithm, in order to avoid the confusion with the original WMMSE algorithm for the case of linear precoding in [4]. In the course of developing the DPC-WMMSE algorithm, we obtain new results on the designs of the transmit beamformers, the receive beamformers, and the weight matrices, compared to the WMMSE algorithm [4]. Unlike the ILA algorithm, which tries to optimize over the transmit covariances $\mathbf{Q}_{q_i}$'s, the optimization in the DPC-WMMSE algorithm is executed over the transmit beamforming matrix $\mathbf{V}_{q_i}$.

With $\mathbf{V}_{q_i}$'s being the variables, the optimization problem (5) can be restated as

$$\begin{aligned} &\underset{\{\mathbf{V}_{q_i}\}_{\forall i,\forall q}}{\text{maximize}} \sum_{q=1}^{Q} w_q \sum_{i=1}^{K} R_{q_i}^{\text{BC}} \\ &\text{subject to } \sum_{i=1}^{K} \text{Tr}\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\} \leq P_{q_i},\ \forall q \end{aligned} \tag{27}$$

where the achievable rate $R_{q_i}^{\text{BC}}$, given in (4), can be restated in (28) at the bottom of the page.

Since DPC with the encoding order from MS-K to MS-1 is applied at each BS, MS-i receives the signal from BS-q as if there was no intra-cell interference from user-$(i+1)$ to user-K. Thus, while treating the ICI as noise, the estimated signal for user-i in BS-q is given by

$$\hat{\mathbf{s}}_{q_i} = \mathbf{U}_{q_i}^H \left(\sum_{j=1}^{i} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{s}_{q_j} + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{s}_{r_j} + \mathbf{z}_{q_i} \right) \tag{29}$$

where $\mathbf{U}_{q_i}^H$ is the receive beamformer at MS-i. Let $\mathbf{E}_{q_i}$ be the expected MSE matrix of MS-i, which is defined as

$$\begin{aligned} \mathbf{E}_{q_i} &= \mathbb{E}\left[(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})(\hat{\mathbf{s}}_{q_i} - \mathbf{s}_{q_i})^H\right] \\ &= \left(\mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i}\right)\left(\mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i}\right)^H \\ &\quad + \sum_{j=1}^{i-1} \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H\mathbf{U}_{q_i} \\ &\quad + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{U}_{q_i}^H\mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H\mathbf{U}_{q_i} + \mathbf{U}_{q_i}^H\mathbf{Z}_{q_i}\mathbf{U}_{q_i}. \end{aligned} \tag{30}$$

Given the transmit beamforming matrices $\mathbf{V}_{q_i}$'s, the optimal receive beamformer $\mathbf{U}_{q_i}$ to minimize the MSE is the Wiener filter, i.e., MMSE receiver

$$\begin{aligned} \mathbf{U}_{q_i} &= \arg\min_{\mathbf{U}_{q_i}} \text{Tr}\{\mathbf{E}_{q_i}\} \\ &= \left(\sum_{j=1}^{i} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H \right. \\ &\quad \left. + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i} \right)^{-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_i}. \end{aligned} \tag{31}$$

As a result, the achieved minimum MSE matrix for user-i in cell-q is then given by

$$\begin{aligned} \mathbf{E}_{q_i}^{\text{MMSE}} &= \mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i} \\ &= \left[\mathbf{I} + \mathbf{V}_{q_i}^H\mathbf{H}_{qq_i}^H \left(\sum_{j=1}^{i-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H \right.\right. \\ &\quad \left.\left. + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H + \mathbf{Z}_{q_i} \right)^{-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_i} \right]^{-1}. \end{aligned} \tag{32}$$

Similar to the case of linear precoding, the relationship between the data rate given in (28) and the MMSE matrix given in(32) with DPC can be expressed as

$$R_{q_i}^{\text{BC}} = \log\left|\left(\mathbf{E}_{q_i}^{\text{MMSE}}\right)^{-1}\right|. \tag{33}$$

Due to this relationship, the equivalence between the WSR maximization problem in the multicell MIMO-BC and the matrix-weighted sum-MSE minimization can be established in the following theorem.

Theorem 4: The multicell MIMO-BC WSR maximization problem (27) is equivalent to the following matrix weighted sum-MSE minimization

$$\begin{aligned} &\underset{\mathbf{W}_{q_i},\mathbf{V}_{q_i},\mathbf{U}_{q_i}}{\text{minimize}} \sum_{q=1}^{Q} w_q \sum_{i=1}^{K} \left[\text{Tr}\left\{\mathbf{W}_{q_i}\mathbf{E}_{q_i}\right\} - \log|\mathbf{W}_{q_i}|\right] \\ &\text{subject to } \sum_{i=1}^{K} \text{Tr}\left\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\right\} \leq P_q, \forall q \end{aligned} \tag{34}$$

where $\mathbf{W}_{q_i} \succeq \mathbf{0}$ is the weight matrix for MS-i at cell-q. In particular, the globally optimal solutions $\{\mathbf{V}\}_{\forall q,\forall i}$ are identical for the two problems.

Proof: The proof for this theorem is similar to that in [4], [10] for the case of linear precoding. Thus, we omit the details for brevity. ■

Since solving problem(27) is equivalent to solving problem(34), we now proceed to numerically obtain the solution to the latter problem. Although the objective function in(34) is not jointly convex over the whole set of variables, it is convex in each set of the variables $\mathbf{U}_{q_i}, \mathbf{V}_{q_i}, \mathbf{W}_{q_i}$. Thus, it is possible to find a locally optimal solution of problem (34) by alternately optimizing one set of the variables while fixing the other two until convergence. First, with fixed transmit beamformers $\mathbf{V}_{q_i}$'s, the receive beamformers $\mathbf{U}_{q_i}$'s are given as in (31). Second, fixing the transmit and receive beamformers $\mathbf{V}_{q_i}$'s and $\mathbf{U}_{q_i}$'s, the weighted-matrices $\mathbf{W}_{q_i}$'s are updated in a closed form solution

$$\mathbf{W}_{q_i} = \mathbf{E}_{q_i}^{-1} = \mathbf{I} - \mathbf{U}_{q_i}^H\mathbf{H}_{qq_i}\mathbf{V}_{q_i}. \tag{35}$$

$$R_{q_i}^{\text{BC}} = \log \frac{\left|\mathbf{Z}_{q_i} + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H + \sum_{j=1}^{i} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H\right|}{\left|\mathbf{Z}_{q_i} + \sum_{r\neq q}^{Q}\sum_{j=1}^{K} \mathbf{H}_{rq_i}\mathbf{V}_{r_j}\mathbf{V}_{r_j}^H\mathbf{H}_{rq_i}^H + \sum_{j=1}^{i-1} \mathbf{H}_{qq_i}\mathbf{V}_{q_j}\mathbf{V}_{q_j}^H\mathbf{H}_{qq_i}^H\right|}. \tag{28}$$

Finally, by decomposing the objective function in (34), the transmit beamformers $\mathbf{V}_{q_i}$'s are updated by solving decoupled optimization problems across the BSs. For example, the optimization performed at BS-q is given in (36) at the bottom of the page. This optimization then can be carried out simultaneously and separately at BS-q. Since problem (36) is a convex quadratic program, its optimal solution can be in a closed-form solution as given in (37) at the bottom of the page. Herein, the optimal Lagrangian multiplier $\mu_q^\star$ associated with the power constraint can be easily found by the bisection method.

We summarize the DPC-WMMSE algorithm for the multicell MIMO-BC as in Algorithm 3. In this algorithm, in each outer-loop iteration, each set of variables (i.e., the receive beamforming matrices $\mathbf{U}_{q_i}$'s, the weight matrices $\mathbf{W}_{q_i}$'s, and the transmit beamforming matrices $\mathbf{V}_{q_i}$'s) can be updated simultaneously across the Q cells. Compared to the ILA algorithm, the DPC-WMMSE algorithm does not require any inner-loop iteration or the BC-MAC transformations because of the direct update of the variables $\mathbf{U}_{q_i}$, $\mathbf{W}_{q_i}$, and $\mathbf{V}_{q_i}$. Compared to the original WMMSE algorithm with linear precoding in [4], [10], the DPC-WMMSE algorithm requires some modifications to the transmit beamforming matrices $\mathbf{V}_{q_i}$'s and the receive beamforming matrices $\mathbf{U}_{q_i}$ to accommodate the DPC.

In the DPC-WMMSE algorithm, the iterative process is executed by alternatively optimizing over each set of variables in $\mathbf{U}_{q_i}$'s, $\mathbf{W}_{q_i}$'s, and $\mathbf{V}_{q_i}$'s. Since the constraint set of problem (34) is decoupled for each set of variables, the alternative optimization over $\mathbf{U}_{q_i}$'s, $\mathbf{W}_{q_i}$'s, and $\mathbf{V}_{q_i}$'s must decrease the objective function monotonically [29]. Moreover, the cost function in (34) is lower-bounded due to the power constraints on $\mathbf{V}_{q_i}$'s. Thus, the DPC-WMMSE must converge to at least a local minimum $(\mathbf{U}_{q_i}^\star, \mathbf{W}_{q_i}^\star, \mathbf{V}_{q_i}^\star)$ of the cost function (34). Note that the cost function of the original sum-rate maximization problem(27) does not necessarily improve after each iteration. However, given $(\mathbf{U}_{q_i}^\star, \mathbf{W}_{q_i}^\star, \mathbf{V}_{q_i}^\star)$ as a local minimizer of problem (34) obtained from the DPC-WMMSE algorithm, $\mathbf{V}_{q_i}^\star$ is also a local optimizer of the original problem (27). This observation was proved for the WMMSE algorithm in case of linear precoding [4]. The same proof can be applied here for the DPC-WMMSE algorithm with the DPC consideration.

As shown in [4], the WMMSE algorithm can be implemented in distributed manner. Under the assumptions given in Section III-B, distributed implementation to the DPC-WMMSE algorithm is also possible with same message exchange mechanism as in the WMMSE algorithm.

Algorithm 3 DPC-WMMSE Algorithm for the Multicell MIMO-BC with DPC

Initialize $\{\mathbf{V}_{q_i}\}_{\forall q, \forall i}$, such that $\sum_{j=1}^{K} \mathrm{Tr}\{\mathbf{V}_{q_i}\mathbf{V}_{q_i}^H\} = P_q$;
repeat
  Set $\bar{\mathbf{V}}_{q_i} \leftarrow \mathbf{V}_{q_i}, \ \forall q, \forall i.$;
  Simultaneously update across Q cells;
  for $q = 1, \ldots, Q$ **do**
    At the K MSs, update the receive beamformers and weight matrices;
    for $i = 1, \ldots, K$ **do**
      Update $\mathbf{U}_{q_i}$ as in **(31)**;
      Update $\mathbf{W}_{q_i}$ as in **(35)**;
    end
    At the BS, update the transmit matrices $\mathbf{V}_{q_i}, \forall i$ as in **(37)**;
  end
until *convergence;*

V. Simulation Results

This section presents some numerical evaluations on the achievable downlink sum-rate of a multicell system with different levels of coordination and on the convergence behavior of the proposed algorithms. We compare the sum-rate between 3 schemes: (i) the *interference coordination* mode with DPC obtained from the ILA and DPC-WMMSE algorithms (with equal weights $w_1 = \cdots = w_Q$), (ii) the *competitive* mode where each BS selfishly maximizes the sum-rate for its connected MSs using DPC, and (iii) the *interference coordination* mode with linear precoding obtained from the WMMSE algorithm in [4].

$$\begin{aligned}
\underset{\mathbf{V}_{q_1},\ldots,\mathbf{V}_{q_K}}{\text{minimize}} \ & \sum_{i=1}^{K} \Bigg[w_q \mathrm{Tr}\left\{ \mathbf{W}_{q_i} \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}\right) \left(\mathbf{I} - \mathbf{U}_{q_i}^H \mathbf{H}_{qq_i} \mathbf{V}_{q_i}\right)^H \right\} + w_q \sum_{j>i}^{K} \mathrm{Tr}\left\{ \mathbf{W}_{q_j} \mathbf{U}_{q_j}^H \mathbf{H}_{qq_j} \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \mathbf{H}_{qq_j}^H \mathbf{U}_{q_j} \right\} \\
& + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} w_r \mathrm{Tr}\left\{ \mathbf{W}_{r_j} \mathbf{U}_{r_j}^H \mathbf{H}_{qr_j} \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \mathbf{H}_{qr_j}^H \mathbf{U}_{r_j} \right\} \Bigg] \\
\text{subject to} \ & \sum_{i=1}^{K} \mathrm{Tr}\left\{ \mathbf{V}_{q_i} \mathbf{V}_{q_i}^H \right\} \leq P_q.
\end{aligned} \tag{36}$$

$$\mathbf{V}_{q_i} = w_q \left(\sum_{j=i}^{K} w_q \mathbf{H}_{qq_j}^H \mathbf{U}_{q_j} \mathbf{W}_{q_j} \mathbf{U}_{q_j}^H \mathbf{H}_{qq_j} + \sum_{r \neq q}^{Q} \sum_{j=1}^{K} w_r \mathbf{H}_{qr_j}^H \mathbf{U}_{r_j} \mathbf{W}_{r_j} \mathbf{U}_{r_j}^H \mathbf{H}_{qr_j} + \mu_q^\star \mathbf{I} \right)^{-1} \mathbf{H}_{qq_i}^H \mathbf{U}_{q_i} \mathbf{W}_{q_i}. \tag{37}$$

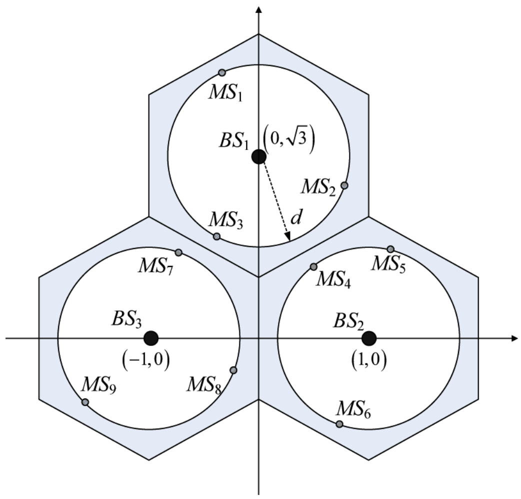

Fig. 1. A multicell system with 3 cells and 3 MSs per cell. Each MS is randomly located at a distance d from its connected BS.

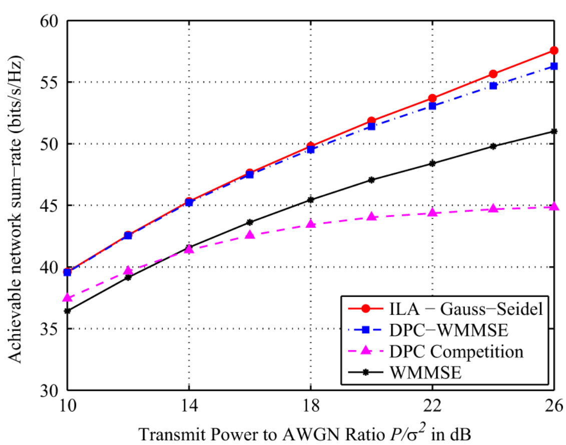

Fig. 2. Network sum-rate versus the transmit power to AWGN ratio for $d = 0.7$. Each algorithm is run until full convergence with the number of iterations capped at 500.

Unless stated otherwise, the ILA scheme is implemented with the Gauss-Seidel update due to its guaranteed convergence. We consider a generic 3-cell system with 3 MSs per cell, as illustrated in Fig. 1. The numbers of antennas at each BS and each MS are assumed to be 4 and 2, respectively. The BSs are located at a normalized distance of 2 and the MSs are randomly located on a circle at distance d from its connected BS. The channels from a BS to a MS are generated from i.i.d. Gaussian random variables using the path-loss model with the path-loss exponent of 3 and the reference distance of 1 corresponding to MSs at the cell edge. The variance of the small-scale fading (shadowing) is set at 0 dB. The additive Gaussian noise at each MS is assumed to be white with the covariance matrix $\mathbf{Z}_{q_i} = \sigma^2 \mathbf{I}$ and σ^2 is set at 0.01. The transmit power P_q at each BS is constrained at 1 W such that the average signal to AWGN ratio at the cell edge is given by $P_q/\sigma^2 = 20$ dB, unless stated otherwise.

Fig. 2 illustrates the network sum-rate versus the transmit power to AWGN ratio P/σ^2 (with same power P at each BS) for $d = 0.7$. As P/σ^2 is varied, 10,000 channel realizations at each value of P/σ^2 are used to achieve the average network sum-rate. Note that the plotted results in the figure are obtained from the full convergence of each algorithm. However, we do set a upper limit on the number of iterations for each algorithm at 500 for practical implementation purposes. Herein, we define an iteration as an instance of message exchange between the BSs. With the ILA algorithm, due to the similar sum-rates obtained at full convergence by the Gauss-Seidel and Jacobi updates, only the results from the former type of update are displayed. It is observed from Fig. 2 that increasing the transmit power at each BS shall increase the network sum-rate for all 3 schemes. However, at the high P/σ^2 region the sum-rate obtained from the DPC competition scheme becomes saturated. This is due to the reason that the competitive design does not attempt to control ICI, and thus increases the ICI relatively with the transmit power. In this case, it is more appealing to implement the coordinated designs obtained by the ILA, DPC-WMMSE, or WMMSE algorithm. It can also be seen from the figure that the non-linear precoding design with DPC (by the ILA and DPC-WMMSE algorithms) can extract extra performance from the multicell network over the linear precoding design (by the WMMSE algorithm). Interestingly, the ILA algorithm outperforms the DPC-WMMSE algorithm at high P/σ^2, corresponding to the high SINR region. This appears to be due to the much slower convergence rate of the DPC-WMMSE algorithm at high SINR region. As noted in a recent work [30], at high SINR, the WMMSE-based solution approach does not sufficiently suppress small interference sources. Thus, this approach only provides small sum-rate improvement after each iteration, compared to the ILA approach.

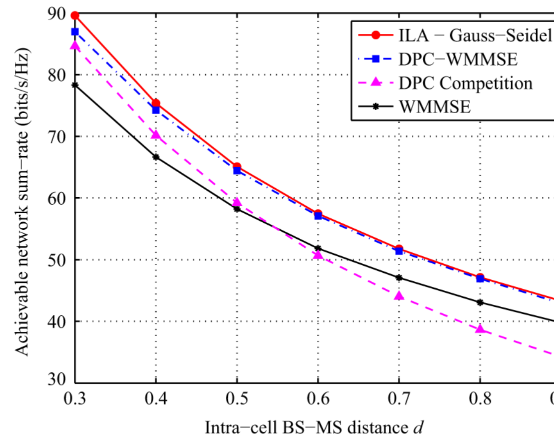

Fig. 3. Network sum-rate versus the intra-cell BS-MS distance d. Each algorithm is run until full convergence with the number of iterations capped at 500.

Fig. 3 illustrates the total network sum-rate versus the intracell MS-BS distance d obtained from the 3 schemes with $P/\sigma^2 = 20$ dB. All other parameters in the simulation setup this figure are the same as those to obtain the results in Fig. 2. Note that the effect of ICI is more apparent with increasing d due to the decreasing gain of intra-cell channels and the increasing gain of inter-cell channels. Thus, the network sum-rate is reduced with increasing d, as observed in the figure for all the schemes. Out of the 3 schemes, the coordination mode with DPC always show a superior sum-rate

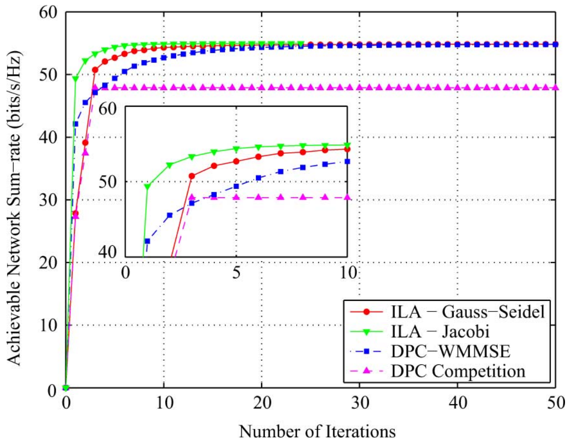

Fig. 4. Convergence of the proposed ILA and DPC-WMMSE algorithm to maximize the network sum-rate with interference coordination.

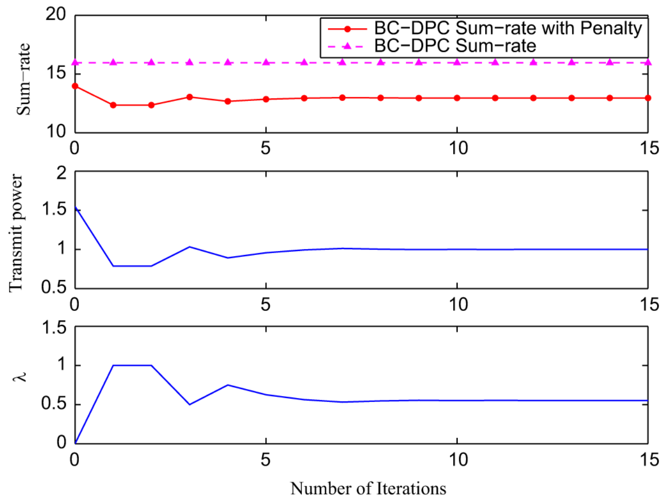

Fig. 5. The convergence of Algorithm 1 to solve Problem (8).

performance, since it takes advantages of both the nonlinear precoding in DPC and the interference coordination. At the low-ICI region, i.e., high SINR, the coordination mode with DPC with the ILA and DPC-WMMSE algorithms significantly outperforms the WMMSE algorithm due to the use of DPC over linear precoding. On the other hand, at the high-ICI region, by implementing the IC with the proposed algorithms, one can significantly improve the network sum-rate over the competitive design. In the coordination mode with DPC, it can be observed that the sum-rates obtained by the ILA and DPC-WMMSE algorithms closely match over whole range of d. However, at low d, the ILA algorithm outperforms the DPC-WMMSE algorithm. This behavior is probably due to the reason that each BS focuses more on maximizing its own sum-rate than limiting the ICI at the low-ICI region. In this case, Algorithm 1 utilized in the ILA algorithm can obtain the optimal solution to the per-cell sum-rate maximization problems, whereas the DPC-WMMSE obtains the sum-rate from the transformed WMMSE problem. Similar to the observation in Fig. 2, the slower convergence of the WMMSE-based algorithm at high SINR [30] is another reason for this performance gap.

It is to be noted that the ILA, DPC-WMMSE, WMMSE algorithms, and the competitive design may require several iterations to fully converge. For a specific channel realization with $d = 0.7$, we illustrate the convergence behavior of the proposed ILA and DPC-WMMSE algorithms in Fig. 4. After each iteration, the network sum-rates obtained from the algorithms are plotted. In general, the ILA algorithm converges faster than the DPC-WMMSE algorithm. As observed from Fig. 4, once a BS updates its covariance matrices, the overall network sum-rates are always improved by the ILA algorithm with the Gauss-Seidel update and the DPC-WMMSE algorithm. This behavior agrees with our analysis on the convergence of the algorithms. Interestingly, the ILA algorithm also experiences the monotonic convergence with the Jacobi update. Both the ILA and DPC-WMMSE algorithms eventually converge to a network sum-rate that is superior than the one obtained by the *competitive* design.

With the same sample channel realization as in Figs. 4, 5 displays the convergence of Algorithm 1 in maximizing the BC sum-rate with a penalty term, i.e., problem(8). After each update of the dual variable λ, the evolutions of the sum-rate and the transmit power at BS-1 are plotted in the figure. As can be observed from the figure, the algorithm converges very fast in a few iterations. Due to the penalty term, BS-1 needs to *balance* its achievable sum-rate with the ICI induced to cell-2 and -3. Thus, its sum-rate is undoubtedly reduced, compared to the one obtained in a conventional BC without the penalty term. Nonetheless, under the IC mode, each BS adopts a less selfish strategy to improve the overall network sum-rate, as shown in Fig. 4.

VI. Conclusion

This work examined the problem of network WSR maximization in the multicell MIMO-BC with DPC. Under the *interference coordination* mode, the network sum-rate maximization problem was shown to be nonconvex. This work then considered two low-complexity solution approaches, namely ILA and DPC-WMMSE to search for locally optimal solutions. In the first approach, successive convex approximation technique was utilized to transform the problem into multiple per-cell problems, which are then optimized distributively at each BS. In particular, each BS attempted to maximize the BC sum-rate to its connected BS with a penalty term on its induced ICI to other cells. A distributed and fast converging algorithm was then proposed to efficiently find a locally optimal solution to the network WSR maximization problem. In the second approach, by establishing the equivalence between the maximization of the WSR and the minimization of the weighted MSE, the WSR problem was locally optimized by alternatively optimizing over the weight matrices and MMSE decoders at the MSs and the MMSE precoders at the BSs. As the proposed algorithms allow the multicell to take advantage of both DPC and coordinating the ICI, they show a significant improvement in the network sum-rate compared to competitive design and the linear precoding.

APPENDIX
PROOF TO THEOREM 3

Suppose that $\mathbf{Q}_q = \bar{\mathbf{Q}}_q, \forall q$ was obtained from the previous iteration, and $\mathbf{Q}_q^\star = \left\{\mathbf{Q}_{q_i}^\star\right\}_{i=1}^K, \forall q$ is the optimal solution after the current iteration. Note that $f_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is a convex function with respect to $\mathbf{Q}_q \in \mathcal{S}_q \triangleq \{\{\mathbf{Q}_{q_i}\}|\mathbf{Q}_{q_i} \succeq \mathbf{0}, \sum_{i=1}^K \mathrm{Tr}\{\mathbf{Q}_{q_i}\} \leq P_q\}$[7], [9]. Thus, the first-order condition of the convex function f_q[28] dictates that

$$f_q(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \geq f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}_q(\mathbf{Q}_{q_i}^\star - \bar{\mathbf{Q}}_{q_i})\} \quad (38)$$

with $\mathbf{A}_q$ being defined in (7) at $\bar{\mathbf{Q}}_{q_i}$.

After one Gauss-Seidel iteration, the network WSR is updated such that

$$\begin{aligned}
&\sum_{q=1}^Q w_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \\
&\quad = w_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) + f_q(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \\
&\quad \geq w_q R_q^{\mathrm{BC}}(\mathbf{Q}_q^\star, \bar{\mathbf{Q}}_{-q}) \\
&\qquad + f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}_q(\mathbf{Q}_{q_i}^\star - \bar{\mathbf{Q}}_{q_i})\} \\
&\quad \geq w_q R_q^{\mathrm{BC}}(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) \\
&\qquad + f_q(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q}) - \sum_{i=1}^K \mathrm{Tr}\{\mathbf{A}_q(\bar{\mathbf{Q}}_{q_i} - \bar{\mathbf{Q}}_{q_i})\} \\
&\quad = \sum_{q=1}^Q w_q R_q^{\mathrm{BC}}(\bar{\mathbf{Q}}_q, \bar{\mathbf{Q}}_{-q})
\end{aligned}$$

where the first inequality is due to the one in (38), and the second inequality is due to $\mathbf{Q}_q^\star$ being the optimal solution of problem (8).

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